The Equation of State Problem

Recent advances in microscopic theories

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on behalf of A. Polls (University of Barcelona)

Compstar Workshop
Catania, 10 May 2011
1 Motivation

2 Nuclear many-body problem

3 Review of many-body techniques

4 Three-body sector

5 Exotic phases of nuclear matter

6 Conclusions
Isotope science
First principles in nuclear physics

~3200 isotopes

Uncharted territory to be explored at RIB facilities
RIKEN, FRIB, FAIR
Isotope science
First principles in nuclear physics

Segré Chart

~3200 isotopes
Isotope science
First principles in nuclear physics

Segré Chart

~3200 isotopes
Isotope science
First principles in nuclear physics

~3200 isotopes

Neutron stars are also nuclear labs!

Segré Chart

- $\tau_{1/2} < 0.1$ s
- $0.1$ s $< \tau_{1/2} < 3$ s
- $3$ s $< \tau_{1/2} < 2$ mins
- $2$ mins $< \tau_{1/2} < 1$ hour
- $1$ hour $< \tau_{1/2} < 1$ day
- $1$ day $< \tau_{1/2} < 1$ year
- $1$ year $< \tau_{1/2} < 1$ Gy
- $\tau_{1/2} > 1$ Gy

Neutron stars are also nuclear labs!
Isotope science
First principles in nuclear physics

~3200 isotopes

Thoennessen & Sherrill, Nature (Comment) 473, 25 (2011)
What do we know about the EoS?
Taylor expansion near symmetric matter

• EoS provides a characterization of bulk properties:

\[ p(\varepsilon) = ? \quad \varepsilon = \rho \frac{E}{A} \quad p(\rho) = \rho^2 \frac{\partial E/A}{\partial \rho} \quad \frac{E}{A}(\rho, \beta) = ? \]

• Taylor expansion
  • Minimum at saturation density, \( \rho_0 \)
  • Minimum in asymmetry: \( \beta = \frac{N-Z}{N+Z} = 0 \)
  • Isospin symmetry \( \Rightarrow \) even powers of \( \beta \)
  • Give the coefficients a name!
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\[
\frac{E}{A}(\rho, \beta) = \frac{E}{A}(\rho_0, \beta) \\
+ 3\rho_0 \left. \frac{\partial E/A}{\partial \rho} \right|_{\rho_0} \left( \frac{\rho - \rho_0}{3\rho_0} \right) \\
+ \frac{9\rho_0^2}{2!} \left. \frac{\partial^2 E/A}{\partial \rho^2} \right|_{\rho_0} \left( \frac{\rho - \rho_0}{3\rho_0} \right)^2 \\
+ \mathcal{O}(3)
\]
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  - Give the coefficients a name!

\[
\frac{E}{A}(\rho, \beta) = \frac{E}{A}(\rho_0, 0) + \frac{1}{2!} \left. \frac{\partial^2 E/A}{\partial \beta^2} \right|_{\rho_0, \beta=0} \beta^2 \\
+ \frac{3\rho_0}{2!} \left. \frac{\partial^3 E/A}{\partial \beta^2 \partial \rho} \right|_{\rho_0, \beta=0} \beta^2 \left( \frac{\rho - \rho_0}{3\rho_0} \right) \\
+ \frac{9\rho_0^2}{2!} \left\{ \left. \frac{\partial^2 E/A}{\partial \rho^2} \right|_{\rho_0, \beta=0} + \frac{1}{2!} \left. \frac{\partial^4 E/A}{\partial \rho^2 \beta^2} \right|_{\rho_0, \beta=0} \beta^2 \right\} \left( \frac{\rho - \rho_0}{3\rho_0} \right)^2 \\
+ O(3, 2)
\]
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Taylor expansion near symmetric matter

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\[
\frac{E}{A}(\rho, \beta) = E_0 + E_{\text{sym}} \beta^2 + L \beta^2 \left( \frac{\rho - \rho_0}{3\rho_0} \right) \\
+ \frac{1}{2!} \left\{ K_0 + K_{\text{sym}} \beta^2 \right\} \left( \frac{\rho - \rho_0}{3\rho_0} \right)^2 \\
+ \mathcal{O}(3, 2)
\]
EoS from basic nuclear properties
An incomplete list

\[
\frac{E}{A}(\rho, \beta) = E_0 + E_{sym} \beta^2 + L \beta^2 \left( \frac{\rho - \rho_0}{3\rho_0} \right) + \frac{1}{2!} \left\{ K_0 + K_{sym} \beta^2 \right\} \left( \frac{\rho - \rho_0}{3\rho_0} \right)^2
\]

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Experimental probes</th>
<th>Value</th>
<th>Ref.</th>
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<tbody>
<tr>
<td>(\rho_0)</td>
<td>((e, e')) elastic scattering</td>
<td>0.16 fm(^{-3})</td>
<td>[1]</td>
</tr>
<tr>
<td>(E_0)</td>
<td>(\frac{E}{A}) bulk systematics</td>
<td>-16 MeV</td>
<td>[1]</td>
</tr>
<tr>
<td>(K_0)</td>
<td>GMR energy in (Z \sim N)</td>
<td>240 (\pm) 20 MeV</td>
<td>[2]</td>
</tr>
<tr>
<td>(E_{sym})</td>
<td>(\frac{E}{A}) bulk systematics + ID</td>
<td>32 (\pm) 2 MeV</td>
<td>[3]</td>
</tr>
<tr>
<td>(L)</td>
<td>ID, IVMR energies, (\delta R)</td>
<td>88 (\pm) 25 MeV</td>
<td>[3]</td>
</tr>
<tr>
<td>(K_{sym})</td>
<td>?</td>
<td>?</td>
<td></td>
</tr>
</tbody>
</table>

EoS from astrophysical observations
Results from isolated NS

Özel, Baym & Güver, arxiv:1002.3153

• Mass-Radius relation from bursts
• Bayesian data analysis to get model-independent EoS
  1. 3 type-I X-ray bursts
  2. 3 transient low mass X-ray binaries
  3. 1 isolated cooling NS, RX J1856-3754
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EoS from astrophysical observations
Results from isolated NS

Good nuclear parameters!

\[
\frac{E}{A} = E_0 + \frac{K_0}{18} (u - 1)^2 + \left[ S_k u^{2/3} + S_p u \gamma \right] \beta^2
\]

\[K_0 = 180 - 280 \text{ MeV}\]
\[u = \frac{\rho}{\rho_0}\]
\[E_{sym} = S_k + S_p = 28 - 38 \text{ MeV}\]
\[\gamma = 0.2 - 1.2\]


- Mass-Radius relation from bursts
- Bayesian data analysis to get model-independent EoS
  1. 3 type-I X-ray bursts
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Outline

1 Motivation

2 Nuclear many-body problem

3 Review of many-body techniques

4 Three-body sector

5 Exotic phases of nuclear matter

6 Conclusions
A philosophical matter

The EoS is unknown \textit{a priori}

**Ab-initio**

- Microscopic NN interaction
- Use many-body theory
- Build the EoS
- **Safest** way to objective

**Phenomenological**

- Fit \textit{effective} interaction
- Rely on \textit{mean-field} or DFT
- Extrapolate the EoS
- **Fastest** way to objective
A philosophical matter

The EoS is unknown a priori

Ab-initio

Microscopic NN interaction

Use many-body theory

Build the EoS

Safest way to objective
Ab initio description of nuclear systems

**Framework**
- Non-relativistic & quantum
- Relativistic, classical?

**Degrees of freedom**
- Nucleons
- Hyperons, quarks, mesons?

**Hamiltonian**
- Phase-shift equivalent NN force
- NNN force?

**Many-body technique**
- BHF, FHNC, AFDMC, SCGF

**Empirical properties**
- Saturation density
- Binding energy
- Compressibility
Complications
The hard life of nuclear many-body physicists

\[ H = -\frac{\hbar^2}{2m} \sum_{i=1}^{A} \nabla^2_i + \sum_{i<j} V_{ij} \]

Different NN potentials

- NN interaction is not uniquely defined...
- Short-range core needs many-body treatment
- Complicated channel structure \( \Rightarrow \) tensor term coupling
- Very different techniques ...
Complications
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\[ H = -\frac{\hbar^2}{2m} \sum_{i=1}^{A} \nabla_i^2 + \sum_{i<j} V_{ij} \]

Different NN potentials

Lattice QCD potential

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GFMC pair distribution function

\[ ^1S_0 \text{ Argonne v18} \]

\[ V_{nn} \text{ [MeV]} \]


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\[ H = -\frac{\hbar^2}{2m} \sum_{i=1}^{A} \nabla^2_i + \sum_{i<j} V_{ij} \]

Deuteron wave-function: S-D mixing

\[ \delta E = \sum_{i,j<F \atop m,n>F} \frac{\langle ij J (LS) | V | mn J (L' S) \rangle \langle mn J (L' S) | V | ij J (LS) \rangle}{E_i + E_j - E_m - E_n - i\eta} \]

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Variational techniques

CBF, FHNC

- Trial many-body wave-function

\[ |\Psi\rangle = F|\Phi\rangle \quad F = \mathcal{A}\left\{ \prod_{i>j} \hat{F}_{ij} \right\} \]

\[ \hat{F}_{ij} = \sum_{p=1}^{6} f^p(r_{ij}) \hat{O}_{ij}^p \quad h(r) = f^c(r)^2 - 1 \]

- \( g(x_1, x_2) \) from Hypernetted Chain expansion
- Massive resummation via integral equations
- Operatorial structure of correlations
- Minimization of the total energy

\[ \min \left\{ \frac{\langle \Psi | \hat{H} | \Psi \rangle}{\langle \Psi | \Psi \rangle} \right\} \geq E \]

Advantages

1. Access to several properties
2. Sums short- & long-range correlations
3. Applied to closed-shell nuclei

Limitations

1. Only local potentials
2. Difficulties with operatorial structure (SOC)
3. Treatment of elementary diagrams
4. Difficult to handle for asymmetric matter

Nodal diagrams in FHNC
Variational techniques
CBF, FHNC

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Recent example: FHNC EoS

**EoS of symmetric nuclear matter**

![Graph showing EoS of symmetric nuclear matter](image)

- **AFDMC fit**
- **AFDMC**
- **FHNC/SOC**
- **FHNC/SOC + elem.**
- **BHF**


**Classic references**

- Pandharipande & Fantoni, PRC **37**, 1697 (1988)

**Latest advances**

- Lovato *et al.*, arxiv:1011.3784
Monte-Carlo techniques
VMC, GFMC, AFDMC

- **VMC**: energy minimization
  \[
  \min \left\{ \frac{\langle \Psi | \hat{H} | \Psi \rangle}{\langle \Psi | \Psi \rangle} \right\} \geq E
  \]

- **DMC**: Schrödinger equation in imaginary time
  \[
i \frac{\partial}{\partial t} |\Psi\rangle = \hat{H} |\Psi\rangle \Rightarrow -\frac{\partial}{\partial \tau} |\Psi\rangle = \hat{H} |\Psi\rangle
  \]

- **GFMC**: trial wave function
  \[
  |\psi\rangle = \sum_{\alpha=0}^{N_\alpha} c_\alpha |\Psi_\alpha\rangle \Rightarrow |\Psi_0\rangle = \lim_{\tau \to \infty} e^{-(H-E_0)\tau} |\psi\rangle
  \]

- **AFDMC**: Hubbard-Stratonovitch for spin-isospin operators

### Advantages
1. Symmetric & asymmetric matter
2. Applied to nuclei
3. Virtually exact

### Limitations
1. Only local potentials
2. Fermion sign limitation
3. Finite-size effects?
Recent example: AFDMC EoS

EoS of neutron matter

Gandolfi, Carlson & Reddy, arxiv:1101.1921

Classic references
Pudliner, Pandharipande et al., PRL 74, 4396 (1995)

Latest advances
Carlson et al., PRC 68, 025802 (2003)
Gandolfi et al., MNRAS 404, 35 (2010)
Gezerlis & Carlson, PRC 81, 025803 (2010)
Wlazlowski & Majierski, PRC 83, 012801 (2011)
Diagrammatic techniques: BHF

- Based on Bethe-Goldstone perturbation theory
- Infinite resummation of two-hole line diagrams
- pp Pauli blocked in-medium interaction (G-matrix)

\[ G(\omega) = V + V \frac{Q}{\omega - \epsilon - \epsilon' + i\eta} G(\omega) \]

\[ U(k) = \sum_{|\vec{k}'| < k_F} \langle \vec{k} \vec{k}' | G(\omega = \epsilon(k) + \epsilon(k')) | \vec{k} \vec{k}' \rangle_A \]

\[ \epsilon(k) = \frac{\hbar^2 k^2}{2m_\tau} + \text{Re}[U(k)] \]

- Expansion for the energy

\[ \frac{E}{A}(\rho, \beta) = \frac{1}{A} \sum_\tau \sum_{|\vec{k}| < k_{F\tau}} \left( \frac{\hbar^2 k^2}{2m_\tau} + \frac{1}{2} \text{Re}[U_\tau(\vec{k})] \right) \]

Advantages
1. Symmetric, asymmetric & exotic matter
2. Also non-local potentials
3. Systematic expansion

Limitations
1. Missing diagrams
2. Thermodynamical inconsistency
Recent example: BHF EoS


Classic references

Brueckner et al., Phys. Rev. 95, 217 (1954)
Brandow, Phys. Rev. 152, 863 (1966)
Day, Rev. Mod. Phys. 39, 719 (1967)

Latest advances

Baldo & Burgio, arxiv:1102.1364
Diagrammatic techniques: SCGF

- Feynman diagrams for many-body propagators
- Truncate hierarchy & get pp+hh Pauli blocking
- Impose self-consistency at all levels
- Characterize medium with spectral function

\[
A^<(k, \omega) = \sum_{n,m} \frac{e^{-\beta(E_n - \mu A)}}{Z} \left| \langle m | a_k | n \rangle \right|^2 \delta[\omega - (E_n^A - E_m^A - 1)]
\]

- Energy from GMK sum rule

\[
E = \sum_k \int \frac{d\omega}{2\pi} \frac{1}{2} \left[ \frac{k^2}{2m} + \omega \right] A(k, \omega) f(\omega)
\]

Advantages
1. Symmetric, asymmetric & exotic matter
2. Also non-local potentials
3. Thermodynamically consistent

Limitations
1. Missing diagrams
2. T=0 instability, meaningful ground state?
Recent example: SCGF EoS

**EoS of hot neutron matter**

**CDBONN**

**Argonne V18**

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>0 0.01 0.02 0.03</td>
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</tr>
<tr>
<td>10 20 30 40</td>
<td></td>
<td></td>
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</table>

**SCGF**

**BHF**

**FP**

**Virial**


**Classic references**


Dickhoff & Van Neck, *Many-Body theory exposed!*

**Latest advances**


Tempering the interaction...
Renormalization group inspired methods

• Use RG arguments to rebuild the NN interaction

\[ T = V + VGT \Rightarrow \frac{d}{d\Lambda} T(k, k'; \Lambda) = 0 \]
\[ \frac{d}{d\Lambda} V^\Lambda(k, k') = \frac{2}{\pi} \frac{V^\Lambda(k', \Lambda)T(\Lambda, k')}{1 - (k/\Lambda)^2} \]

• Universal forces up to scale, \( \Lambda \)
• Softer potentials become perturbative
• Need of three-body forces!
• New technique: similarity renormalization group

\[ \frac{dH_s}{ds} = [[T, H_s], H_s] \]

Advantages
1. Symmetric, asymmetric & exotic matter
2. Also non-local potentials
3. Applied to nuclei

Limitations
1. Many-body forces
2. Dressing of operators
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Recent example: EoS of neutron matter

3BF needed for saturation

\[ V_{\text{low } k} \text{ NN from N}^3\text{LO (500 MeV)} \]

3NF fit to \( E_3H \) and \( r_{4\text{He}} \) \( \Lambda_{\text{3NF}} = 2.0 \text{ fm}^{-1} \)

NN + 3N

\[ \chi\text{PT allowed } M \text{ vs. } R \]

\( \chi \text{PT} \) allowed \( M \) vs. \( R \)

Hebeler et al., PRC 83, 031301 (2011)

Hebeler et al., PRL 105, 161102 (2010)

Classic references

Bogner et al., Prog. Part. Nucl. Phys. 65, 94 (2010)

Latest advances

A benchmark calculation
Local interactions with operatorial structure

How can we **gauge the quality of different** many-body techniques?

Benchmark calculations with:
1. Non-relativistic quantum mechanics
2. Same degrees of freedom: nucleons
3. Same NN interactions

Argonne refitted NN potentials


\[ V_{ij}^M (r) = \sum_{p=1}^{M} v_p (r) \hat{O}_{ij}^p \]

\[ O_{ij}^{p=1, \ldots, 8} = \{ \mathbb{I}, \sigma_i \cdot \sigma_j, S_{ij}, L \cdot S \} \otimes \{ \mathbb{I}, \tau_i \cdot \tau_j \} \]
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Argonne v4’

\[ V_{ij}^4 = v_1(r) + v_2(r) \sigma_i \cdot \sigma_j + v_3(r) \tau_i \cdot \tau_j + v_4(r) (\sigma_i \cdot \sigma_j \otimes \tau_i \cdot \tau_j) \rightarrow \text{Central} \]
A benchmark calculation
Local interactions with operatorial structure

How can we gauge the quality of different many-body techniques?

Benchmark calculations with:

1. Non-relativistic quantum mechanics
2. Same degrees of freedom: nucleons
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Argonne refitted NN potentials


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Argonne v6'

\[ V_{ij}^6 = v_1 (r) + v_2 (r) \sigma_i \cdot \sigma_j + v_3 (r) \tau_i \cdot \tau_j + v_4 (r) (\sigma_i \cdot \sigma_j \otimes \tau_i \cdot \tau_j) \rightarrow \text{Central} \]

\[ + v_5 (r) S_{ij} + v_6 (r) (S_{ij} \otimes \tau_i \cdot \tau_j) \rightarrow \text{Tensor} \]
A benchmark calculation
Local interactions with operatorial structure

How can we gauge the quality of different many-body techniques?

Benchmark calculations with:
1. Non-relativistic quantum mechanics
2. Same degrees of freedom: nucleons
3. Same NN interactions

Argonne refitted NN potentials

\[ V_{ij}^M (r) = \sum_{p=1}^{M} v_p(r) \hat{O}_{ij}^p \]

\[ O_{ij}^{p=1,\ldots,8} = \{1, \sigma_i \cdot \sigma_j, S_{ij}, L \cdot S\} \otimes \{1, \tau_i \cdot \tau_j\} \]

Argonne v8’

\[ V_{ij}^8 = v_1(r) + v_2(r) \sigma_i \cdot \sigma_j + v_3(r) \tau_i \cdot \tau_j + v_4(r)(\sigma_i \cdot \sigma_j \otimes \tau_i \cdot \tau_j) \rightarrow \text{Central} \]
\[ + v_5(r) S_{ij} + v_6(r)(S_{ij} \otimes \tau_i \cdot \tau_j) \rightarrow \text{Tensor} \]
\[ + v_7(r) L \cdot S + v_8(r)(L \cdot S \otimes \tau_i \cdot \tau_j) \rightarrow \text{Spin-orbit} \]
The Compstar Equation of State

Preliminary comparisons

T=0 extrapolation of SCGF EoS

SCGF, neutron matter

SCGF, symmetric matter

• NN interaction dependence
• Many-body dependence of EoS
• First comparison of non-relativistic approaches
The Compstar Equation of State
Preliminary comparisons

- NN interaction dependence
- Many-body dependence of EoS
- First comparison of non-relativistic approaches
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• Many-body dependence of EoS
• First comparison of non-relativistic approaches

Klähn et al., PRC 74 035802 (2006)
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Why 3 body forces?

\[ H = -\frac{\hbar^2}{2m} \sum_{i=1}^{A} \nabla^2_i + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} \]


- Properties of light nuclei
- Saturation of nuclear matter
- Origin of 3BF
Why 3 body forces?

\[ H = -\frac{\hbar^2}{2m} \sum_{i=1}^{A} \nabla_i^2 + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} \]

SNM saturation BHF

Coester line with BHF

- Properties of light nuclei
- Saturation of nuclear matter
- Origin of 3BF

Why 3 body forces?

\[
H = -\frac{\hbar^2}{2m} \sum_{i=1}^{A} \nabla_i^2 + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk}
\]

3NF in \(\chi PT\)

<table>
<thead>
<tr>
<th>LO (\sigma (q^2 / \Lambda^2))</th>
<th>NN</th>
<th>3N</th>
<th>4N</th>
</tr>
</thead>
<tbody>
<tr>
<td>NLO (\sigma (q^2 / \Lambda^2))</td>
<td>X</td>
<td>X</td>
<td>-</td>
</tr>
<tr>
<td>N^2LO (\sigma (q^2 / \Lambda^2))</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>N^3LO (\sigma (q^2 / \Lambda^2))</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

Phenomenological 3NF

(a) \(N^*\) \(\pi \rho\) \(\sigma \omega\)

(b) \(\pi \rho\) \(\omega\) \(\pi \rho\) \(\omega\)

(c) \(\pi \rho\) \(\sigma \rho\) 

(d) \(\pi \rho\) \(\sigma \omega\) 


- Properties of light nuclei
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### 3BFs in many-body approaches

#### Direct

<table>
<thead>
<tr>
<th>Method</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>BHF</td>
<td>?</td>
</tr>
<tr>
<td>SCGF</td>
<td>?</td>
</tr>
</tbody>
</table>

#### Average over 3rd particle

<table>
<thead>
<tr>
<th>Method</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>FHNC</td>
<td>Lovato et al., arxiv:1011.3784</td>
</tr>
<tr>
<td>Monte Carlo</td>
<td>Gandolfi et al., MNRAS 404, 35 (2010)</td>
</tr>
</tbody>
</table>
Recent examples

**BHF**

\[
\bar{V}_{ij}(r) = \rho \int d^3 r_k \sum_{\alpha_k, \alpha_i} g(r_{ik})^2 g(r_{jk})^2 V_{ijk}
\]

Li, Lombardo, Shulze & Zuo, PRC 77, 034316 (2008)

**FHNC**

![Energy per nucleon (MeV)](image)

Lovato et al., arxiv:1011.3784

**SCGF**

![E / A | MeV)](image)

Soma & Bozek, PRC 78, 054003 (2008)

**RG**

![Energy/nucleon (MeV)](image)

Hebeler et al., PRC 82, 014314 (2010)
Hot nuclear matter

<table>
<thead>
<tr>
<th>T=0</th>
<th>FHNC</th>
<th>Monte Carlo</th>
<th>BHF</th>
<th>SCGF</th>
<th>RG</th>
</tr>
</thead>
<tbody>
<tr>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>?</td>
<td>✓</td>
</tr>
</tbody>
</table>

- **FHNC**: Free-energy minimization
- **Monte Carlo**: Monte Carlo
- **BHF**: Bloch-de Dominicis
- **SCGF**: Luttinger-Ward
- **RG**: Finite temperature perturbation theory

**Free-energy minimization**

- Schmidt & Pandharipande, PLB 87, 11 (1979)
- Friedman & Pandharipande, NPA 361 (1981)

**Microcanonical ensemble**


**Hybrid approach**

- Baldo & Ferreira, PRC 59, 682 (1999)

**FT-BHF**

- Rios et al., PRC 72, 024316 (2005)
- Burgio et al., PRC 83, 025804 (2011)

**Finite temperature perturbation theory**

- Rios et al., PRC 74, 054317 (2006)
- Soma & Bozek, PRC 78, 054003 (2008)
- Tolos et al., NPA 74, 054317 (2006)
Hypernuclear matter

**FHNC**

- LOCV
  - Pandharipande, NPA 178, 123 (1971)

**Monte Carlo**

- ✓

**SCGF**

- ✓

**Impurity**
- Robertson, PRC 70, 044301 (2004)

**RG**

- ✓

**YN interactions**
- Dapo et al, PRC 81, 035803 (2010)

---

**Issues**

- Uncertainties in NY & YY interactions
- Hyperonic 3BF
- Softening of the EoS
- Other effects: response, transport, viscosities

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**10^3 BHF data & fits for hot hyperonic matter EoS**

Burgio, Schulze & Li, PRC 83, 025804 (2011)

**BHF data & fits for hot hyperonic matter EoS**

BHF data & fits for hot hyperonic matter EoS

**Q=0, S=-2 channels**

\[
\begin{align*}
\frac{\partial \sigma_{n-n}}{\partial \rho} & \frac{\partial \sigma_{p-p}}{\partial \rho} \\
\frac{\partial \sigma_{n-p}}{\partial \rho} & \frac{\partial \sigma_{n-p}}{\partial \rho} \\
\frac{\partial \sigma_{p-p}}{\partial \rho} & \frac{\partial \sigma_{p-p}}{\partial \rho}
\end{align*}
\]

**Schulze et al., PLB 355, 21 (1995)**
Baldo, Burgio & Schulze, PRC 61, 055801 (1999)
Vidaña et al., PRC 61, 025802 (2000)
Vidaña et al., PRC 62, 035801 (2000)
Schulze, Polls, Ramos, Vidaña, PRC 73, 058801 (2006)
Exotic phases of nuclear matter

\[ \Delta = \frac{\rho_\uparrow - \rho_\downarrow}{\rho} \]

\[ \beta = \frac{\rho_n - \rho_p}{\rho} \]

- Spin & isospin polarized nuclear matter
- Which phases are favored and why?
- Need of safe theoretical estimations!
Exotic phases of nuclear matter

\[ \beta = \rho_n - \rho_p \]

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Exotic phases of nuclear matter

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- Spin & isospin polarized nuclear matter
- Which phases are favored and why?
- Need of safe theoretical estimations!
Ferromagnetism?
Instabilities in phenomenological & microscopic models

\[ S_2 = \frac{1}{2} \left( \frac{\partial^2 E/A}{\partial \beta^2} \right)_{\beta=0} \]

\[ \frac{1}{\chi} = \frac{1}{\mu^2 \rho} \left( \frac{\partial^2 E/A}{\partial \Delta^2} \right)_{\Delta=0} \]

Symmetry energy

Neutron matter susceptibility

Group I Skyrme forces

- Skyrme mean-field calculations predict instabilities
- Microscopic calculations do not predict transition
Ferromagnetism?
Instabilities in phenomenological & microscopic models

- Skyrme mean-field calculations predict instabilities
- Microscopic calculations do not predict transition
Beyond the EoS

- Don’t stick to EoS only, aim at complete models
- Better if experimentally testable: mfp, viscosity, symmetry energy

See also Omar Benhar’s talk

Nucleon mean-free path within SCGF

\[ \lambda = \frac{k}{m^* \Gamma} \]

CDBONN: \( \rho = 0.16 \text{ fm}^{-3}, T = 5 \text{ MeV} \)

Rios & Somà, preliminary
Conclusions

• Nuclear physics is an exciting field
• but nuclear many-body problem is difficult!
• Combined with empirical knowledge, a powerful method
• Joint effort from Compstar nuclear theorists
• A certain degree of agreement... But also disagreement!
• Still work to do for exotic phases
Thank you!