Physics 217
Problem Set 5
Due: Friday, October 5, 2018

1. (5 points) Determine the expression for the wavelength of a photon emitted when an electron in an infinite well of length a makes a transition from a state with quantum number n to the ground state.

2. (15 points) Prove that the normalized eigenfunctions of the infinite-well potential have the property that

$$\int_{-a/2}^{a/2} \psi_n^*(x)\psi_m(x)\ dx = 0 \tag{1}$$

for $n \neq m$. You can use the fact that

$$\int_0^{\pi/2} \sin(nz)\sin(mz)\,dz = \frac{m\sin(\frac{n\pi}{2})\cos(\frac{m\pi}{2}) - n\cos(\frac{n\pi}{2})\sin(\frac{m\pi}{2})}{n^2 - m^2}$$

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3. (10 points) Consider a wave function of the form

$$\psi(x) = A\psi_1(x) + B\psi_2(x),$$

where ψ_1 and ψ_2 are eigenfunctions of the infinite square well. **(a)** Use the requirement that ψ is properly normalized to show that $|A|^2 + |B|^2 = 1$ (use the results of the previous problem). **(b)** Calculate the expectation value of the energy for this wave function. In addition, use the measurement postulates to write this result down directly.

4. (10 points) Consider the wave function

$$\psi(x,t) = \frac{1}{\sqrt{2}}\{\psi_2(x)e^{-iE_2t/\hbar} + \psi_3(x)e^{-iE_3t/\hbar}\},$$

where ψ_2 and ψ_3 are again eigenfunctions of the infinite square well. Calculate the probability that the electron is in the domain $[-a/2, 0]$ as a function of time. Determine the period of oscillation of this probability.