Self-consistent Green’s function and SRC (ladders) -> nuclear matter

Single-particle Green’s function $G$

Dyson equation: $G = G^{(0)} + G^{(0)} \Sigma G$

$$G(k, E) = \frac{1}{E - \varepsilon_k - \Sigma(k, E)}$$

spectral function $\sim \text{Im } G(k, E)$

Self-energy $\Sigma$, $\Gamma$-matrix

- Pairing instability possible
- Finite temperature calculation can avoid this
- $T=0$ extrapolation of normal self-energy OK

- Rios
- Polls
- Carbone (NNN)
The Bethe-Goldstone theory described above still differs in principle from the Brueckner theory because the Brueckner theory relies on a self-consistent single-particle potential. In terms of Green’s functions, this result can be achieved by replacing \( G_0(p) \) with a \( G(p) \) that includes self-energy effects associated with \( \Gamma \). Furthermore, \( \Gamma \) must itself be determined with \( G \) and not \( G_0 \). The equations for this self-consistent theory are shown schematically in Fig. 42.4.

\[ \begin{align*}
\text{Self-consistent } G: & \\
\text{Self-consistent } \Gamma: & \\
\end{align*} \]

As they stand, these equations are quite intractable because the frequency dependence of \( \Sigma^{*}(p,\epsilon_0) \) complicates the integral equation for \( \Gamma \) immensely. (This difficulty is sometimes known as \textit{propagation off the energy shell}.) The simpler Brueckner-Goldstone theory can be obtained from these equations in a series of approximations. First, the self-consistency is treated only on the average, and we use a frequency-independent self-energy \( \Sigma_{rt}^{*}(p) \equiv \Sigma^{*}(p,\epsilon_0/h) \), obtained by setting \( p_0 = \epsilon_0/h \), where \( \epsilon_0 \) satisfies the self-consistent equation

\[ \epsilon_0 = \epsilon_0^0 + h\Sigma^{*}(p,\epsilon_0/h) \equiv \epsilon_0^0 + h\Sigma_{rt}^{*}(p) \]  

(42.13)

In this way, the Green’s function is given approximately as

\[ \begin{align*}
G_{rt}(p, p_0) &= \frac{\theta(|p| - k_F)}{p_0 - \epsilon_0/h + i\eta} + \frac{\theta(k_F - |p|)}{p_0 - \epsilon_0/h - i\eta} \\
\end{align*} \]

(42.14)

Second, this Green’s function is used to evaluate both the proper self-energy [Eq. (42.4)] and the scattering amplitude [Eqs. (42.5) and (42.6)]. We again obtain \( \chi_{t} \) by omitting the hole-hole scattering, which is presumed small in the low-density limit. The only effect on the self-consistent wave function is to change the denominator in Eq. (42.6) from \( mP_0/h - \frac{3}{4}(\mathbf{P} + \mathbf{q})^2 - \frac{3}{4}(\mathbf{P} - \mathbf{q})^2 + i\eta \) to \( mP_0/h - \frac{3}{4}(\mathbf{P} + \mathbf{q})^2 - \frac{3}{4}(\mathbf{P} - \mathbf{q})^2 + i\eta \)
Nucleons in nuclear matter

- NN interaction requires summation of ladder diagrams
- Study influence of SRC on sp propagator
- Formulate in self-consistent form

\[ \langle k m_\alpha m_{\alpha'} | \Gamma_{pphh}(K, E) | k' m_\beta m_{\beta'} \rangle \]
\[ = \langle k m_\alpha m_{\alpha'} | V | k' m_\beta m_{\beta'} \rangle + \langle k m_\alpha m_{\alpha'} | \Delta \Gamma_{pphh}(K, E) | k' m_\beta m_{\beta'} \rangle \]
\[ = \langle k m_\alpha m_{\alpha'} | V | k' m_\beta m_{\beta'} \rangle + \frac{1}{2} \sum_{m_\gamma m_{\gamma'}} \int \frac{d^3 q}{(2\pi)^3} \langle k m_\alpha m_{\alpha'} | V | q m_\gamma m_{\gamma'} \rangle \]
\[ \times G_{pphh}^f (K, q; E) \langle q m_\gamma m_{\gamma'} | \Gamma_{pphh}(K, E) | k' m_\beta m_{\beta'} \rangle \]
- employs noninteracting but dressed convolution of sp propagators

\[ G_{pphh}^f (K, q; E) = \int_{\epsilon_F}^{\infty} dE' \int_{\epsilon_F}^{\infty} dE'' \frac{S_p(q + K/2; E') S_p(K/2 - q; E'')}{E - E' - E'' + i\eta} \]
\[ - \int_{-\infty}^{\epsilon_F} dE' \int_{-\infty}^{\epsilon_F} dE'' \frac{S_h(q + K/2; E') S_h(K/2 - q; E'')}{E - E' - E'' - i\eta} \]
Lehmann representation

- Dispersion relation (arrows for location of poles in energy plane)
  \[
  \langle km_\alpha m_{\alpha'} | \Delta \Gamma_{pphh}(K, E') | k' m_\beta m_{\beta'} \rangle \\
  = -\frac{1}{\pi} \int_{2\varepsilon_F}^{\infty} dE' \left( \frac{\text{Im} \langle km_\alpha m_{\alpha'} | \Delta \Gamma_{pphh}(K, E') | k' m_\beta m_{\beta'} \rangle}{E - E' + i\eta} \right) \\
  + \frac{1}{\pi} \int_{-\infty}^{2\varepsilon_F} dE' \left( \frac{\text{Im} \langle km_\alpha m_{\alpha'} | \Delta \Gamma_{pphh}(K, E') | k' m_\beta m_{\beta'} \rangle}{E - E' - i\eta} \right)
  \equiv \langle km_\alpha m_{\alpha'} | \Delta \Gamma_{\downarrow}(K, E) | k' m_\beta m_{\beta'} \rangle + \langle km_\alpha m_{\alpha'} | \Delta \Gamma_{\uparrow}(K, E) | k' m_\beta m_{\beta'} \rangle
  \]

- Corresponding self-energy
  \[
  \Sigma_{\Delta \Gamma}(k; E) = -i \frac{1}{\nu} \sum_{\alpha m_\alpha m_{\alpha'}} \int \frac{d^3 k'}{(2\pi)^3} \int \frac{dE'}{2\pi} G(k'; E') \\
  \times \langle \frac{1}{2}(k - k') m_\alpha m_{\alpha'} | \Delta \Gamma_{pphh}(K, E + E') | \frac{1}{2}(k - k') m_\alpha m_{\alpha'} \rangle
  \]

- Energy integral can be performed (note arrows)
  \[
  \Sigma_{\Delta \Gamma}(k; E) = \frac{1}{\nu} \sum_{\alpha m_\alpha m_{\alpha'}} \int \frac{d^3 k'}{(2\pi)^3} \int_{-\infty}^{\varepsilon_F} dE' \langle \frac{1}{2}(k - k') m_\alpha m_{\alpha'} | \Delta \Gamma_{\downarrow}(E + E') | \frac{1}{2}(k - k') m_\alpha m_{\alpha'} \rangle S_h(k', E') \]
  \[+ \frac{1}{\nu} \sum_{\alpha m_\alpha m_{\alpha'}} \int \frac{d^3 k'}{(2\pi)^3} \int_{\varepsilon_F}^{\infty} dE' \langle \frac{1}{2}(k - k') m_\alpha m_{\alpha'} | \Delta \Gamma_{\uparrow}(E + E') | \frac{1}{2}(k - k') m_\alpha m_{\alpha'} \rangle S_p(k', E') \]
  \[\equiv \Delta \Sigma_{\downarrow}(k; E) + \Delta \Sigma_{\uparrow}(k; E)\]
Final steps and implementation

- **Total self-energy**

\[
\Sigma(k; E) = \Sigma_V(k) - \frac{1}{\pi} \int_{E_F}^{\infty} dE' \frac{\text{Im} \Sigma(k; E')}{E - E' + i\eta} + \frac{1}{\pi} \int_{-\infty}^{E_F} dE' \frac{\text{Im} \Sigma(k; E')}{E - E' - i\eta}
\]

\[
\Sigma_V(k) = \frac{1}{\nu} \sum_{m_\alpha m_{\alpha'}} \langle \frac{1}{2}(k - k')m_\alpha m_{\alpha'} | V | \frac{1}{2}(k - k')m_\alpha m_{\alpha'} \rangle n(k')
\]

- **includes HF-like term**

- **Practical calculations first done with mean-field sp propagators in scattering equation**

\[
\Sigma^{(0)}_{\Delta \Gamma}(k; E) = \frac{1}{\nu} \sum_{m_\alpha m_{\alpha'}} \int \frac{d^3k'}{(2\pi)^3} \langle \frac{1}{2}(k - k')m_\alpha m_{\alpha'} | \Delta \Gamma^{(0)}_{\downarrow}(E + \varepsilon(k')) | \frac{1}{2}(k - k')m_\alpha m_{\alpha'} \rangle \theta(k_F - k')
\]

\[
- \frac{1}{\nu} \sum_{m_\alpha m_{\alpha'}} \int \frac{d^3k'}{(2\pi)^3} \langle \frac{1}{2}(k - k')m_\alpha m_{\alpha'} | \Delta \Gamma^{(0)}_{\uparrow}(E + \varepsilon(k')) | \frac{1}{2}(k - k')m_\alpha m_{\alpha'} \rangle \theta(k' - k_F)
\]
Propagator

- **DE** \[ G(k; E) = G^{(0)}(k; E) + G^{(0)}(k; E)\Sigma(k; E)G(k; E) \]
  \[= \frac{E - \varepsilon(k) - \text{Re} \Sigma(k; E) + i\text{Im} \Sigma(k; E)}{(E - \varepsilon(k) - \text{Re} \Sigma(k; E))^2 + (\text{Im} \Sigma(k; E))^2} \]

- only magnitude of wave vector needed

- **Noninteracting propagator** \[ G^{(0)}(k; E) = \frac{\theta(k - k_F)}{E - \varepsilon(k) + i\eta} + \frac{\theta(k_F - k)}{E - \varepsilon(k) - i\eta} \]
  with \( \varepsilon(k) = \frac{\hbar^2 k^2}{2m} + U(k) \)

- **Particle spectral function** \[ S_p(k; E') = \frac{-1}{\pi} \frac{\text{Im} \Sigma(k; E)}{(E - \varepsilon(k) - \text{Re} \Sigma(k; E))^2 + (\text{Im} \Sigma(k; E))^2} \]

- **Hole spectral function** \[ S_h(k; E) = \frac{1}{\pi} \frac{\text{Im} \Sigma(k; E)}{(E - \varepsilon(k) - \text{Re} \Sigma(k; E))^2 + (\text{Im} \Sigma(k; E))^2} \]

- **Dispersion relation (check)** \[ G(k; E) = \int_{\varepsilon_F}^{\infty} dE' \frac{S_p(k; E')}{E - E' + i\eta} + \int_{-\infty}^{\varepsilon_F} dE' \frac{S_h(k; E')}{E - E' - i\eta} \]
Spectral functions

- Near $k_F$ about 70% of sp strength is contained in QP peak
- additional ~13% distributed below the Fermi energy
- remaining strength (~17%) above the Fermi energy
- Note “gap”
- Distribution narrows
- towards $k_F$
Spectral functions above the Fermi energy

- Wave vectors 0.79 (dotted), 1.74 (solid), and 3.51 fm\(^{-1}\) (dashed)
- Common distribution -> SRC
- For 0.79 -> 17%
- \(\varepsilon_F + 100\) MeV -> 13%
- above 500 MeV -> 7%
- Without tensor force:
  - strength only 10.5%
- Momentum distribution
- \(n(0)\) same for different methods
- and interactions (not at \(k_F\))
- Solid: self-consistent
Self-consistent results

- Wave vectors 0.0 (solid), 1.36 (dotted), and 2.1 fm\(^{-1}\) (dashed)
- Limited set of Gaussians --> self-consistent
- Spectral functions
- Main difference:
  - common strength
- Slightly less correlated
- \(Z_F\) from 0.72 to 0.75
Nuclear Matter

- Nuclear masses near stability
  \[ M(N, Z) = \frac{E(N, Z)}{c^2} = N m_n + Z m_p - \frac{B(N, Z)}{c^2} \]
- Data
- Each A most stable
  N,Z pair
Nuclear Matter

- Smooth curve

\[ B = b_{vol} A - b_{surf} A^{2/3} - \frac{1}{2} b_{sym} \frac{(N - Z)^2}{A} - \frac{3}{5} \frac{Z^2 e^2}{R_c} \]

- volume \( b_{vol} = 15.56 \text{ MeV} \)
- surface \( b_{surf} = 17.23 \text{ MeV} \)
- symmetry \( b_{sym} = 46.57 \text{ MeV} \)
- Coulomb \( R_c = 1.24 A^{1/3} \text{ fm} \)

Great interest in limit: N=Z; no Coulomb; A \( \rightarrow \infty \)

Two most important numbers in Nuclear Physics

\[ \frac{B}{A} \approx 16 \text{ MeV} \quad \rho_0 \approx 0.16 \text{ fm}^3 \]
Physics of saturation

• How do we determine the saturation density
  - role of SRC
  - role of LRC
  - what are LRC in nuclei and nuclear matter

• How do we extract the binding energy at saturation
  - Can this be done with a liquid drop model?
Saturation density and SRC

- Saturation density related to nuclear charge density at the origin. Data for $^{208}\text{Pb} \rightarrow A/Z * \rho_{ch}(0) = 0.16 \text{ fm}^{-3}$
- Charge at the origin determined by protons in s states
- Occupation of 0s and 1s totally dominated by SRC as can be concluded from recent analysis of $^{208}\text{Pb}(e,e'p)$ data and theoretical calculations of occupation numbers in nuclei and nuclear matter.
- Depletion of 2s proton also dominated by SRC:
  15% of the total depletion of 25% ($n_{2s} = 0.75$)

Conclusion: Nuclear saturation dominated by SRC
and therefore $\rightarrow$ presence of high-momentum components
Elastic electron scattering from $^{208}\text{Pb}$

B. Frois et al.
Nuclear density distribution

- Central density \((A/Z^* \, \text{charge density})\) about the same for nuclei heavier than \(^{16}\text{O}\), corresponding to \(0.16\, \text{nucleons/fm}^3\)

- Important quantity

- Shape roughly represented by

\[
\rho_{ch}(r) = \frac{\rho_0}{1 + \exp\left(\frac{r-c}{z}\right)}
\]

\(c \approx 1.07A^{1/3}\, \text{fm}\)

\(z \approx 0.55\, \text{fm}\)

- Potential similar shape
Recent work on $^{208}$Pb (thesis Mack Atkinson)

- Analysis of data to constrain the nucleon self-energy using dispersion relations
Nuclear Matter

- Smooth curve

\[ B = b_{vol} A - b_{surf} A^{2/3} - \frac{1}{2} b_{sym} \frac{(N - Z)^2}{A} - \frac{3}{5} \frac{Z^2 e^2}{R_c} \]

- volume \( b_{vol} = 15.56 \text{ MeV} \)
- surface \( b_{surf} = 17.23 \text{ MeV} \)
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Great interest in limit: \( N=Z; \) no Coulomb; \( A \to \infty \)

Two most important numbers in Nuclear Physics

\[ \frac{B}{A} \approx 16 \text{ MeV} \quad \rho_0 \approx 0.16 \text{ fm}^3 \]
Saturation problem of nuclear matter

Given $V_{NN}$ ⇒ explain correct minimum of $E/A$ in nuclear matter as a function of density inside empirical box

Describe the infinite system of neutrons
⇒ properties of neutron stars
Saturation properties of nuclear matter

- Colorful and continuing story
- Initiated by Brueckner: proper treatment of SRC in medium -> ladder diagrams but only include pp propagation

\[
\langle k_m a_{\alpha'} | G(K, E) | k' m_{\beta'} \rangle = \langle k_m a_{\alpha'} | V | k' m_{\beta'} \rangle + \frac{1}{2} \sum_{m_{\gamma'} m_{\gamma}} \int \frac{d^3 q}{(2\pi)^3} \langle k_m a_{\alpha'} | V | q m_{\gamma} m_{\gamma'} \rangle \frac{\theta(|q + K/2| - k_F) \theta(|K/2 - q| - k_F)}{E - \varepsilon(q + K/2) - \varepsilon(K/2 - q) + i\eta} \langle q m_{\gamma} m_{\gamma'} | G(K, E) | k' m_{\beta'} \rangle
\]

- Brueckner G-matrix but Bethe-Goldstone equation...

- Dispersion relation

\[
\langle k_m a_{\alpha'} | G(K, E) | k' m_{\beta'} \rangle = \langle k_m a_{\alpha'} | V | k' m_{\beta'} \rangle - \frac{1}{\pi} \int_{2\varepsilon_F}^{\infty} dE' \frac{\text{Im} \langle k_m a_{\alpha'} | \Delta G(K, E') | k' m_{\beta'} \rangle}{E - E' + i\eta} \equiv \langle k_m a_{\alpha'} | V | k' m_{\beta'} \rangle + \langle k_m a_{\alpha'} | \Delta G_1(K, E) | k' m_{\beta'} \rangle
\]

- Include HF term in “BHF” self-energy

\[
\Sigma_{BHF}(k; E) = \int \frac{d^3 k'}{(2\pi)^3} \frac{1}{\nu} \sum_{m_{\alpha'} m_{\alpha}} \theta(k_F - k') \langle \frac{1}{2} (k - k') m_{\alpha} m_{\alpha'} | G(k + k'; E + \varepsilon(k') | \frac{1}{2} (k - k') m_{\alpha} m_{\alpha'} \rangle
\]

- Below Fermi energy: no imaginary part
BHF

- DE for $k < k_F$ yields solutions at
  \[ \varepsilon_{BHF}(k) = \frac{\hbar^2 k^2}{2m} + \Sigma_{BHF}(k; \varepsilon_{BHF}(k)) \]
- with strength $< 1$
- Since there is no imaginary part below the Fermi energy, no momenta above $k_F$ can admix -> problem with particle number
- Only sp energy is determined self-consistently

- Choice of auxiliary potential
  - Standard \[ U_s(k) = \Sigma_{BHF}(k; \varepsilon_{BHF}(k)) \] only for $k < k_F$ (0 above)
  - Continuous \[ U_c(k) = \Sigma_{BHF}(k; \varepsilon_{BHF}(k)) \] all $k$

- Only one calculation of $G$-matrix for standard choice
- Iterations for continuous choice
Propagator
\[ G^{BHF}(k; E) = \frac{\theta(k - k_F)}{E - \varepsilon_{BHF}(k) + i\eta} + \frac{\theta(k_F - k)}{E - \varepsilon_{BHF}(k) - i\eta} \]

Energy
\[ \frac{E^A_0}{A} = \frac{\nu}{2\rho} \int \frac{d^3k}{(2\pi)^3} \left( \frac{\hbar^2 k^2}{2m} + \varepsilon_{BHF} \right) \theta(k - k_F) \]

Rewrite using on-shell self-energy
\[ \frac{E^A_0}{A} = \frac{4}{\rho} \int \frac{d^3k}{(2\pi)^3} \left( \frac{\hbar^2 k^2}{2m} \right) + \frac{1}{2\rho} \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3k'}{(2\pi)^3} \sum_{m\alpha m\alpha'} \theta(k_F - k)\theta(k_F - k') \]
\[ \langle \frac{1}{2}(k - k') m\alpha m\alpha' | G(k + k'; \varepsilon_{BHF}(k) + \varepsilon_{BHF}(k')) | \frac{1}{2}(k - k') m\alpha m\alpha' \rangle \]

First term: kinetic energy free Fermi gas
\[ E^{HF} = \frac{1}{2} \sum_p \theta(p_F - p) \left[ \frac{p^2}{2m} + \varepsilon^{HF}(p) \right] \]
\[ = T_{FG} + \frac{1}{2} \sum_{pp'} \theta(p_F - p)\theta(p_F - p') \langle pp' | V | pp' \rangle \]

so BHF obtained by replacing V by G
• Binding energy usually within 10 MeV from empirical volume term in the mass formula even for very strong repulsive cores

• Repulsion always completely cancelled by higher-order terms

• Minimum in density never coincides with empirical value when binding OK -> Coester band

Location of minimum determined by deuteron D-state probability
**Historical perspective**

- First attempt using scattering in the medium  
  Brueckner 1954

- Formal development (linked cluster expansion)  
  Goldstone 1956

- Reorganized perturbation expansion (60s)  
  Bethe & students  
  BBG-expansion

- Variational Theory vs. Lowest Order BBG (70s)  
  Clark (also crisis paper)  
  Pandharipande

- Variational results & next hole-line terms (80s)  
  Day, Wiringa

- New insights from experiment & theory  
  NIKHEF Amsterdam  
  about what nucleons are up to in the nucleus (90s)

- 3-hole line terms with continuous choice (90s)  
  Baldo et al.

- Ongoing...
Some remarks

• Variational results gave more binding than $G$-matrix calculations

• Interest in convergence of Brueckner approach

• Bethe et al.: hole-line expansion

• $G$-matrix: sums all energy terms with 2 independent hole lines (noninteracting ...)

• Dominant for low-density

• Phase space arguments suggests to group all terms with 3 independent hole lines as the next contribution

• Requires technique from 3-body problem first solved by Faddeev
  -> Bethe-Faddeev summation

• Including these terms generates minima indicated by * in figure

• Better but not yet good enough
More

- Variational results and 3-hole-line results more or less in agreement
- Baldo et al. also calculated 3-hole-line terms with continuous choice for auxiliary potential and found that results do not depend on choice of auxiliary potential, furthermore 2-hole-line with continuous choice is already sufficient!
- Conclusion: convergence OK for a given realistic two-body interaction for the energy per particle
- Also: for other observables no such statements can be made
- Still nuclear matter saturation problem!
Based on results from (e,e'p) reactions

- nucleons are dressed (substantially) and this should be included in the description of nuclear matter (depletion, high-momentum components in the ground state, propagation w.r.t. correlated ground state \(\leftrightarrow\) BHF?)

- SRC dominate actual value of saturation density
  
  - from \(^{208}\text{Pb}\) charge density: 0.16 nucleons/fm\(^3\)
  
  - determined from s-shell proton occupancy at small radius
  
  - occupancy determined mostly by SRC

- Earlier result for SCGF of ladders do not include LRC!!

So why can’t we get it right?

• Must be LRC?!

• Look at hole-line expansion

• Identify LRC contribution to the energy
Ingredients hole-line expansion

- Wiggle: $G$-matrix
- $a) + b) = 2$ hole-line = BHF
- $c) + d) + e) + f) = 3$ hole-line
- c) bubble
- d) U insertion for C choice
- e) ring
- f) summed in Bethe-Faddeev
Results hole-line expansion 2+3

- Important confirmation Baldo et al. PRL 81, 1584 (1998)
Continuous choice

What about long-range correlations in nuclear matter?

• Collective excitations in finite nuclei very different from those in nuclear matter

• Long-range correlations normally associated with small $q$

• Contribution to the energy like $dq \, q^2 \Rightarrow$ very small (except for e-gas)

• Contributions of collective excitations to the binding energy of nuclear matter dominated by pion-exchange induced excitations and not small?!?
Pion-exchange channel dominates 3rd order ring

- Decomposition in spin-isospin excitations at normal density

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Inclusion of $\Delta$-isobars as 3N- and 4N-force

Rings with $\Delta$-isobars:

- PPN Phys. 11, 529 (1983)

$\Rightarrow$ No sensible convergence with $\Delta$-isobars

Must do nuclear saturation without $\pi$-collectivity
LRC in finite nuclei

Remember:

- LRC in infinite nuclear matter —> no counterpart in finite nuclei
- BUT: LRC in finite nuclei —> no counterpart in nuclear matter
- They will contribute some binding!
- How much: nobody has really looked into this
Nuclear Saturation without $\pi$-collectivity

- Variational calculations treat LRC (on average) and SRC simultaneously (Parquet equivalence) so difficult to separate LRC and SRC
- Remove 3-body ring diagram from Catania hole-line expansion calculation ⇒ about the correct saturation density
- Hole-line expansion doesn’t treat Pauli principle very well
- Present results improve treatment of Pauli principle by self-consistency of spectral functions ⇒ more reasonable saturation density and binding energy acceptable

- Neutron matter: pionic contributions must be included ($\Delta$)
Recent result SCGF & SRC compared to BHF and BBG

- BBG requires a repulsive NNN at high density to improve density
Latest work

- Maybe 16 MeV binding is not needed!
Effect of 3N attractive $\leftrightarrow$ AV18

Argonne $v_{18}$
With Illinois-2
GFMC Calculations

Energy (MeV)

$^2\text{H}$ $^3\text{H}$ $^4\text{He}$ $^6\text{Li}$ $^7\text{Li}$ $^8\text{Li}$ $^8\text{Be}$ $^9\text{Be}$ $^{10}\text{Be}$ $^{10}\text{B}$ $^{12}\text{C}$

AV18
IL2
Exp

nuclear matter