

QUANTUM MECHANICS I (523)
 PROBLEM SET 8 (hand in November 5)

- 29) Consider a system with angular momentum $j = 1$, whose ket space is spanned by the basis $|+1\rangle, |0\rangle, |-1\rangle$ of three eigenkets common to \mathbf{J}^2 (eigenvalue $2\hbar^2$) and J_z (respective eigenvalues $+\hbar, 0$ and $-\hbar$). The state of the system is given by

$$|\psi\rangle = \alpha|+1\rangle + \beta|0\rangle + \gamma|-1\rangle,$$

where α, β and γ are three complex parameters.

- a) Calculate the mean value $\langle \mathbf{J} \rangle$ of the angular momentum in terms of α, β and γ .
- b) Give the expression for the three mean values $\langle J_x^2 \rangle, \langle J_y^2 \rangle$, and $\langle J_z^2 \rangle$ in terms of the same quantities.

- 30) Consider a physical system whose four-dimensional vector space is spanned by a basis of four eigenkets $|jm\rangle$ common to \mathbf{J}^2 and J_z with $j = 0$ or 1 and $-j \leq m \leq j$.

- a) Determine the eigenkets common to \mathbf{J}^2 and J_x in terms of these basis states.
- b) Consider the normalized state

$$|\psi\rangle = \alpha|j=1, m=1\rangle + \beta|j=1, m=0\rangle + \gamma|j=1, m=-1\rangle + \delta|j=0, m=0\rangle.$$

Determine the probability of finding $2\hbar^2$ and \hbar if \mathbf{J}^2 and J_x are measured simultaneously.

- c) Calculate the mean value of J_z when the system is in the state $|\psi\rangle$. Do the same for the operators \mathbf{J}^2 and J_x .
- d) Now J_z^2 is measured. Determine the possible results, their probabilities and their mean value.

- 31) An angular momentum eigenstate $|j, m = j\rangle$ is rotated by an infinitesimal angle ϵ about the y -axis. Calculate to order ϵ^2 the probability that a measurement on the rotated state will yield the original state.
- 32) The wave function of a particle in a spherically symmetric potential $V(r)$ is given by
- $$\psi(\mathbf{r}) = (x + y + 3z)f(r)$$
- a) Is ψ an eigenfunction of ℓ^2 ? If yes, determine the ℓ -value. If no, what are the possible ℓ -values one may obtain upon measuring ℓ^2 ?
- b) What are the probabilities for the particle to be found in various m_ℓ states?