09/16/16

QUANTUM MECHANICS (471)
PROBLEM SET 3 (hand in September 23)

9) (5 points) Let A and B be observables. Suppose the simultaneous eigenkets of A and B $\{|a_i b_j\rangle\}$ form a *complete* orthonormal set of basis kets. Can one always conclude that $[A, B] = 0$? If yes, prove it. If no, give a counterexample.

10) (25 points) Consider a three-dimensional ket space. In a certain orthonormal basis with kets $|1\rangle$, $|2\rangle$, and $|3\rangle$ the operators A and B are represented by

$$A \Rightarrow \begin{pmatrix} a & 0 & 0 \\ 0 & -a & 0 \\ 0 & 0 & -a \end{pmatrix}$$

and

$$B \Rightarrow \begin{pmatrix} b & 0 & 0 \\ 0 & 0 & -ib \\ 0 & ib & 0 \end{pmatrix},$$

with a and b both real.

a) Clearly A has a degenerate spectrum. Does B also have a degeneracy?

b) Show that A and B commute.

c) Find a new set of orthonormal kets which are simultaneous eigenkets of A and B. Specify the eigenvalues of A and B for each of these three eigenkets. Does this specification completely chracterize each eigenket or is there another commuting observable?

11) (10 points) Construct the transformation matrix that connects the basis in which S_z is diagonal to the one in which S_x is diagonal. Demonstrate that your result is consistent with the general relation

$$U = \sum_i |b_i\rangle \langle a_i|$$

which was discussed in class.