Physics of Neutron Stars 3

Equation of State of Neutron Stars

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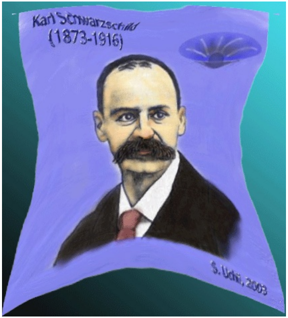
Outline

Neutron Stars, Work of many people

Chadwick, Baade and Zwicky, Schwarzschild (photo below), Oppenheimer and Volkoff, Tolman, Hewish and Bell (photo below)...

Jocelyn Bell



Karl Schwarzschild

and many more.

Tolman-Oppenheimer-Volkoff Equations

General Relativity

$$G^{\mu\nu} = 8\pi T^{\mu\nu}$$

- Assume spherical symmetry
- Perfect fluid (Neglect viscosity).
- Non-rotating body in hydrostatic equilibrium

TOV Equations

$$\frac{dM(r)}{dr} = 4\pi\mathcal{E}r^2,$$

$$\frac{d\mathcal{P}(r)}{dr} = -\frac{\mathcal{E}M}{r^2}\left(1 + 4\pi r^3\frac{\mathcal{P}}{M}\right)\left(1 + \frac{\mathcal{P}}{\mathcal{E}}\right)\left(1 - 2\frac{M}{r}\right)^{-1}.$$

and a supplemented equation for the baryon number:

$$\frac{d\mathcal{A}(r)}{dr} = 4\pi r^2 n_b \frac{1}{1 - 2M(r)/r}$$

From GR to Newton

$$\frac{dM(r)}{dr} = 4\pi\mathcal{E}r^2,$$
$$\frac{d\mathcal{P}(r)}{dr} = -\frac{\mathcal{E}M}{r^2}\frac{\left(1+4\pi r^3\frac{\mathcal{P}}{M}\right)\left(1+\frac{\mathcal{P}}{\mathcal{E}}\right)}{\left(1-2\frac{M}{r}\right)}.$$

The TOV equations reduce to Newtons if:

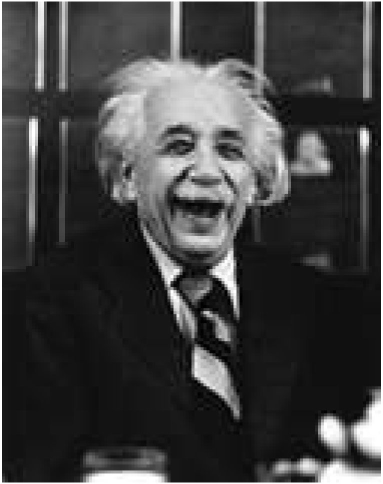
1. Speed of sound is much smaller than c: $\mathcal{P} \ll \mathcal{E}$.
2. Compactness: $2\frac{M(r)}{r} \ll 1$.
3. $4\pi r^3\mathcal{P} \ll M(r)$.

Newtonian Gravity

$$\frac{dM(r)}{dr} = 4\pi\mathcal{E}r^2,$$
$$\frac{d\mathcal{P}(r)}{dr} = -\frac{\mathcal{E}M}{r^2}.$$

$G = 1$ and $c = 1$

Very happy



for Neutron Stars

G.R. to N.G.

1. $\mathcal{P} \ll \mathcal{E}$
2. $2\frac{M(r)}{r} \ll 1$ or $\frac{2GM(r)}{r} \ll c^2$
3. $4\pi r^3 \mathcal{P} \ll M(r)$

N.S.s in few numbers:

- $M \sim 1.4 M_\odot$
- $R \sim 12$ to 14 km
- $n_c \sim 5$ to $10 \times n_0$ ($n_0 \simeq 0.16 \text{fm}^{-3}$)
- $T \sim 0.5$ MeV

N.S.s are compact objects

1. $\mathcal{P} \leq \mathcal{E}$
2. $g_{NS} \sim 10^{11} g$

$$\frac{2GM(r)}{r} \sim \frac{2GM}{R^2} R \sim 2 g_{NS} R \sim 2 \times 10^{16} \text{m}^2/\text{s}^2$$

3. If $\mathcal{P} \sim 10^{34} \text{erg/cm}^3$ for $r \sim R/2$ then: $4\pi r^3 \mathcal{P} \sim 10^{53}$erg, and $M(R/2) \sim 10^{54}$erg.

need full GR to describe N.S.s

Equation of State

The input of the TOV Equations is the Equation of State:

$$\mathcal{P} = \mathcal{P}(\mathcal{E}, n_b, Y_p)$$

Want to find the mass and the radius of the star.

Do these $\mathcal{M} - R$ agree with astronomical observations ?

- Astronomical observations in principal can rule out EoS and therefore nuclear models.
- One EoS has a unique $\mathcal{M} - R$ family curve.

Pure Neutron Stars

Will see:

- Algorithm to solve the TOV Eqns.
- Why we need interactions.

Relativistic Fermi Gas at $T = 0$

$$E = \int d\mathbf{x} \langle \psi^\dagger \left(i\boldsymbol{\alpha} \cdot \nabla + m \right) \psi \rangle$$

Find the energy density:

$$\mathcal{E} = \frac{m^4}{8\pi^2} \left(2\frac{\epsilon_f k_f^3}{m^4} - \frac{\epsilon_f k_f}{m^2} - \log\left(\frac{\epsilon_f + k_f}{m} \right) \right)$$

and the pressure by:

$$\mathcal{P} = n^2 \frac{\partial}{\partial n} \left(\frac{\mathcal{E}}{n} \right)$$

with $\epsilon_f = \sqrt{k_f^2 + m^2}$ and $n = \frac{k_f^3}{3\pi^2}$

Solving the TOV Eqns.

Boundary conditions

$$
\begin{aligned}
M(r=0) &= 0 \\
\mathcal{E}(r=0) &= \mathcal{E}_c \quad \text{A guess} \\
\mathcal{P}(r=0) &= \mathcal{P}_c \quad \text{From EoS}
\end{aligned}
$$

Algorithm

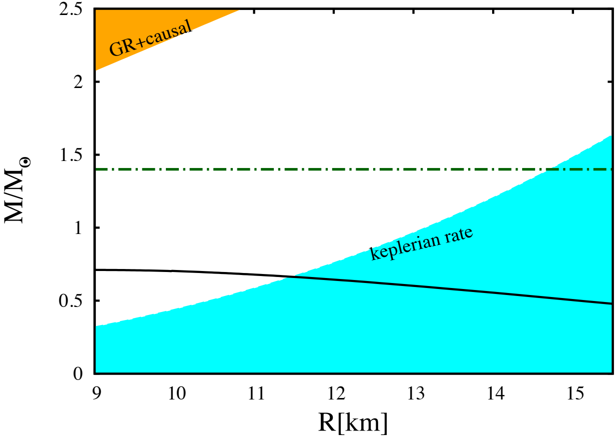
1. Use favourite method to advance one step: M_{i+1} and $\mathcal{P}_{i+1}$
2. From $\mathcal{P}_{i+1}$ use EoS to get $\mathcal{E}_{i+1}$
3. These values are initial points for next iteration.
4. If $\mathcal{P}_{k+1} \leq 0$ then stop $\Rightarrow M_k = M(r_k) = M$ and $r_k = R$.

Notes

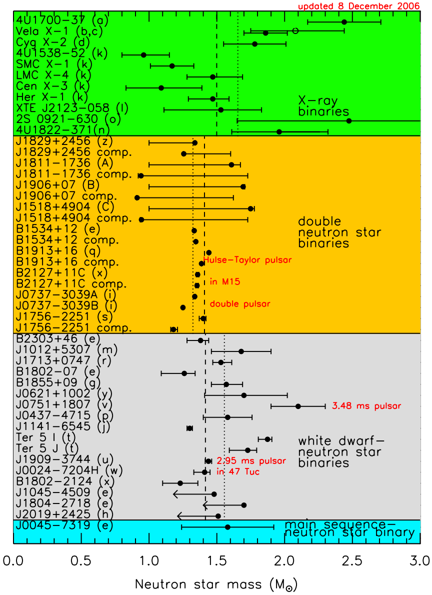
- Each central density $\mathcal{E}_c \Rightarrow (M, R)$ point.
- This complete family of stars, with masses and radii are determined by the EoS.
- Technically is better to put everything in km.

$$
\begin{aligned}
1 &= G = 6.6726 \times 10^{-8} \frac{\text{cm}^3}{\text{gs}^2} \\
1 &= c = 2.9979 \times 10^{10} \frac{\text{cm}}{\text{s}}
\end{aligned}
$$

Free Neutron Gas Star



Measured and Estimated Masses



Including interactions

Many ways to describe nuclear systems because we do not know the form of the interaction.

- *Ab initio* methods: Use a N-N interaction (obtained from N-N scattering) ⇒ solve the many-body problem (very hard).
- Effective theories: Parametrising our ignorance (short distance physics) with coupling constants ⇒ solve self-consistent mean field equations (easier but very hard to go beyond mean field).

In general any nuclear model must

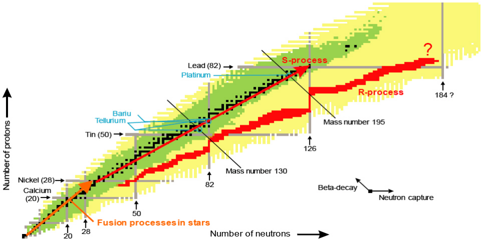
... reproduce Nuclear Matter properties

$$n_0 = 0.16\ \mathrm{fm}^{-3}$$

$$\left(\frac{\mathcal{E}}{n_b}\right)_0 = -16\ \mathrm{MeV}$$

$$K = 234\ \mathrm{MeV}$$

...reproduce nuclei properties:



Relativistic Mean Field Formalism

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- Nucleons interact by the exchange of mesons: Sigma (σ), Omega (ω), Rho (ρ), and the photon.

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Parametrising what we do not know...

- Hadron degrees of freedom:

$$\psi = \begin{pmatrix} p \\ n \end{pmatrix}.$$

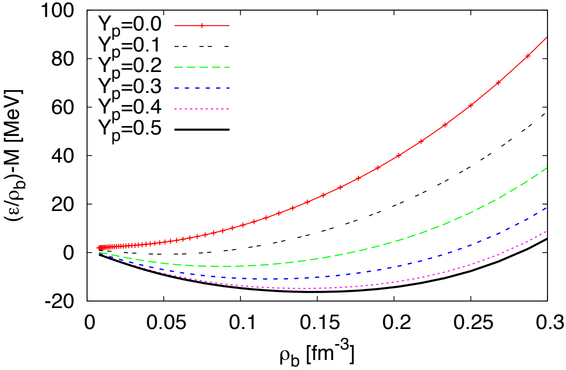
- Nucleons interact by the exchange of mesons: Sigma (σ), Omega (ω), Rho (ρ), and the photon.
- The lagrangian of the theory is:

$$\mathcal{L} = \mathcal{L}_B + \mathcal{L}_e + \mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_\rho + \mathcal{L}_\gamma + \mathcal{L}_{M-M},$$

- Coupling constants are found by fitting to masses of magic nuclei and properties of nuclear matter.

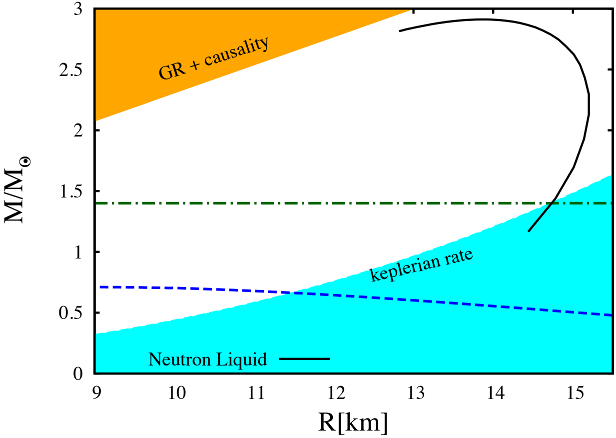
Relativistic Mean Field Formalism

Uniform Nuclear Matter



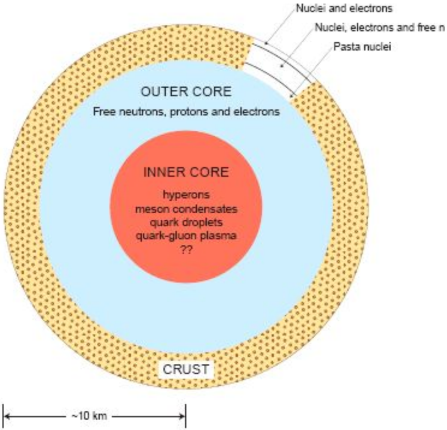
- Using $Y_p = 0$, means EoS of a liquid of neutrons.
- Using our algorithm to solve the TOV equations
- Find the $M - R$.

Neutron Star (Pure Neutron Matter)



Neutron Star Structure

Because of cooling, glitches phenomena, spin down, it is possible that the neutron star is not only made of neutrons...



Layers

- Inner Core ⇒ Exotic Matter?
- Outer Core ⇒ Nuclear liquid.
- Inner Crust ⇒ Coulomb Crystal +Neutron liquid
- Outer Crust ⇒ Coulomb Crystal
- Ocean
- Atmosphere

Chemical Equilibrium

Conserved quantities

- Baryon number $\Rightarrow \mu_n$
- Electric charge $\Rightarrow \mu_e$

Any other chemical potentials can be expressed as a linear combination of them. νs and γs are radiated away, $\mu_\nu = 0$ and $\mu_\gamma = 0$.
For a particle χ:

$$\mu_\chi = q_\chi^b \mu_n - q_\chi^{el} \mu_e$$

q_χ^b: Baryon number.
q_χ^{el}: Electric charge.

Examples:

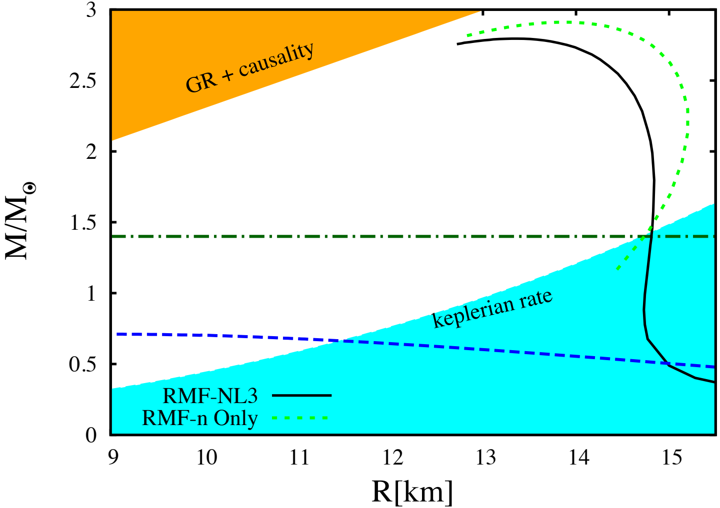
- $n \leftrightarrow p + e + \nu_e$, then $\mu_p = (+1)\mu_n - (+1)\mu_e$
- $e \leftrightarrow \mu + \nu_e + \bar{\nu}_\mu$ then $\mu_\mu = (0)\mu_n - (-1)\mu_e$

EoS at β Equilibrium

For a given baryon density n_B
Find the composition Y_p for which $\Delta = \mu_n - \mu_p - \mu_e = 0$:

- Guess the composition (Y_P).
- By charge neutrality $n_e = n_p = Y_p n_b$.
- Solve the nuclear many body problem
  $\Rightarrow$ Means we have found $\mathcal{E}(n_b, Y_p)$ and $\mathcal{P}(n_b, Y_p)$
- If no β equilibrium yet: If $\Delta \neq 0$ $\Rightarrow$ change Y_p.
- Otherwise sing victory.

RMF with NL3 parametrisation



Douchin and Haensel EoS

Try to use a unified nuclear model to describe solid crust and liquid core.

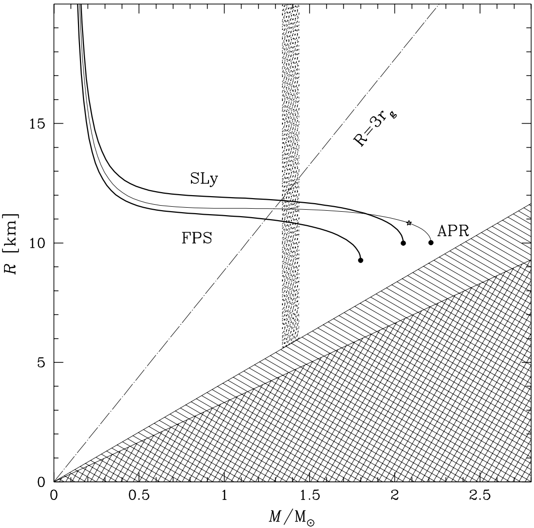
Skyrme forces:Effective N-N interaction, an average 3-body force was added.

Using Skyrme type forces ...

- Parametrise a liquid drop model.
- Use this liquid drop model to describe the crust.
- Use Skyrme interaction to describe the liquid core.

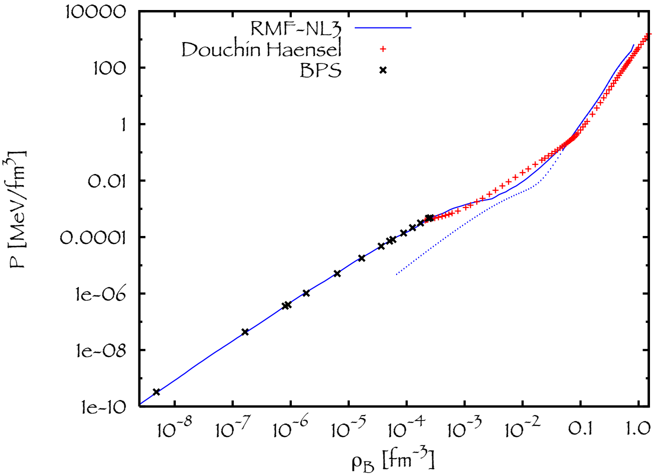
F. Douchin and P. Haensel, A& A **380, 151(2001)**

Douchin and Haensel EoS



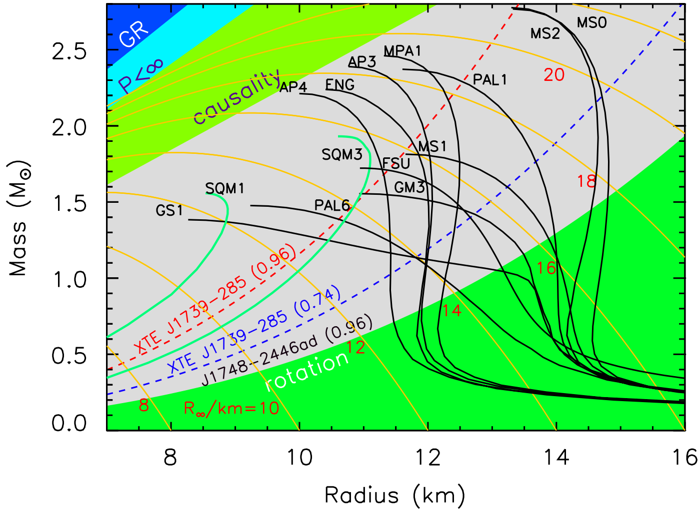
F. Douchin and P. Haensel, A& A **380, 151(2001)**

Parallel with D&H EoS

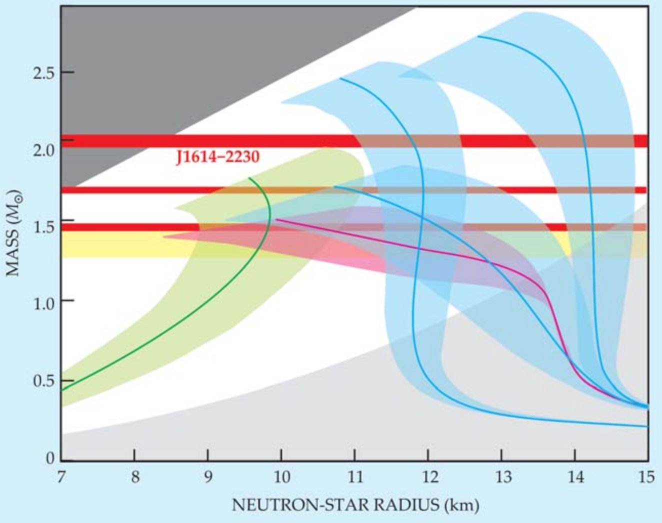


Still early

$M - R$ Compilation thanks to Lattimer and Prakash



$2M_{\odot}$ N.S. observed [7]



Constraints from Heaven

As we said last talk, cooling observations, also ...

$M - R$ from Observations, Steiner et al. [4]

- Combine observations of 6 neutron stars.
- Two types: X-ray bursts and Neutron Stars in globular clusters.
  ⇒ Observations favour stiff high density EoS.
  ⇒ EoS does not favour a phase transition to exotic matter.
  ⇒ EoS can support $2M_\odot$.
  ⇒ EoS is soft at low densities, then a $1.4M_\odot$ star has a 12km radius.
  ⇒ EoS has associated a $R_n - R_p = 0.15 \pm 0.02$fm for ^{208}Pb.

$M - R$ from Observations, Ozel et al. [5]

- Same than above, but with a different model for X-ray bursts.
  ⇒ EoS favour softer high density EoS.
  ⇒ EoS can support $2M_\odot$.
  ⇒ EoS is soft at low densities, then a $1.4M_\odot$ star has a 10km radius.

Constraints on Earth

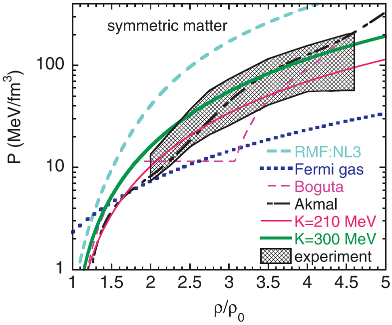
Parametrising what we do not know here too:

$$\mathcal{E}(n_b, Y_p) = \mathcal{E}(n_b, Y_p = 1/2) + S(n_b)(1 - 2Y_p)^2 + \mathcal{O}(Y_p)$$

Heavy Ion Collisions (HIC)

- Au+Au collisions ($^{197}79 + ^{197}79$).
- Incident kinetic energy of 394GeV.
- Using a transport model, extract dS/dn_b.
- Not completely conclusive.

Figure: Constraint on compression modulus with HIC



P. Danielewicz, R. Lacey, W. G. Lynch, Science **298**, 1592(2002) and M. B. Tsang et al., PRL **102**, 122701(2009)

Constraints on Earth

Neutron skin of ^{208}Pb:

$$\delta R = R_n - R_p$$

Lead Radius Experiment

- Correlation between δR and R_{NS} $\Rightarrow$ constrains the EoS at about $^2/_3\ n_0$
- Since it is an electron nucleus scattering, unambiguous interpretation.
- Complement HIC constraints on EoS.

Last but not least

- Gravitational wave observations: advance LIGO, advance VIRGO, LISA, etc., observations can impose constrains on the shape of N.S.s, as well as the structure of the crust. How strong is the crust depends on EoS.
- Accreting NS.s: There are observations of accreting neutron stars, particularly, after quiescence, related to how the crust cools down.
- Rotating N.S.s and EoS: The general case of rotating stars has been studied, when rotation is added larger masses are possible.
- Experiments of neutron-rich nuclei: Experimental efforts towards the neutron drip line can help to improve our current nuclear models and make the interpolation to 10^{14} orders of magnitude, a bit more reliable.
- Other layers: The crust, ocean, and the atmosphere are important to interpret observations.
- Exotic matter: Part of the work is what are the signatures of exotic matter, depends strongly on interpretations of the astronomical observations.

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