Physics 217
Problem Set 1
Due: Friday, September 8th, 2006

1. Chemical processes typically occur at an energy scale on the order of 1 eV. Calculate the corresponding wavelength of electromagnetic radiation that is emitted in such processes. Nuclear processes involve energies on the order of 1 MeV. Locate the wavelength of the corresponding radiation in the electromagnetic spectrum.

2. Sunlight has a typical wavelength of 500 nm. What is the energy (in eV) of a typical photon in sunlight? Can sunlight produce photoelectrons from metals, given that most metals have work functions in the range 2 to 5 eV?

3. Eisberg and Resnick: Chapter 2, problem 5.

4. Eisberg and Resnick: Chapter 2, problem 11.

Physics 217
Problem Set 2
Due: Friday, September 15th, 2006

1. Eisberg and Resnick: Chapter 2, problem 15.
2. Eisberg and Resnick: Chapter 2, problem 18.
3. Eisberg and Resnick: Chapter 2, problem 23.
4. Eisberg and Resnick: Chapter 3, problem 12.

Physics 217
Problem Set 3
Due: Friday, September 22rd, 2006

1. Eisberg and Resnick: Chapter 4, problems 12 and 14.
2. Eisberg and Resnick: Chapter 4, problem 16.
3. Eisberg and Resnick: Chapter 4, problem 23.
4. Eisberg and Resnick: Chapter 4, problem 29.

Physics 217
Problem Set 4
Due: Friday, September 29th, 2006

In this homework you will use MATLAB to do some visualization and numerical integration. MATLAB 7.0 is available on the ArtSci computers in the basement of Eads Hall, For help, read the "MATLAB Hints" available on the course webpage

`http://www.physics.wustl.edu/~ wimd/index217.html`

Feedback on the hints is welcomed.

1. (20 points) Consider a particle in one dimension. Its wavefunction at $t = 0$ is

$$\psi(x) = \frac{C}{1 + (x/a)^2}$$

The constant a (which is a length) gives the width of the wavefunction, i.e. it tells us how tightly localized the particle is around $x = 0$. The normalization constant C is a function of a.

(a) Calculate $C(a)$ by normalizing the wavefunction. What are the dimensions of C?
You may use the fact that

$$\int_{-\infty}^{\infty} \frac{1}{(1+u^2)^2}\, du = \frac{\pi}{2} \ .$$

(b) Create a MATLAB function called "`bump.m`" (to do this click the icon of the blank page (upper left) and type in the new "editor" window) that gives the value of the normalized wavefunction as a function of position:

```
function psi = bump(x,a)
 psi = C./(1+(x./a).^2);
return
```

where "`C`" is to be replaced with the expression that you found previously. Make sure you understand this MATLAB function, and could have constructed it yourself (see the "MATLAB Hints"). If you type `bump(1,2.5)` into MATLAB it will return the value at $x = 1$ of the wavefunction with width $a = 2.5$. *What is this value?*

(c) Use MATLAB to check that our function `bump(x,a)` is properly normalized (i.e. that we got the expression for `C` right). We will first check it for $a = 2.5$. The simplest way is to use an "anonymous function" (see the MATLAB hints) to construct the probability density function from the wavefunction, We then numerically integrate that over all x (actually, -10^6 to $+10^6$ since MATLAB can't handle infinities) using the `quad` function:

```
quad( @(x) conj(bump(x,2.5)).*bump(x,2.5), -10^6, 10^6)
```

– An alternative way to do the same thing is to create a new function that gives the probability density, "bump_prob.m":

  ```
  function psisq = bump_prob(x)
   psisq = conj(bump(x,2.5)).*bump(x,2.5)
  return
  ```

  Then integrate it:

  ```
  quad(@bump_prob, -10^6, 10^6)
  ```

What should the answer be? Is that what you got? (If not, you either made a mistake in part (a), or in your MATLAB function: find the error and correct it.) Do the numerical integration again for several different values of a, and make sure that you always get the expected answer.

(d) Make plots of the normalized wave function for $a = 1, 3, 5$. For example

```
xrange = -10 : 0.1 : 10;       % x = -10 to +10 in steps of 0.1
plot(xrange,bump(xrange,1));   % wavefunction of width 1
plot(xrange,bump(xrange,3));   % wavefunction of width 3
plot(xrange,bump(xrange,5));   % wavefunction of width 5
```

(Everything after "%" is just my comment.) Do several plots on the same set of axes, eg:

```
xrange = -10 : 0.1 : 10;
plot( xrange,bump(xrange,1), xrange,bump(xrange,3), ...
   xrange,bump(xrange,5));
```

(The "..." tells MATLAB that the command continues on the next line.) Print this and and hand it in with your homework. If you can't print it, sketch the curves, showing the numerical scales on the axes. *How does the value of "a" affect the shape of the wavefunction?*

2. (20 points)

(a) Calculate the expectation value of the position in the correctly-normalized "bump" state of question 1, for arbitrary width a:

$$\langle \hat{x} \rangle = \int \psi^*(x) x \psi(x)\, dx \ .$$

Now check your answer using MATLAB. For a bump wavefunction of width 2, $\langle \hat{x} \rangle$ is

```
quad( @(x) conj(bump(x,2)).*x.* bump(x,2), -10^6, 10^6)
```

Try different widths. Do the answers agree with your analytic calculation?

(b) Calculate the expectation value of the position-squared in the "bump" state of question 1, for arbitrary width a. You may use the fact that

$$\int_{-\infty}^{\infty} \frac{u^2}{(1+u^2)^2} = \frac{\pi}{2} \ .$$

Now check your answer using MATLAB, for various widths a.

3. Consider a free particle in one dimension. At $t = 0$ its wavefunction is

$$\psi(x) = \cos(kx)$$

(a) Is ψ an eigenfunction of the position operator? If so, what is its eigenvalue?

(b) Is ψ an eigenfunction of the momentum operator? If so, what is its eigenvalue?

(c) Is ψ an eigenfunction of the Hamiltonian? If so, what is its eigenvalue? How will it evolve over time?

Physics 217
Problem Set 5
Due: Friday, October 6th, 2005

1. Determine the expression for the wavelength of a photon emitted when an electron in an infinite well of length a makes a transition from a state with quantum number n to the ground state.

2. Prove that the normalized eigenfunctions of the infinite-well potential have the property that

$$\int_{-a/2}^{a/2} \psi_n^*(x)\psi_m(x)\; dx = 0 \tag{1}$$

   for $n \neq m$. You can use the fact that

$$\int_0^{\pi/2} \sin(nz)\sin(mz)\,dz = \frac{m\sin(\frac{n\pi}{2})\cos(\frac{m\pi}{2}) - n\cos(\frac{n\pi}{2})\sin(\frac{m\pi}{2})}{n^2-m^2}$$

$$\int_0^{\pi/2} \cos(nz)\cos(mz)\,dz = \frac{n\sin(\frac{n\pi}{2})\cos(\frac{m\pi}{2}) - m\cos(\frac{n\pi}{2})\sin(\frac{m\pi}{2})}{n^2-m^2}.$$

3. Consider a wave function of the form

$$\psi(x) = A\psi_1(x) + B\psi_2(x),$$

   where ψ_1 and ψ_2 are eigenfunctions of the infinite square well. **(a)** Use the requirement that ψ is properly normalized to show that $|A|^2 + |B|^2 = 1$ (use the results of the previous problem). **(b)** Calculate the expectation value of the energy for this wave function. In addition, use the measurement postulates to write this result down directly.

4. Consider the wave function

$$\psi(x,t) = \frac{1}{\sqrt{2}}\{\psi_2(x)e^{-iE_2t/\hbar} + \psi_3(x)e^{-iE_3t/\hbar}\},$$

   where ψ_2 and ψ_3 are again eigenfunctions of the infinite square well. Calculate the probability that the electron is in the domain $[-a/2, 0]$ as a function of time. Determine the period of oscillation of this probability.

Physics 217
Problem Set 6
Due: Friday, October 13th, 2006

These questions all involve a particle of mass m in an infinite potential well of length a, as discussed in class. The eigenstates of the Hamiltonian are $\psi_n(x)$, $n = 1, 2, \ldots$. At time $t = 0$ the wavefunction of the system is

$$\psi(x,0) = \frac{1}{\sqrt{2}}\Big(\psi_1(x) + \psi_2(x)\Big)$$

1. Sketch (or plot) the two lowest eigenstates $\psi_1(x)$ and $\psi_2(x)$. Calculate the expectation value of the position in these states. In order to identify which state is employed in determining the expectation value, we will employ the notation $\langle\psi_1|\hat{x}|\psi_1\rangle$ and $\langle\psi_2|\hat{x}|\psi_2\rangle$. You may use symmetry arguments. **{5 points}**

2. (a) Create a MATLAB M-function that calculates the wavefunction of the system at $t = 0$, for a well of width $a = 100$. Call it "wavefn_t0(x)":

```
function psi = wavefn_t0(x)
a=100;
k1=pi/a;    % lowest energy state (n=1)
k2=2*pi/a;  %   next energy state (n=2)
psi = 1/sqrt(2)*( sqrt(2/a)*cos(k1*x) + sqrt(2/a)*sin(k2*x) );
return
```

   Use MATLAB to plot the wavefunction over the range $x = -50$ to 50. Now plot the probability density $|\psi(x)|^2$,

```
a=100;
fplot( @(x) conj(wavefn_t0(x)).*wavefn_t0(x),[-a/2 a/2]);
```

   Print out these plots and hand them in with your homework (or sketch them including numbers on the axes). In what region would you be most likely to find the particle (at time $t = 0$) if you measured its position? **{4 points}**

   (b) Use MATLAB to check that the wavefunction is correctly normalized, for example by typing

```
a=100;
quad( @(x) conj(wavefn_t0(x)).*wavefn_t0(x),-a/2,a/2)
```

   Now use MATLAB to calculate the expectation value of the position. Does this answer make sense when you look at the plot of $|\psi(x,0)|^2$? **{6 points}**

3. (a) What are the angular frequencies ω_1 and ω_2 of the two lowest eigenstates, in terms of m (mass) and a? How do $\psi_1(x)$ and $psi_2(x)$ evolve in time? **{3 points}**

(b) Create a new MATLAB function, "wavefn(x,t)", that gives the time-dependence of our wavefunction, for $L = 100$ and $m = 0.1$:

```
function psi = wavefn(x,t)
a=100;
m=0.1;
hbar=1;  % we are working in ''particle physics units''
k1=pi/a;
k2=2*pi/a;
omega1 = % you fill in the correct expression here
omega2 = % you fill in the correct expression here
psi = 1/sqrt(2)* ( sqrt(2/a)*cos(k1*x)*XXX
                 + sqrt(2/a)*sin(k2*x)*YYY );
return
```

where the values of omega1 and omega2 and XXX and YYY are to be filled in by you. Note that $\exp(izt)$ is written exp(i*z*t). Check that wavefn(x,0) gives the same values as wavefn_t0(x). **{4 points}**

(c) Download the MATLAB function "animate_wavefn.m" from the course website. It makes a movie of your time-dependent wavefunction, wavefn(x,t) (actually it plots the probability density, $|\psi(x,t)|^2$). Run it for $t = 0$ to 600 by typing

```
animate_wavefn(600)
```

Describe the behavior of the particle. What, roughly, is the period and amplitude of its oscillations? (Remember the amplitude is half of the peak-to-trough variation.) **{2 points}**

(d) To analyse the behavior more precisely, use MATLAB to calculate how the expected position $\langle\psi|\hat{x}|\psi\rangle$ varies in time. Make an M-file "expected_x.m":

```
function xavg = expected_x(t)
  a=100;
  xavg = quad( @(x) ZZZ ,-a./2,a./2);
return
```

replacing ZZZ with the correct expression. Check that $\langle\hat{x}\rangle$ at $t = 0$ agrees with what you got in the last part of question 2.

Plot expected_x(t) for $t = 0$ to 600. You will need to use the fplot function,

```
fplot(@expected_x,[0,600])
```

(it may take a while to calculate). Print out the plot and hand it in with your homework, or sketch it, showing the numerical scale of the axes. Describe physically how the particle is behaving. From the plot, make a more accurate estimate of the amplitude and period of its oscillations. **{6 points}**

4. Analytically calculate the expected position, $\langle\psi|\hat{x}|\psi\rangle$, as a function of time, for general a, m, etc. You may use the results of question 1 and the fact that
$$\int_{-\pi/2}^{\pi/2} \cos(\theta)\theta\sin(2\theta)\,d\theta = \frac{8}{9}\ .$$
Obtain the oscillation period and the amplitude as a function of a and m. Plug in the values used in the MATLAB functions, $\hbar = 1$, $a = 100$ and $m = 0.1$, and check that your answer agrees with what you found in question 3. **{10 points}**

Physics 217
Problem Set 7
Due: Fri, October 27th, 2006

Whenever the question asks you to make a plot, you should print out the plot and hand it in as part of your homework. If you can't print it, sketch it, including the number labels on the axes.

Consider a particle in the ground state of an infinite well of width a,

$$\begin{aligned} \psi(x) &= \sqrt{\tfrac{2}{a}}\cos(\pi x/a) \quad (-a/2 < x < a/2) \\ &= 0 \quad\quad\quad\quad\quad\quad (x < -a/2,\ x > a/2) \end{aligned}$$

At time $t = 0$ we suddenly take away the "walls" of the well, setting $V(x) = 0$ everywhere. The state is still $\psi(x)$, but it is now appropriate to write this state as a superposition of plane waves,

$$\psi(x) = \int_{-\infty}^{\infty} \tilde{\psi}(k) \frac{\exp(ikx)}{\sqrt{2\pi}}\, dk\ . \qquad (*)$$

1. Calculate $\tilde{\psi}(k)$, the Fourier transform of $\psi(x)$. (Remember that $\sin\theta = (\exp(i\theta) - \exp(-i\theta)/(2i)$.) [You may want to jump ahead and do question 3 at this point, since that will tell you whether you have got the right expression.] Calculate $|\tilde{\psi}(k)|^2$. Check that $|\tilde{\psi}(k)|^2$ is normalized correctly: the easiest way is to evaluate the integral numerically (using MATLAB or other software) for various values of a. Make a plot of $|\tilde{\psi}(k)|^2$ for k from 0 to 0.2, for $a = 100$.

2. Calculate k_0, the lowest value of k at which $|\tilde{\psi}(k)|^2 = 0$. As the infinite well becomes wider (increasing a), what happens to k_0? Note from your plot of $|\tilde{\psi}(k)|^2$ that the dominant contribution to $\tilde{\psi}$ comes from $0 < k < k_0$, so k_0 is an estimate of Δk, the range of wavenumbers of the plane waves that constitute $\psi(x)$. The uncertainty in position is $\Delta x = a$. Write down $\Delta x \Delta k$ and hence $\Delta x \Delta p$. Is your result consistent with Heisenberg's uncertainty relation?

3. Equation (*) above shows how, if you add together plane waves of *all* wavenumbers k with the appropriate weights, you can build any function you want to. In this question we will see how the function becomes closer to what we wanted as we include more wavenumbers.

   First, create a MATLAB function `psitilde(k)` (M-file `psitilde.m`) that evaluates your expression for $\tilde{\psi}(k)$ from question 1, for the case where $a = 100$. Now create another function `psi_approx(x)` that makes a crude approximation to the integral in equation (*) by just using the value of $\tilde{\psi}(k)$ at $k = 0.03$:

```
function result = psi_approx(x)
  psi = 0;
  k1 = 0.03;
  psi = psi + 0.03*( psitilde( k1)*exp( j*k1*x)/sqrt(2*pi)  ...
                    +psitilde(-k1)*exp(-j*k1*x)/sqrt(2*pi)  );
  result=psi;
return
```

Plot this function: `fplot( @psi_approx, [-400,600,-0.1,0.2]);`. You see that you get a cos wave with a bump as desired between $x = -50$ and $x = 500$, but also bumps elsewhere. *What determined the wavelength of the sine wave?* Improve things by including the contribution from $k = 0.06$: add to `psi_approx(x)` the lines

```
  k2 = 0.06;
  psi = psi + 0.03*( psitilde( k2)*exp( j*k2*x)/sqrt(2*pi)  ...
                    +psitilde(-k2)*exp(-j*k2*x)/sqrt(2*pi)  );
```

Plot this version too, and see that it is closer to what we wanted. To continue this by including the contributions from many different wavenumbers, download the file `plot_approx.m` from the course website. It calls your `psitilde(k)` function. First reproduce your previous result with

```
plot_approx([0.03 0.06]);
```

Note that it shows the desired wavepacket in red. Now add more wavenumbers:

```
plot_approx([0.02 0.03 0.04 0.06]);
plot_approx([0.01 0.02 0.03 0.04 0.06 ]);
plot_approx([0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08]);
```

See how the sum of plane waves approximates the desired wavepacket more and more closely as we allow contributions from plane waves with a wider range of wavelengths.
Note that the last line can be written in a more economical notation,

```
plot_approx(0.01:0.01:0.08)
```

Why is it wavenumbers k of order 0.05 that are important? What goes wrong if you leave out the lower wavenumbers? What goes wrong if you leave out the higher wavenumbers?

Physics 217
Problem Set 8
Due: Friday, November 3th, 2006

1. A beam of electrons traveling with speed 0.8×10^8 m/s passes through a slit of width 10^{-5} m. Because of the uncertainty in the lateral position of the beam, there will be an uncertainty in the transverse (perpendicular to the beam) momentum as well. Estimate this uncertainty, and use it to calculate the spread of the image of the electron beam on a photographic plate at a distance of 2.0 m beyond the slit.

2. The size of a lead nucleus is approximately 7.8×10^{-15} m. Use the uncertainty relation to estimate the energy of an electron that can be emitted from the nucleus (in a weak interaction decay), assuming the electron was localized inside the nucleus before it emerged. Express your answer in MeV.

3. Eisberg and Resnick: Chapter 3, problem 32.

4. Eisberg and Resnick: Chapter 3, problem 35.

Physics 217
Problem Set 9
Due: Friday, Nov 10th, 2006

1. A beam of electrons is sent along the x-axis from $x = -\infty$ with kinetic energy $E = 6$ eV. It encounters a potential barrier of height $V = 4$ eV and width $2a = 2$ nm. What fraction of the beam is reflected back to $x = -\infty$?

2. In class we discussed the reflection amplitude r of a square barrier. Show that $|r| \leqslant 1$ for any E and V_0, i.e. for any k_1 and k_2. Note that k_1 is always real, but k_2 may be real or imaginary, and you may want to treat those cases separately. Why would one expect that $0 \leqslant |r| \leqslant 1$, even without doing a calculation?

3. Consider the step potential discussed in Sec. 6-4 in the textbook. Solve the Schrödinger equation for the case where $V_0 > E > 0$, and obtain the reflection amplitude r. What is $|r|^2$? What is the physical meaning of this result?

4. Make a quantitative calculation of the transmission coefficient for an unbound particle ($E > V_0$) moving over a finite square well potential (in the usual way: coming from $-\infty$). Determine the condition on the energy of the particle for which $T = 1$.

Physics 217
Problem Set 10
Due: Friday, Nov 17th, 2006

Note: this problem set has two pages.

In class we derived the energy eigenvalue equation for a particle in a square well of width a and depth V_0. We only treated the even solutions, for which $\psi = A\cos(k_I x)$ inside the well and $\psi = C\exp(-k_{II}|x|)$ outside the well. We matched these at $x = a/2$, and obtained

$$k_I \tan(k_I a/2) = k_{II}$$

$$\text{where} \quad k_{II}^2 = \frac{2mV_0}{\hbar^2} - k_I^2$$

1. The eigenvalue equation as written above is an equation that can be solved for allowed values of k_I. How do we then obtain the allowed values of the energy E? **{4 points}**

2. Show that the eigenvalue equation for the even solutions can be written in terms of a dimensionless variable θ and dimensionless parameter Υ (capital Greek "Upsilon"),

$$\theta \tan\theta = \sqrt{\Upsilon^2 - \theta^2}\ . \qquad (*)$$

   What is Υ in terms of m, V_0, a? For $\Upsilon = 1$, use MATLAB or another plotting tool to make superimposed plots of the right-hand side and left-hand side of this equation. Make a separate plot for $\Upsilon = 2$, and a third plot for $\Upsilon = 6$. (You should plot from $\theta = 0$ to 1.1Υ, with a vertical range from 0 to 2Υ.) How many even bound states are there in each of the cases $\Upsilon = 1, 2, 6$? **{12 points}**

   [Note that when MATLAB plots $\theta\tan\theta$, it includes a vertical downward line at $\theta = \pi/2, 3\pi/2\ldots$ where $\tan\theta$ changes discontinuously from $+\infty$ to $-\infty$. However, at these precise values of θ the function is really undefined, so they are not additional solutions of equation (*).]

3. Derive the eigenvalue equation for the odd solutions, for which $\psi = A\sin(k_I x)$ inside the well and $\psi = C\exp(-k_{II}|x|)$ outside the well. How is the value of C for the solution in region I ($x < -a/2$) related to the value for the solution in region III ($x > a/2$)? Write the eigenvalue equation in a dimensionless form, analogous to equation (*) above. Make superimposed plots in this case too, for $\Upsilon = 1, 2, 6$. How many odd bound states are there for each of these values of Υ? **{12 points}**

4. Suppose we fix the width a of the well, and we want to study the limit $\Upsilon \to 0$. In this limit, what is happening to the depth V_0 of the well? What does the spectrum of bound states look like in this case? Does this make sense?

   {8 points}

5. Show that for large Υ, the values of θ at which there are *even* bound states are approximately

$$\theta = (j + \tfrac{1}{2})\pi, \qquad j = 0, 1, 2 \dots \ .$$

   What are the energies of these states? Show that $\Upsilon \to \infty$ at fixed a corresponds to $V_0 \to \infty$, i.e. the infinite well that we studied earlier. The way to do this question is to show that these even solutions correspond to the even eigenstates of the infinite well, i.e. that they have the same energies. **{14 points}**

Physics 217
Problem Set 11
Due: Friday, Dec 1st, 2006

1. Consider two particles of mass $m = 1$ g, connected by a light rigid rod of length $r = 1$ cm, rotating about their center of mass at an angular frequency $\omega = 500$ radians/s. What is the value of the angular momentum quantum number ℓ? How many possible values of m_ℓ are there?

2. Show that the $\ell = 1, m_\ell = 1$ spherical harmonic $Y_{11}(\theta, \phi)$ is a solution of the angular part of the Schrödinger equation for a central potential

$$\frac{d^2 Y_{\ell m_\ell}}{d\theta^2} + \cot\theta \, \frac{dY_{\ell m_\ell}}{d\theta} + \frac{1}{\sin^2\theta} \frac{d^2 Y_{\ell m_\ell}}{d\phi^2} = -\ell(\ell+1) Y_{\ell m_\ell}$$
$$\frac{d^2 Y_{\ell m_\ell}}{d\phi^2} = -m_\ell^2 Y_{\ell m_\ell}$$

3. Eisberg and Resnick: Chapter 7, problem 8.

4. Eisberg and Resnick: Chapter 7, problem 14.

Physics 217
Problem Set 12
Due: Friday, Dec 8th, 2006

Please supply your evaluation of the course at the course evaluation website, `http://evals.wustl.edu`.

1. (a) If the radial part of a particle's wavefunction is $R(r)$, what is the probability of finding the particle somewhere between radius r_1 and r_2?

   (b) Write down the radial wavefunction $R_{10}(r)$ for the $n = 1, \ell = 0$ state of the Hydrogen atom. The nucleus of the Hydrogen aton is a proton, which has a radius $r_p = 10^{-15}$ m. Write down an approximate expression for $R_{10}(r)$ which is valid for $r \lesssim r_p$. What is the probability of finding the electron inside the proton?

   (c) Repeat part (b) for the $n = 2, \ell = 1$ state of Hydrogen. Explain the difference between your results.

2. This is a problem about the Zeeman splitting, so you should ignore the effects of the electron spin (even though they can't be ignored in reality).

   (a) Draw an energy-level diagram showing the $n = 3$ and $n = 2$ energy levels of a Hydrogen atom. How many allowed values of ℓ are there for each?

   (b) Now consider a Hydrogen atom in a magnetic field. Show in an energy level diagram how the $n = 3, \ell = 2$ level and the $n = 2, \ell = 1$ level each split up. How many levels does each split into?

   (c) Show on your diagram all the possible transitions between the $n = 3, \ell = 2$ and $n = 2, \ell = 1$ levels. How many are there?

   (d) Now assume that only transitions that change m_ℓ by 0,$\pm$1 are allowed. Draw a diagram showing those. How many are there? How many different frequencies of light will be emitted?

   (e) If the magnetic field is $B = 1$ T, what will those frequencies be?

3. Consider a hydrogen atom with an electon that has a magnetic dipole moment of magnitude μ_B. Including the contribution of the spin of the electron, determine the angular momentum quantum number of the electron. A beam of these atoms passes through a Stern-Gerlach apparatus. Estimate the field gradient $(\partial B_z/\partial z)$ necessary to make the force on the atom 100 times the weight of the atom. Give a numerical value. Determine the energy (kinetic energy) of the atoms in the beam

such that a measurable deflection on a photographic plate is possible. Make necessary assumptions, if needed.