Physics 217
Problem Set 6
Due: Friday, October 19th, 2018

These questions all involve a particle of mass $m = 0.1$ in an infinite potential well of length $a = 100$, as discussed in class. This time we will use the particle physics units, where $\hbar = 1$. The eigenstates of the Hamiltonian are $\psi_n(x)$, $n = 1, 2, \ldots$. At time $t = 0$ the wavefunction of the system is

$$\psi(x,0) = \frac{1}{\sqrt{2}}\Big(\psi_1(x) + \psi_2(x)\Big)$$

1. (5 points) Sketch (or plot) the two lowest eigenstates $\psi_1(x)$ and $\psi_2(x)$. Calculate the expectation value of position in these states. In order to identify which state is employed in determining the expectation value, we will employ the notation $\langle\psi_1|\hat{x}|\psi_1\rangle$ and $\langle\psi_2|\hat{x}|\psi_2\rangle$. You may use symmetry arguments.

2. (a) (4 points) Create a Mathematica function that calculates the wavefunction of the system at $t = 0$, for the well of width $a = 100$. Call it "`wavefn$t0(x)`". Use Mathematica to plot the wavefunction over the range $x = -50$ to 50. Now plot the probability density $|\psi(x)|^2$. Print out these plots and hand them in with your homework (or sketch them including numbers on the axes). In what region would you be most likely to find the particle (at time $t = 0$) if you measured its position?

   (b) (6 points) Use Mathematica to check that the wavefunction is correctly normalized. Then use Mathematica to calculate the expectation value of the position. Does this answer make sense when you look at the plot of $|\psi(x,0)|^2$?

3. (a) (3 points) What are the angular frequencies ω_1 and ω_2 of the two lowest eigenstates, in terms of m (mass) and a (do not plug in the values)? How do $\psi_1(x)$ and $\psi_2(x)$ evolve in time?

   (b) (4 points) Create a new Mathematica function, "`wavefn(x,t)`", that gives the time-dependence of our wavefunction $\psi(x,t)$, for $a = 100$ and $m = 0.1$. Check that `wavefn(x,0)` gives the same values as `wavefn$t0(x)`.

   (c) (2 points) Add this line to your code:

```
DynamicModule[{t},
{Slider[Dynamic[t], {0, 600, 1}], "t =" Dynamic[t],
 Dynamic[Plot[Conjugate[wavefn[x, t]] wavefn[x, t],
 {x, -50, 50}]]}]
```

This will generate a slider which allows you to change `t` dynamically and see how the probability density of your wavefunction $|\psi(x,t)|^2$ changes with time. Describe the behavior of the particle. What, roughly, is the period and amplitude of its oscillations? (Remember the amplitude is half of the peak-to-trough variation.)

(d) (6 points) To analyse the behavior more precisely, use Mathematica to calculate how the expected position $\langle\psi|\hat{x}|\psi\rangle$ varies in time. Create a new function "`expected$x(t)`". Check that $\langle\hat{x}\rangle$ at $t = 0$ agrees with what you got in the last part of question 2.

Plot `expected$x(t)` for $t = 0$ to 600. Print out the plot and hand it in with your homework, or sketch it, showing the numerical scale of the axes. Describe physically how the particle is behaving. From the plot, make a more accurate estimate of the amplitude and period of its oscillations.

4. (10 points) Analytically calculate the expected position, $\langle\psi|\hat{x}|\psi\rangle$, as a function of time, for general a, m, etc. You may use the results of question 1 and the fact that

$$\int_{-\pi/2}^{\pi/2} \cos(\theta)\theta\sin(2\theta)\,d\theta = \frac{8}{9}\ .$$

Obtain the oscillation period and the amplitude as a function of a and m. Plug in the values used in the Mathematica functions, $\hbar = 1$, $a = 100$, and $m = 0.1$, and check that your answer agrees with what you found in question 3.