

**Physics 217**  
**Problem Set 9**  
**Due: Friday, Nov 9th, 2018**

Note: this problem set has two pages.

In class we derived the energy eigenvalue equation for a particle in a square well of width  $a$  and depth  $V_0$ . We only treated the even solutions, for which  $\psi = A \cos(k_I x)$  inside the well and  $\psi = C \exp(-k_{II}|x|)$  outside the well. We matched these at  $x = a/2$ , and obtained

$$k_I \tan(k_I a/2) = k_{II}$$

$$\text{where } k_{II}^2 = \frac{2mV_0}{\hbar^2} - k_I^2$$

1. (4 points) The eigenvalue equation as written above is an equation that can be solved for allowed values of  $k_I$ . How do we then obtain the allowed values of the energy  $E$ ?
2. (12 points) Show that the eigenvalue equation for the even solutions can be written in terms of a dimensionless variable  $\theta$  and dimensionless parameter  $\Upsilon$  (capital Greek “Upsilon”),

$$\theta \tan \theta = \sqrt{\Upsilon^2 - \theta^2} . \quad (*)$$

What is  $\Upsilon$  in terms of  $m$ ,  $V_0$ ,  $a$ ? For  $\Upsilon = 1$ , use Mathematica to make superimposed plots of the right-hand side and left-hand side of this equation (i.e. plot both the left-hand side and right-hand side as a function of  $\theta$  in the same figure). Make a separate plot for  $\Upsilon = 2$ , and a third plot for  $\Upsilon = 6$ . (You should plot from  $\theta = 0$  to  $1.1\Upsilon$ , with a vertical range from 0 to  $2\Upsilon$ .) How many even bound states are there in each of the cases  $\Upsilon = 1, 2, 6$ ?

[Note that when Mathematica plots  $\theta \tan \theta$ , it includes a vertical downward line at  $\theta = \pi/2, 3\pi/2 \dots$  where  $\tan \theta$  changes discontinuously from  $+\infty$  to  $-\infty$ . However, at these precise values of  $\theta$  the function is really undefined, so they are not additional solutions of equation (\*).]

3. (12 points) Derive the eigenvalue equation for the odd solutions, for which  $\psi = A \sin(k_I x)$  inside the well and  $\psi = C \exp(-k_{II}|x|)$  outside the well. How is the value of  $C$  for the solution in region I ( $x < -a/2$ ) related to the value for the solution in region III ( $x > a/2$ )? Write the eigenvalue equation in a dimensionless form, analogous to equation (\*) above. Make superimposed plots in this case too, for  $\Upsilon = 1, 2, 6$ . How many odd bound states are there for each of these values of  $\Upsilon$ ?

4. (8 points) Suppose we fix the width  $a$  of the well, and we want to study the limit  $\Upsilon \rightarrow 0$ . In this limit, what is happening to the depth  $V_0$  of the well? What does the spectrum of bound states look like in this case? Does this make sense?
5. (14 points) Show that for large  $\Upsilon$ , the values of  $\theta$  at which there are *even* bound states are approximately

$$\theta = (j + \frac{1}{2})\pi, \quad j = 0, 1, 2 \dots .$$

What are the energies of these states? Show that  $\Upsilon \rightarrow \infty$  at fixed  $a$  corresponds to  $V_0 \rightarrow \infty$ , i.e. the infinite well that we studied earlier. The way to do this question is to show that these even solutions correspond to the even eigenstates of the infinite well, i.e. that they have the same energies.