09/09/16

QUANTUM MECHANICS (471)
PROBLEM SET 2 (hand in September 16)

5) (10 points) Using the orthonormality of $|+\rangle$ and $|-\rangle$, prove

$$[S_i, S_j] = i\epsilon_{ijk}\hbar S_k$$

and

$$\{S_i, S_j\} = \left(\frac{\hbar^2}{2}\right)\delta_{ij},$$

where

$$S_x = \tfrac{\hbar}{2}\left(|+\rangle\langle-| + |-\rangle\langle+|\right)$$
$$S_y = \tfrac{i\hbar}{2}\left(-|+\rangle\langle-| + |-\rangle\langle+|\right)$$
$$S_z = \tfrac{\hbar}{2}\left(|+\rangle\langle+| - |-\rangle\langle-|\right).$$

6) (10 points) Construct kets $|\boldsymbol{S}\cdot\hat{\boldsymbol{n}};+\rangle$ such that

$$\boldsymbol{S}\cdot\hat{\boldsymbol{n}}\,|\boldsymbol{S}\cdot\hat{\boldsymbol{n}};+\rangle = \frac{\hbar}{2}\,|\boldsymbol{S}\cdot\hat{\boldsymbol{n}};+\rangle\,,$$

where $\hat{\boldsymbol{n}}$ is characterized by two angles α and β. α corresponds to the azimuthal angle (measured from the x-axis) and β is the polar angle. Express your answer as a linear combination of $|+\rangle$ and $|-\rangle$ kets. Make sure you solve the actual eigenvalue problem!

7) (10 points) A beam of spin 1/2 atoms goes through a series of Stern-Gerlach-type measurements as follows:

1) The first measurement accepts $s_z = \hbar/2$ atoms and rejects $s_z = -\hbar/2$ atoms, where s_z denotes the eigenvalue of the operator S_z.
2) The second measurement accepts $s_n = \hbar/2$ atoms and rejects $s_n = -\hbar/2$ atoms, where s_n is the eigenvalue of the operator $\boldsymbol{S}\cdot\hat{\boldsymbol{n}}$, with $\hat{\boldsymbol{n}}$ making an angle β in the xz-plane with respect to the z-axis.
3) The third measurement accepts $s_z = -\hbar/2$ atoms and rejects $s_z = \hbar/2$ atoms.

What is the intensity of the final $s_z = -\hbar/2$ beam when the $s_z = \hbar/2$ beam surviving the first measurement is normalized to unity? How must one orient the second measuring apparatus in order to maximize the intensity of the final $s_z = -\hbar/2$ beam?

8) (10 points) A certain observable O has a 3×3 matrix representation in some basis as follows:

$$O \Rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}.$$

Find all the normalized eigenvectors and the corresponding eigenvalues.