

QUANTUM MECHANICS (471)

PROBLEM SET 7 (hand in October 28)

This problem set is worth 40 bonus points and involves filling in the steps that were only outlined in the lecture of 10/14/2016.

23) (40 points)

- a) Proof that the square of the orbital angular momentum can also be written as

$$\ell^2 = \mathbf{p}^2 (\mathbf{r}^2 - r_p^2).$$

- b) Demonstrate that for the Hydrogen Hamiltonian the following result holds

$$\mathbf{r}^2 [H, \mathbf{p}^2] = \frac{2i\hbar^3}{ma_0} (\mathbf{p} \cdot \mathbf{r} + 2i\hbar) \frac{1}{r}.$$

- c) Using part b), demonstrate that the Λ operator yields the same effect on an eigenstate of the Hamiltonian as the operator $\mathbf{r}^2(\mathbf{p}^2 - 2mH)^2$.
- d) Confirm the expression for the Λ_ℓ operator and check that

$$\Lambda_\ell = P_{\ell\pm 1}^\mp P_\ell^\pm - 4\hbar^2 \left(\ell + \frac{1}{2} \pm \frac{1}{2} \right)^2 2mE.$$