Direct knockout reactions

- Atoms: \((e,2e)\) reaction
- Nuclei: \((e,e'p)\) reaction [and others like \((p,2p), (d,^3He), (p,d), etc.]\]
- Physics: transfer large amount of momentum and energy to a bound particle; detect ejected particle together with scattered projectile → construct spectral function
- Simple analysis
- Initial state: ground state \( |\Psi_i\rangle = |\Psi_0^N\rangle \)
- Final state:
  \( |\Psi_f\rangle = a_p^{\dagger} |\Psi_{n}^{N-1}\rangle \)
- Probe: acts as one-body excitation operator transferring momentum \(\hbar q\) to a particle
  \[ \rho(q) = \sum_{j=1}^{N} \exp(\imath q \cdot r_j) \]
- 2nd quantization (no spin) \( \hat{\rho}(q) = \sum_{p,p'} \langle p | \exp(\imath q \cdot r) | p' \rangle a_p^{\dagger} a_{p'} = \sum_{p} a_p^{\dagger} a_{p-hq} \)
Transition matrix element

- Impulse approximation: struck particle is ejected

\[
\langle \Psi_f \mid \hat{\rho}(q) \mid \Psi_i \rangle = \sum_{p'} \langle \Psi_{n}^{N-1} \mid a_p a_{p'}^\dagger a_{p'-\hbar q} \mid \Psi_{0}^{N} \rangle
\]

\[
= \sum_{p'} \langle \Psi_{n}^{N-1} \mid \delta_{p',p} a_{p'-\hbar q} + a_{p'}^\dagger a_{p'-\hbar q} a_p \mid \Psi_{0}^{N} \rangle
\]

\[
\approx \langle \Psi_{n}^{N-1} \mid a_{p-\hbar q} \mid \Psi_{0}^{N} \rangle
\]

- Other assumption: final state ~ plane wave on top of N-1 particle eigenstate (more serious in practical experiments) but good approximation if ejectile momentum large enough

- Write \( H_N = \sum_{i=1}^{N} \frac{p_i^2}{2m} + \sum_{i<j=1}^{N} V(i, j) = H_{N-1} + \frac{p_N^2}{2m} + \sum_{i=1}^{N-1} V(i, N) \)

- last term FSI: interaction between ejected particle and others

- If relative momentum large enough, interaction can be neglected:

- PWIA = plane wave impulse approximation
Cross section

- Fermi’s Golden Rule  \[ d\sigma \sim \sum_n \delta(\hbar \omega + E_i - E_f) |\langle \Psi_f | \hat{\rho}(q) | \Psi_i \rangle|^2 \]

- with energy transfer \( \hbar \omega \) linking initial and final state energy

- Define  \[ p_{miss} = p - \hbar q \]
  \[ E_{miss} = \frac{p^2}{2m} - \hbar \omega = E_0^N - E_{n}^{N-1} \]

- Rewrite knockout cross section
  \[ d\sigma \sim \sum_n \delta(E_{miss} - E_0^N + E_{n}^{N-1}) |\langle \Psi_{n}^{N-1} | a_{p_{miss}} | \Psi_0^N \rangle|^2 \]
  \[ = S_h(p_{miss}; E_{miss}) \]

- More comprehensive treatment requires inclusion of FSI
(e,2e) data for atoms

- Start with Hydrogen
- Ground state wave function \( \phi_{1s}(p) = \frac{2^{3/2}}{\pi} \frac{1}{(1 + p^2)^2} \)
- (e,2e) removal amplitude

\[
\langle 0 | a_p | n = 1, \ell = 0 \rangle = \langle p | n = 1, \ell = 0 \rangle = \phi_{1s}(p)
\]

Hydrogen 1s wave function “seen” experimentally
Helium

- IPM description is very successful
- Closed-shell configuration $1s^2$
- Reaction more complicated than for Hydrogen
- DWIA (distorted wave impulse approximation)

$$S = \int dp \left| \langle \Psi_{n}^{N-1} | a_p | \Psi_0^N \rangle \right|^2$$

agreement with IPM!

$\rightarrow 1$

Other closed-shell atoms

- Spectroscopic factor become less than 1
- Neon $2p$ removal: $S = 0.92$ with two fragments each 0.04
- IPM not the whole story: fragmentation of $sp$ strength
- Summed strength: like IPM
- IPM wave functions still excellent
- Example: Argon $3p$ $S = 0.95$
- Rest in 3 small fragments
Fragmentation in atoms

- All the strength remains below (above) the Fermi energy in closed-shell atoms

- Fragmentation can be interpreted in terms of mixing between

\[ a_\alpha |\Phi_0^N\rangle \]

and

\[ a_\beta a_\gamma a_\delta^\dagger |\Phi_0^N\rangle \]

- with the same “global” quantum numbers

- Example: Argon ground state

\[ |\Phi_0^N\rangle = |(3s)^2(3p)^6(2s)^2(2p)^6(1s)^2\rangle \]

- \( \text{Ar}^+ \) ground state

\[ |(3p)^{-1}\rangle = a_{3p} |\Phi_0^N\rangle = |(3s)^2(3p)^5(2s)^2(2p)^6(1s)^2\rangle \]

- excited state

\[ |(3s)^{-1}\rangle = a_{3s} |\Phi_0^N\rangle = |(3s)^1(3p)^6(2s)^2(2p)^6(1s)^2\rangle \]

- also

\[ |(3p)^{-2}4s\rangle = a_{3p}a_{3p}a_{4s}^\dagger |\Phi_0^N\rangle = |(4s)^1(3s)^2(3p)^4(2s)^2(2p)^6(1s)^2\rangle \]

- and

\[ |(3p)^{-2}nd\rangle = a_{3p}a_{3p}a_{nd}^\dagger |\Phi_0^N\rangle = |(nd)^1(3s)^2(3p)^4(2s)^2(2p)^6(1s)^2\rangle \]
Argon spectroscopic factors

- s strength also in the continuum: $\text{Ar}^{++} + e$
- note vertical scale
- red bars: 3s fragments exhibit substantial fragmentation

![Graph showing spectroscopic factors for Argon with notable peaks at $3s^23p^5$, $3s3p^6$, $3p^44s$, $8\%$]
(e,e’p) data for nuclei

- Requires DWIA
- Distorted waves required to describe elastic proton scattering at the energy of the ejected proton
- Consistent description requires that cross section at different energy for the outgoing proton is changed accordingly
- Requires substantial beam energy and momentum transfer
- Initiated at Saclay and perfected at NIKHEF, Amsterdam
- Also done at Mainz and currently at Jefferson Lab, VA
- Momentum dependence of cross section dominated by the corresponding sp wave function of the nucleon before it is removed
Momentum profiles for nucleon removal

- Closed-shell nuclei
But...

- Spectroscopic factors substantially smaller than simple IPM
Remember

- $^{208}\text{Pb}$ sp levels
**Fragmentation patterns**

- $^{208}\text{Pb}(e,e'p)$ NIKHEF data: Quint thesis

- $S(2s_{1/2})=0.65$

- other data:
  - $n(2s_{1/2})=0.75$

- very different from atoms
Fragmentation patterns

- $^{208}\text{Pb}(e,e'p)$ NIKHEF data: Quint thesis (1988)

- Start of strong fragmentation
- Also very different from atoms
Fragmentation patterns

- $^{208}\text{Pb}(e,e'p)$ NIKHEF data: Quint thesis

- deeply bound states: strong fragmentation
- again different from atoms
$^{16}\text{O}$ data from Saclay

- Simple interpretation!
Recent Pb experiment

- 100 MeV missing energy
- 270 MeV/c missing momentum
- complete IPM domain
Reading

• Read one of:
  - Rev. Mod. Phys. 69, 981 (1997) --> nuclei (e,e'p)
  - Rev. Mod. Phys. 67, 713 (1995) --> solids (e,2e)