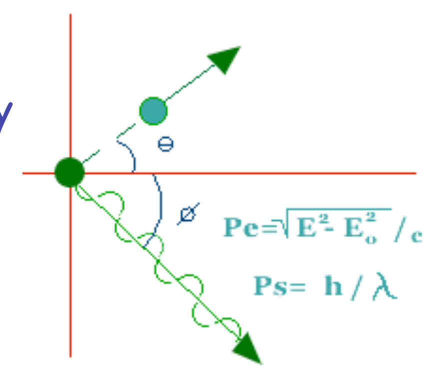
Inspiration …

CISS07 8/28/2007

Comprehensive treatment of correlations at different energy scales in nuclei using Green's functions

Lecture 1: 8/28/07	Propagator description of single-particle motion and the link with experimental data
Lecture 2: 8/29/07	From Hartree-Fock to spectroscopic factors < 1: inclusion of long-range correlations
Lecture 3: 8/29/07	Role of short-range and tensor correlations associated with realistic interactions
Lecture 4: 8/30/07	Dispersive optical model and predictions for nuclei towards the dripline
Adv. Lecture 1: 8/30/07	Saturation problem of nuclear matter & pairing in nuclear and neutron matter
Adv. Lecture 2: 8/31/07	Quasi-particle density functional theory



Wim Dickhoff
Washington University in St. Louis

Some questions …

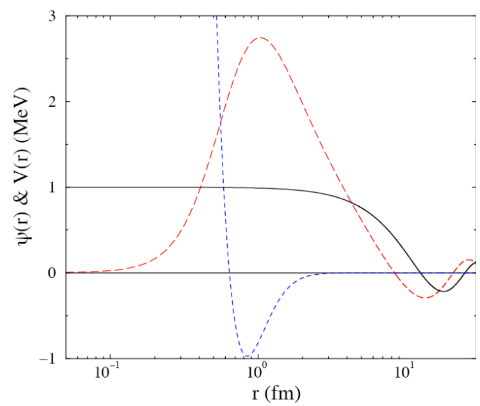
What does a nucleon do in the nucleus?

Is this a legitimate question?

Speculations …

What is the dependence on *N* and *Z*?

Energy scales:



As high as a realistic V_{NN} will take you

…

Δ-isobars, pions

…

As low as the first excited state

⇒ ALL OF THEM! HOW?

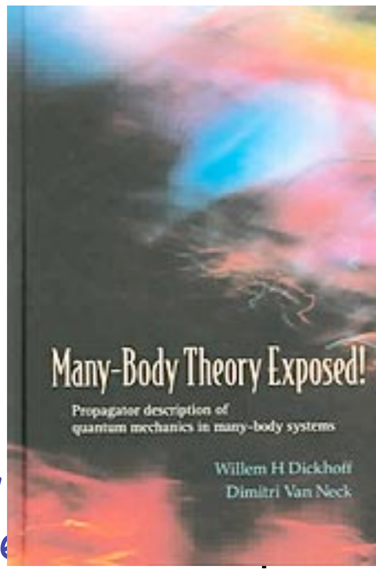
⇒ Time-dependent formulation not surprising

Description of the nuclear many-body problem

Ingredients: Nucleons interacting by "realistic interactions"
Nonrelativistic many-body problem

Method: Green's functions (Propagators)
⇨ amplitudes instead of wave functions
keep track of all nucleons, including the high-momentum ones

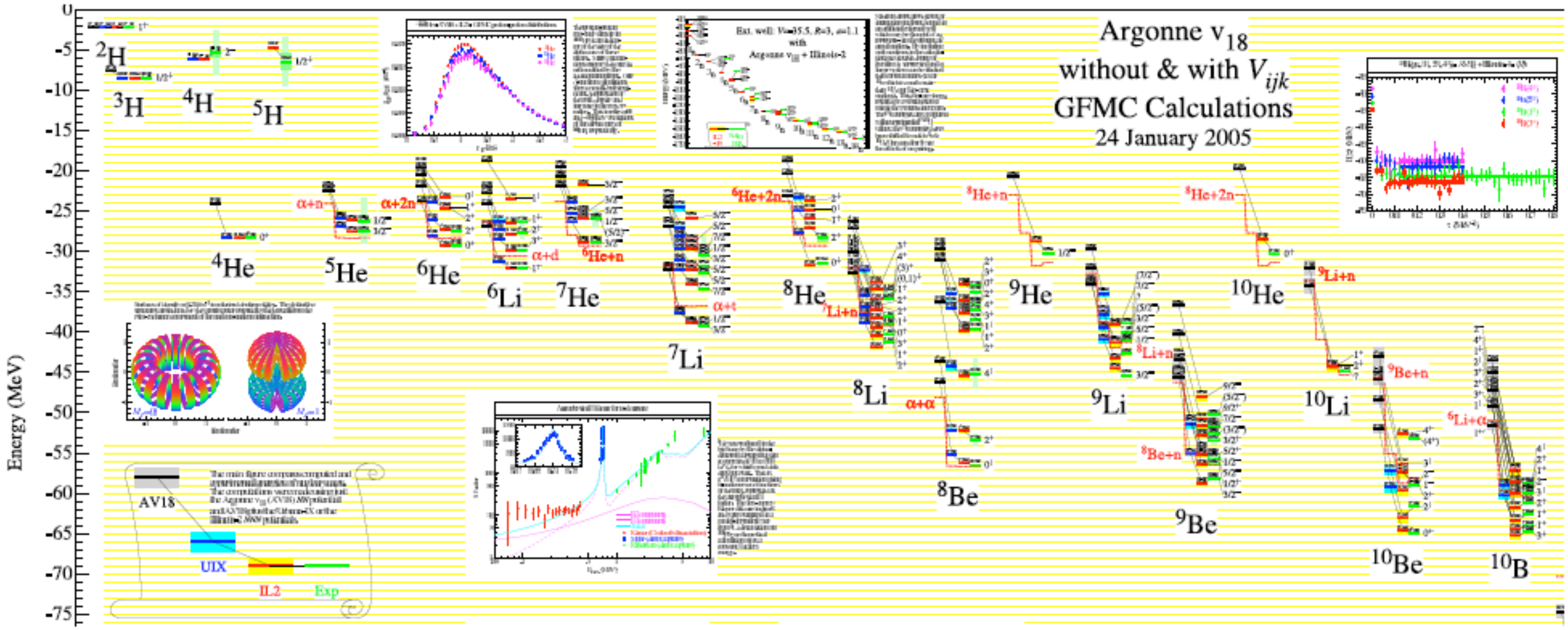
Book: Dimitri Van Neck & W.D.

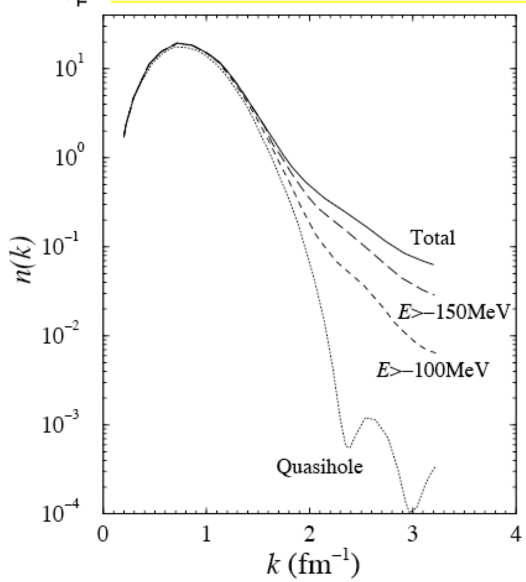


Why: Physical insight and useful for all many-body systems
Link between experiment and theory clear
Can include all energy scales
Efficient: generates amplitudes not wave functions

Review C. Barbieri, Prog. Part. Nucl. Phys. **52**, 377 (2004)
Lecture ww.nscl.msu.edu/~brown/theory-group/lecture-notes.html

Good stuff …





⇐ Physics of this picture requires different approach

Outline

- What is a propagator
- Propagator in the many-body problem
- Information contained in propagator
- Spectral functions
- Relation with experimental data
- Experimental results
- Outline of perturbation theory

What is a propagator or Green's function?

Time evolution is governed by the Hamiltonian *H*. For a single particle the state

$$|\alpha,t_0;t\rangle = e^{-\frac{i}{\hbar}H(t-t_0)}|\alpha,t_0\rangle$$

is indeed a solution of $$i\hbar\frac{\partial}{\partial t}|\alpha,t_0;t\rangle = H|\alpha,t_0;t\rangle$$

Relation between wave function at t and t_0 can then be written as

$$\psi(\vec{r},t) = \langle\vec{r}|\alpha,t_0;t\rangle = \langle\vec{r}|e^{-\frac{i}{\hbar}H(t-t_0)}|\alpha,t_0\rangle = \int d\vec{r}\,'\langle\vec{r}|e^{-\frac{i}{\hbar}H(t-t_0)}|\vec{r}\,'\rangle\langle\vec{r}\,'|\alpha,t_0\rangle$$
$$= i\hbar\int d\vec{r}\,'G(\vec{r},\vec{r}\,';t-t_0)\psi(\vec{r}\,',t_0)$$

with the propagator or Green's function defined by

$$G(\vec{r},\vec{r}\,';t-t_0) = -\frac{i}{\hbar}\langle\vec{r}|e^{-\frac{i}{\hbar}H(t-t_0)}|\vec{r}\,'\rangle$$

Recall Huygens' principle!

Alternative expressions

Using $\theta(t-t_0) = -\int \frac{dE'}{2\pi i} \frac{e^{-\frac{i}{\hbar}E'(t-t_0)}}{E'+i\eta}$ (Note $\frac{d}{dt}\theta(t-t_0) = \delta(t-t_0)$)

the Fourier transform of the propagator can be written as

$$G(\vec{r},\vec{r}\,';E) = \int_{-\infty}^{\infty} d(t-t_0)e^{\frac{i}{\hbar}E(t-t_0)}G(\vec{r},\vec{r}\,';t-t_0)\theta\left(t-t_0\right)$$

$$= \sum_n \frac{\langle 0|a_{\vec{r}}|n\rangle\langle n|a^+_{\vec{r}\,'}|0\rangle}{E-\varepsilon_n+i\eta}$$

$$= \langle 0|a_{\vec{r}} \frac{1}{E-H+i\eta} a^+_{\vec{r}\,'}|0\rangle$$

with $H|n\rangle = \varepsilon_n|n\rangle$

Also $\langle 0|a_{\vec{r}}|n\rangle = \langle \vec{r}|n\rangle = u_n(\vec{r})$

So numerator yields information on wave functions and denominator on eigenvalues of *H*.

How is G calculated?

"Simple" for the case of one particle. Can proceed by splitting

$H = H_0 + V$ and using the operator identity $\frac{1}{A-B} = \frac{1}{A} + \frac{1}{A} B \frac{1}{A-B}$

for the operator $G = \frac{1}{E - H + i\eta}$ with $A = E - H_0 + i\eta$

and $B = V$ to obtain G in terms of $G^{(0)}$ and V:

$$G = G^{(0)} + G^{(0)} V G$$
$$= G^{(0)} + G^{(0)} V G^{(0)} + G^{(0)} V G^{(0)} V G^{(0)} + \cdots$$

or in a particular basis

$$G(\alpha,\beta;E) = G^{(0)}(\alpha,\beta;E) + \sum_{\gamma\delta} G^{(0)}(\alpha,\gamma;E) \langle \gamma | V | \delta \rangle G(\delta,\beta;E)$$

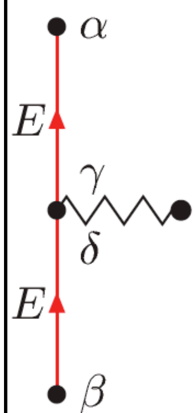
with $G(\alpha,\beta;E) = \langle \alpha | \frac{1}{E - H + i\eta} | \beta \rangle$ and $G^{(0)}(\alpha,\beta;E) = \langle \alpha | \frac{1}{E - H_0 + i\eta} | \beta \rangle$

Diagrams

Lowest order

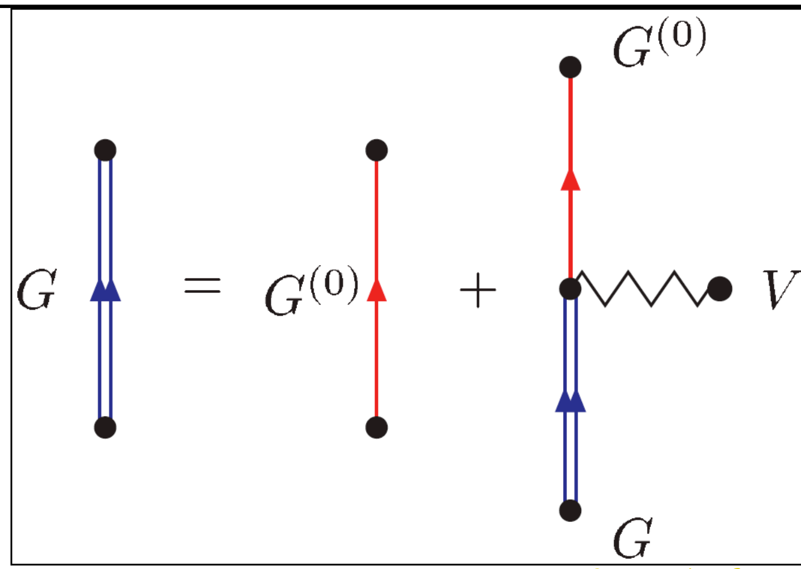
$$G^{(0)}(\alpha, \beta; E)$$

First order



$$\sum_{\gamma,\delta} G^{(0)}(\alpha, \gamma; E)\, \langle\gamma|\, V\, |\delta\rangle\, G^{(0)}(\delta, \beta; E)$$

All orders summed by



Single-particle propagator in the medium

Definition $$G(\alpha,\beta;t-t') = -\frac{i}{\hbar}\left\langle \Psi_0^N \right| T\left[a_{\alpha_H}(t) a_{\beta_H}^{+}(t') \right] \left| \Psi_0^N \right\rangle$$

with $$\hat{H}\left|\Psi_0^N\right\rangle = E_0^N \left|\Psi_0^N\right\rangle$$ for the exact ground state

and $$a_{\alpha_H}(t) = e^{\frac{i}{\hbar}\hat{H}t} a_\alpha e^{-\frac{i}{\hbar}\hat{H}t}$$ (Heisenberg picture)

while *T* orders the operators with larger time on the left including a sign change

$$G(\alpha,\beta;t-t') = -\frac{i}{\hbar}\left\{ \theta(t-t') e^{\frac{i}{\hbar}E_0^N(t-t')} \left\langle \Psi_0^N \right| a_\alpha e^{-\frac{i}{\hbar}\hat{H}(t-t')} a_\beta^{+} \left| \Psi_0^N \right\rangle \right.$$ particle

$$\left. -\theta(t'-t) e^{\frac{i}{\hbar}E_0^N(t'-t)} \left\langle \Psi_0^N \right| a_\beta^{+} e^{-\frac{i}{\hbar}\hat{H}(t'-t)} a_\alpha \left| \Psi_0^N \right\rangle \right\}$$ hole

Fourier transform of G (Lehmann representation)

$$G(\alpha,\beta;E) = \sum_m \frac{\langle \Psi_0^N | a_\alpha | \Psi_m^{N+1} \rangle \langle \Psi_m^{N+1} | a_\beta^+ | \Psi_0^N \rangle}{E - \left(E_m^{N+1} - E_0^N\right) + i\eta} \quad \Leftarrow \text{Particle part}$$

$$+ \sum_n \frac{\langle \Psi_0^N | a_\beta^+ | \Psi_n^{N-1} \rangle \langle \Psi_n^{N-1} | a_\alpha | \Psi_0^N \rangle}{E - \left(E_0^N - E_n^{N-1}\right) - i\eta} \quad \Leftarrow \text{Hole part}$$

Numerator contains information about "wave functions"

$$\langle \Psi_n^{N-1} | a_\alpha | \Psi_0^N \rangle \quad \text{and} \quad \langle \Psi_m^{N+1} | a_\beta^+ | \Psi_0^N \rangle$$

while denominator identifies eigenvalues of H for the $N\pm1$ states

Note $$\hat{H} | \Psi_n^{N\pm1} \rangle = E_n^{N\pm1} | \Psi_n^{N\pm1} \rangle$$

has been used for exact $N\pm1$ states of H

Spectral functions

Probability density for the removal of a particle with quantum numbers represented by α from the ground state, while leaving the remaining system at an energy $E_n^{N-1} = E_0^N - E$

$$S_h(\alpha;E) = \sum_n \left| \left\langle \Psi_n^{N-1} \middle| a_\alpha \middle| \Psi_0^N \right\rangle \right|^2 \delta\left(E - \left(E_0^N - E_n^{N-1}\right)\right)$$

for energies $E \le \varepsilon_F^- = E_0^N - E_0^{N-1}$

Relation of "hole" spectral function to propagator

$$S_h(\alpha;E) = \frac{1}{\pi} \mathrm{Im}\, G(\alpha,\alpha;E)$$

based on $\dfrac{1}{x \pm i\eta} = P\dfrac{1}{x} \mp i\pi\delta(x)$

Occupation number: $n(\alpha) = \int_{-\infty}^{\varepsilon_F^-} S_h(\alpha;E)\, dE = \left\langle \Psi_0^N \middle| a_\alpha^\dagger a_\alpha \middle| \Psi_0^N \right\rangle$

Relation with experimental data

Direct knockout reaction:
Transfer a large amount of momentum and energy to a bound *N*-particle system leaving an ejected fast particle and a bound *N*-1 system. By observing the momentum of the ejected particle one can reconstruct the hole spectral function.

Initial state $\left|\Psi_i\right\rangle = \left|\Psi_0^N\right\rangle$ Final state $\left|\Psi_f\right\rangle = a_{\vec{p}}^{+}\left|\Psi_n^{N-1}\right\rangle$

External probe transfers momentum $\hat{\rho}(\vec{q}) = \sum_{\vec{p}} a_{\vec{p}}^{+} a_{\vec{p}-\vec{q}}$

Transition matrix element $\left\langle \Psi_f \middle| \hat{\rho}(\vec{q}) \middle| \Psi_i \right\rangle \approx \left\langle \Psi_n^{N-1} \middle| a_{\vec{p}-\vec{q}} \middle| \Psi_0^N \right\rangle$

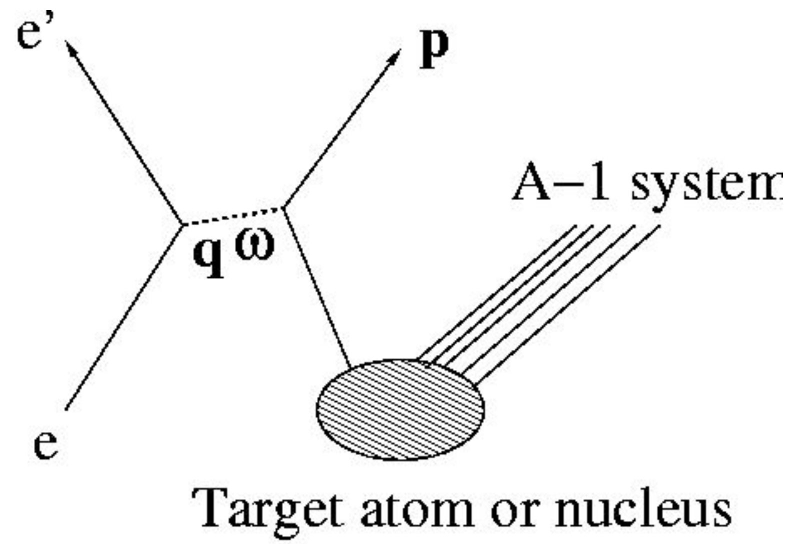
(*Plane Wave*) *Impulse Approximation* $\Rightarrow$ ejected particle absorbs ***q***

Cross section from Fermi's Golden Rule

$$d\sigma \propto \sum_n \left| \left\langle \Psi_f \middle| \hat{\rho}(\vec{q}) \middle| \Psi_i \right\rangle \right|^2 \delta\left(E + E_i - E_f\right) = S_h\left(\vec{p}_{miss}; E_{miss}\right)$$

with $\vec{p}_{miss} = \vec{p} - \vec{q}$ and $E_{miss} = \frac{\vec{p}^{\,2}}{2m} - E = E_0^N - E_n^{N-1}$

Basic idea of (e,2e) or (e,e´p)



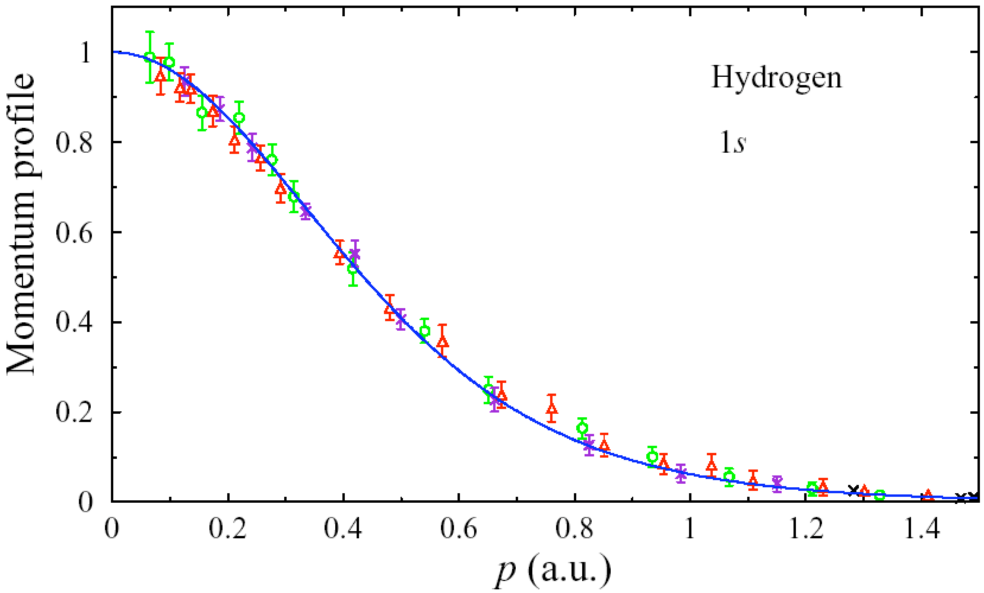
$$d\sigma_L \propto \left| \left\langle \Psi_f \left| \hat{\rho}_c(\vec{q}) \right| \Psi_i \right\rangle \right|^2 \delta(E - E_i - E_f)$$

Simplest case: $\left\langle \vec{p}, \Psi_n^{N-1} \left| \hat{\rho}_c(\vec{q}) \right| \Psi_0^N \right\rangle \Rightarrow \left\langle \Psi_n^{N-1} \left| a_{\vec{p}-\vec{q}} \right| \Psi_0^N \right\rangle$

$$\Rightarrow d\sigma_L \propto \sum_n \left\langle \Psi_0^N \left| a^+_{\vec{p}-\vec{q}} \right| \Psi_n^{N-1} \right\rangle \left\langle \Psi_n^{N-1} \left| a_{\vec{p}-\vec{q}} \right| \Psi_0^N \right\rangle \delta\left(E_{miss} - \left(E_0^N - E_n^{N-1}\right)\right)$$

Realistic case : distorted waves / more realistic description of knocked out particle

Atoms studied with the (e,2e) reaction

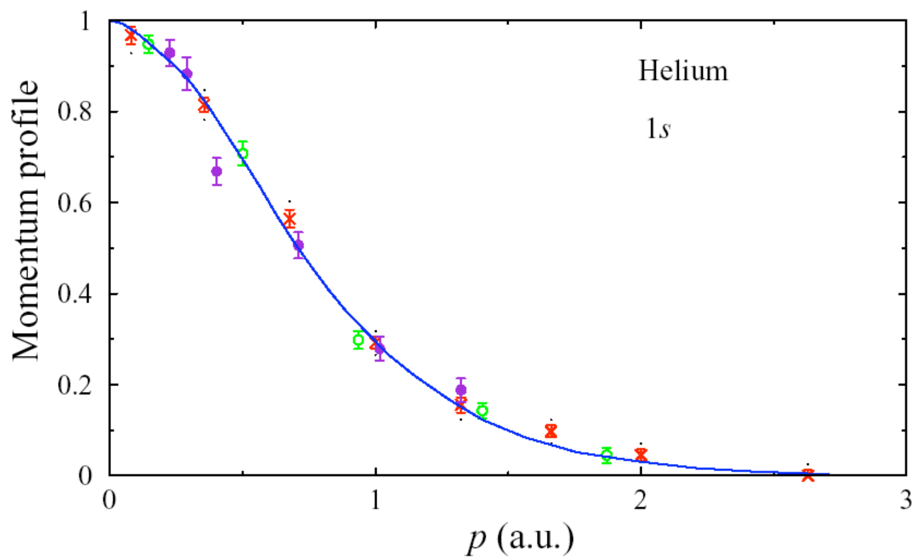


$$\varphi_{1s}(p) = 2^{3/2}\pi \frac{1}{(1+p^2)^2}$$

Hydrogen 1s wave function
"seen" experimentally
Phys. Lett. 86A, 139 (1981)

And so on for other atoms …

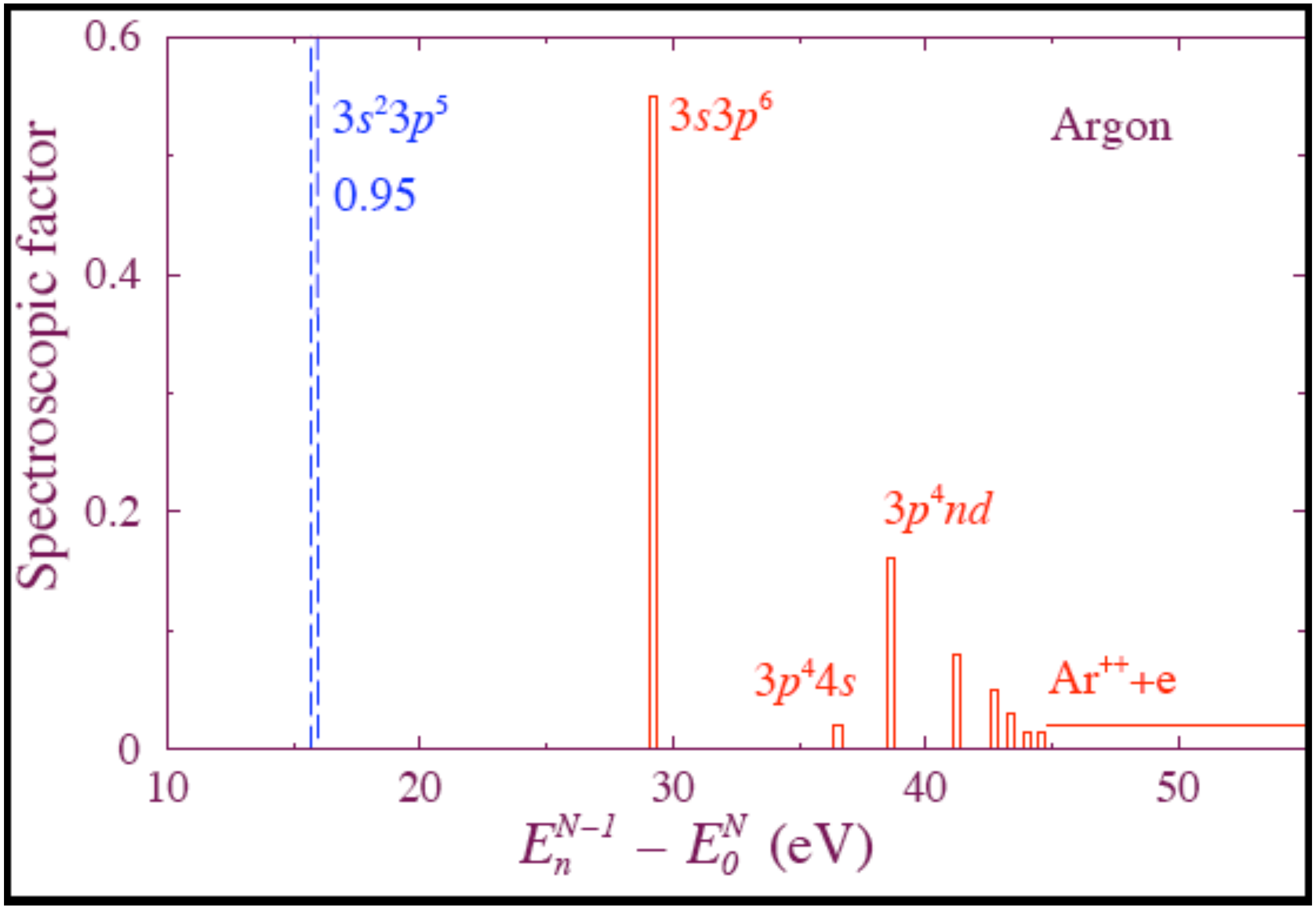
Helium
in Phys. Rev. A8, 2494 (1973)



Spectroscopic factors in atoms

For a bound final *N*-1 state the spectroscopic factor is given by $S = \int d\vec{p} \left| \left\langle \Psi_n^{N-1} \middle| a_{\vec{p}} \middle| \Psi_0^N \right\rangle \right|^2$

For H and He the 1*s* electron spectroscopic factor is 1
For Ne the valence 2*p* electron has *S*=0.92 with two additional fragments, each carrying 0.04, at higher energy.

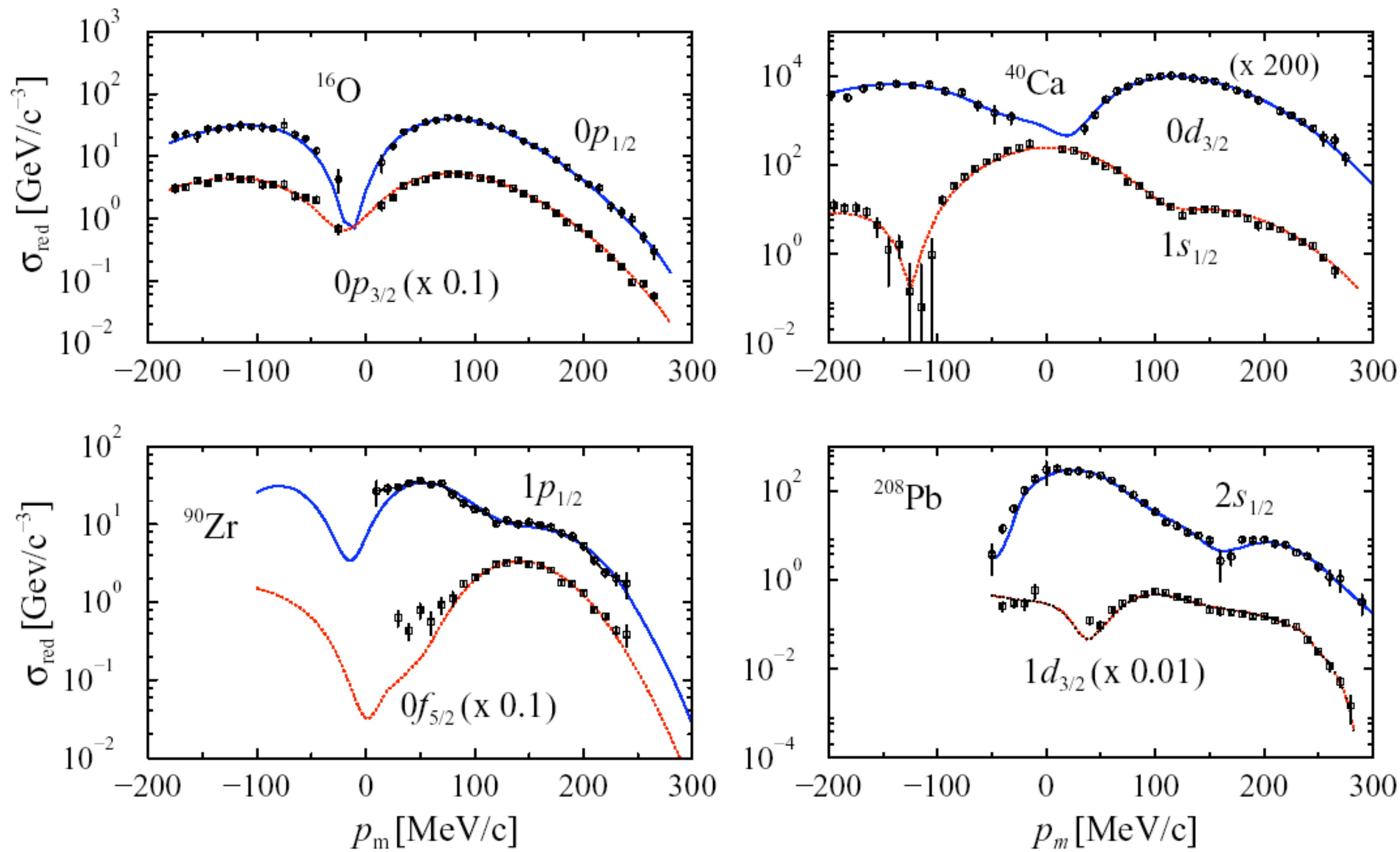


Argon
3*p* and 3*s* strength

Closed-shell atoms
n(α) = 0 or 1

(e,e´p) cross sections for closed-shell nuclei

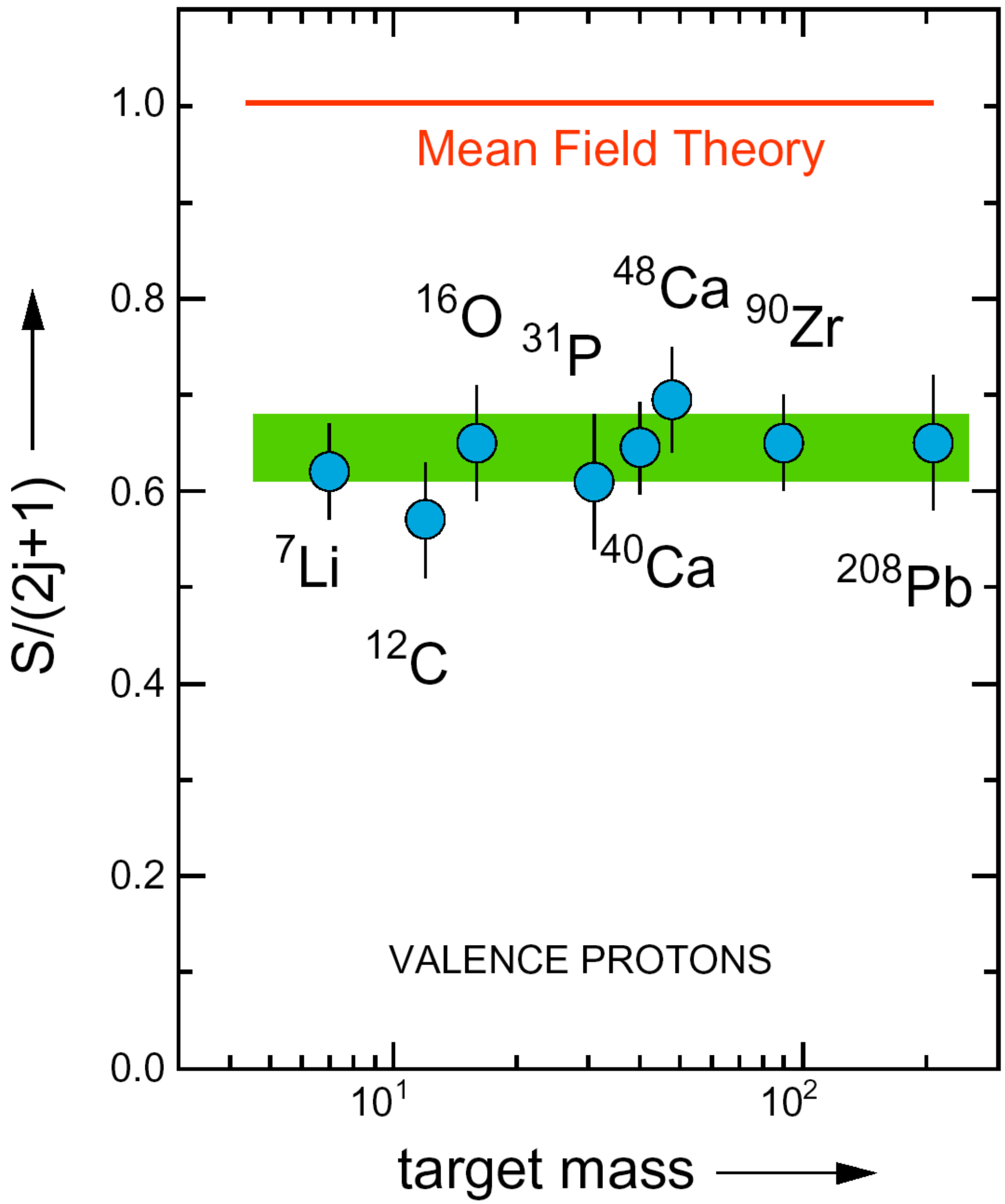
NIKHEF data, L. Lapikás, Nucl. Phys. A553, 297c (1993)



Except ….

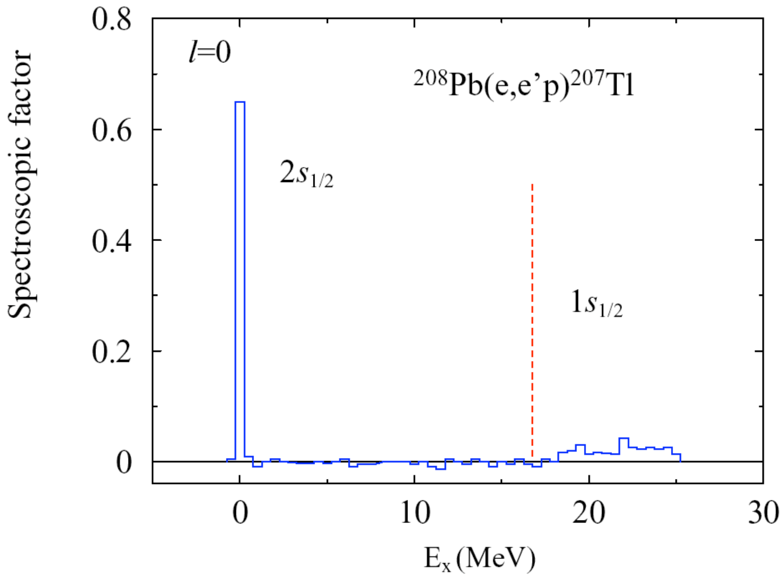
Removal probability for valence protons from NIKHEF data

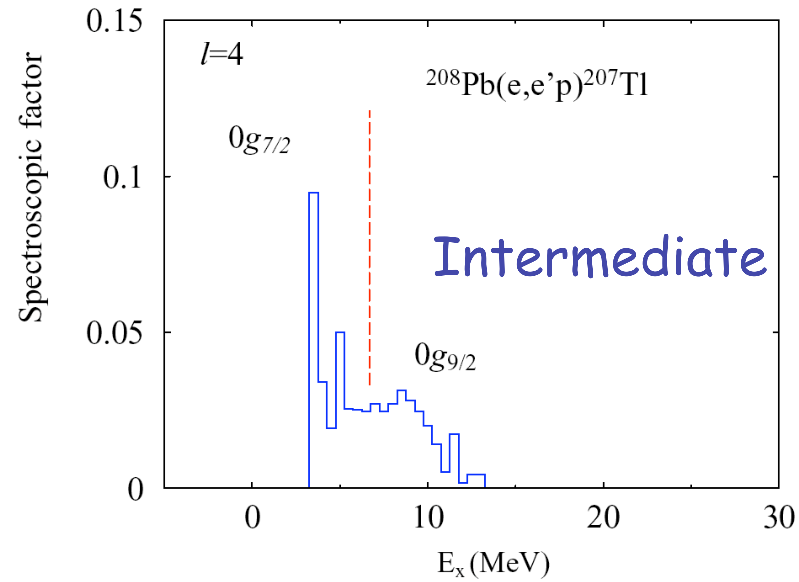
Note:
We have seen mostly data for removal of **valence protons**



and …

E. Quint, Ph.D.thesis NIKHEF, 1988



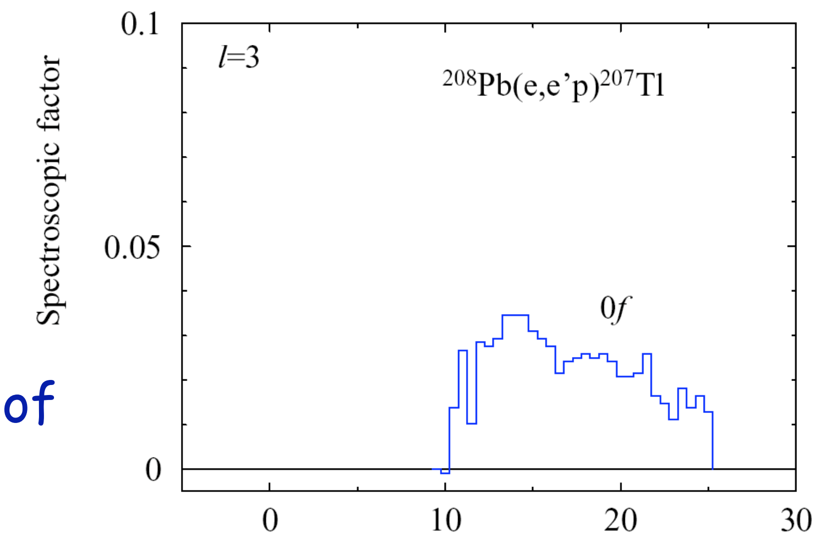


Intermediate

Quasihole strength or spectroscopic factor $Z(2s_{1/2})$ =0.65

$n(2s_{1/2})$ = 0.75

from elastic electron scattering



Strong fragmentation of deeply-bound states

Many-body perturbation theory for *G*

- Identify solvable problem by considering $\hat{H}_0 = \hat{T} + \hat{U}$ where *U* is a suitable auxiliary potential.
- Develop expansion in $\hat{H}_1 = \hat{V} - \hat{U}$
- Employs time-evolution, Heisenberg, Schrödinger, and interaction picture of quantum mechanics.
- Once established, this expansion (expressed in Feynman diagrams) is organized in such a way that nonperturbative results can be obtained leading to the Dyson equation. The Dyson equation describes sp motion in the medium under the influence of the self-energy which is an energy-dependent complex sp potential.
- Further insight into the proper description of sp motion in the medium is obtained by studying the relation between sp and two-particle propagation. This allows the selection of appropriate choices of the relevant ingredients for the system under study.

How to calculate G?

Rearrange Hamiltonian $$\hat{H} = \hat{T} + \hat{V} = \left(\hat{T} + \hat{U}\right) + \left(\hat{V} - \hat{U}\right) = \hat{H}_0 + \hat{H}_1$$

Many-body problem with H_0 can be exactly solved
when U is a one-body potential like a Woods-Saxon or HO potential.
Corresponding sp propagator (replace H by H_0)

$$G^{(0)}(\alpha,\beta;E) = \sum_m \frac{\left\langle \Phi_0^N \middle| a_\alpha \middle| \Phi_m^{N+1} \right\rangle \left\langle \Phi_m^{N+1} \middle| a_\beta^\dagger \middle| \Phi_0^N \right\rangle}{E - \left(E_m^{A+1} - E_{\Phi_0^N}\right) + i\eta} + \sum_n \frac{\left\langle \Phi_0^N \middle| a_\beta^\dagger \middle| \Phi_n^{N-1} \right\rangle \left\langle \Phi_n^{N-1} \middle| a_\alpha \middle| \Phi_0^N \right\rangle}{E - \left(E_{\Phi_0^N} - E_n^{A-1}\right) - i\eta}$$

$$= \delta_{\alpha,\beta}\left[\frac{\theta(\alpha - F)}{E - \varepsilon_\alpha + i\eta} + \frac{\theta(F - \alpha)}{E - \varepsilon_\alpha - i\eta}\right]$$

using the sp basis associated with H_0. Note that $\hat{H}_0 a_\alpha^+ \left|\Phi_0^N\right\rangle = \left(E_{\Phi_0^N} + \varepsilon_\alpha\right) a_\alpha^+ \left|\Phi_0^N\right\rangle$

$$\hat{H}_0 a_\alpha \left|\Phi_0^N\right\rangle = \left(E_{\Phi_0^N} - \varepsilon_\alpha\right) a_\alpha \left|\Phi_0^N\right\rangle$$

So that e.g. $S_h^{(0)}(\alpha;E) = \frac{1}{\pi} \mathrm{Im}\, G^{(0)}(\alpha,\alpha;E) = \delta(E - \varepsilon_\alpha)\theta(F - \alpha)$ ~ like in atoms

and $n^{(0)}(\alpha) = \int_{-\infty}^{\varepsilon_F^{(0)-}} dE\, \delta(E - \varepsilon_\alpha)\theta(F - \alpha) = \theta(F - \alpha)$

Perturbation expansion using $G^{(0)}$ and H_1

Use "interaction picture" $\hat{H}_1(t) = e^{\frac{i}{\hbar}\hat{H}_0 t}\hat{H}_1 e^{-\frac{i}{\hbar}\hat{H}_0 t}$

then ……

$$G(\alpha,\beta;t-t') = -\frac{i}{\hbar}\sum\left(\frac{-i}{\hbar}\right)^m \frac{1}{m!}\int dt_1 \cdots \int dt_m \left\langle \Phi_0^N \left| T\left[\hat{H}_1(t_1)\cdots\hat{H}_1(t_m)a_\alpha(t)a_\beta^+(t')\right]\right| \Phi_0^N \right\rangle_{connected}$$

Can be calculated order by order using diagrams and Wick's theorem.
Yields expressions involving $G^{(0)}$ and matrix elements
of the two-body interaction V (and the auxiliary potential U)

Simple diagram rules in time formulation.

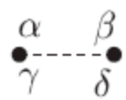
For practical calculations use energy formulation. Diagrams

Diagram rules in energy formulation

Rule 1 Draw all topologically distinct (direct) and connected diagrams with m horizontal interaction lines for V (dashed) and $2m+1$ directed (using arrows) Green's functions $G^{(0)}$

Rule 2 Label external points only with sp quantum numbers, *e.g.* α and β

Label each interaction with sp quantum numbers



$$\Rightarrow \langle\alpha\beta|\, V\, |\gamma\delta\rangle = (\alpha\beta|V|\gamma\delta) - (\alpha\beta|V|\delta\gamma)$$

For each arrow line one writes

$$\Rightarrow G^{(0)}(\mu,\nu;E)$$

but in such a way that energy is conserved for each V

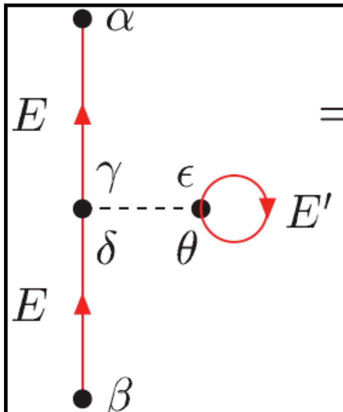
Rule 3 Sum (integrate) over all internal sp quantum numbers and integrate over all m internal energies

For each closed loop an independent energy integration occurs over the contour $C\uparrow$

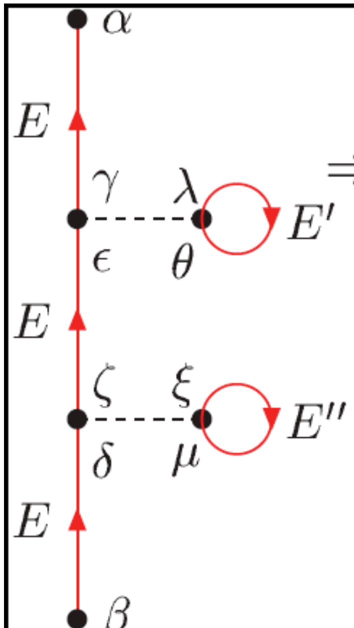
Rule 4 Include a factor $(i/2\pi)^m$ and $(-1)^F$ where F is the number of closed fermion loops

Rule 5 Include a factor of $\frac{1}{2}$ for each equivalent pair of lines

Examples of diagrams

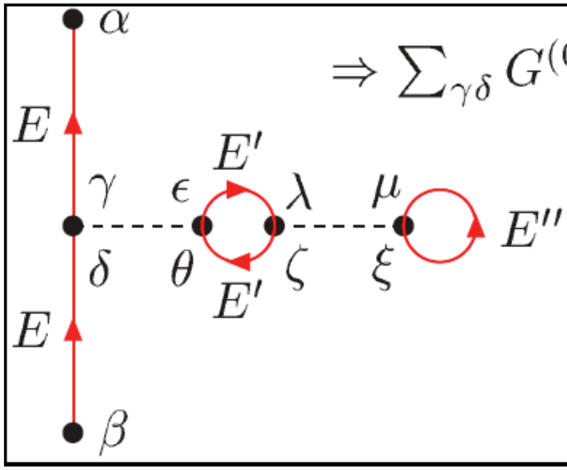


$$\Rightarrow \sum_{\gamma\delta} G^{(0)}(\alpha,\gamma;E)$$
$$\times \; -i\sum_{\epsilon\theta}\langle\gamma\epsilon|\,V\,|\delta\theta\rangle \int_{C\uparrow}\frac{dE'}{2\pi}G^{(0)}(\theta,\epsilon;E')$$
$$\times \; G^{(0)}(\delta,\beta;E)$$



$$\Rightarrow \; \sum_{\gamma\delta} G^{(0)}(\alpha,\gamma;E)$$
$$\times \; (-1)^2 i^2 \sum_{\epsilon,\zeta}\sum_{\lambda,\theta}\int_{C\uparrow}\frac{dE'}{2\pi}\langle\gamma\lambda|\,V\,|\epsilon\theta\rangle\, G^{(0)}(\theta,\lambda;E')$$
$$\times \; G^{(0)}(\epsilon,\zeta;E)\sum_{\xi,\mu}\int_{C\uparrow}\frac{dE''}{2\pi}\langle\zeta\xi|\,V\,|\delta\mu\rangle\, G^{(0)}(\mu,\xi;E'')$$
$$\times \; G^{(0)}(\delta,\beta;E)$$

More diagrams

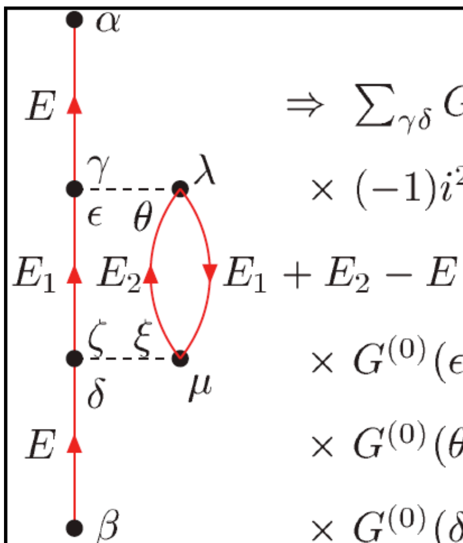


$$\Rightarrow \sum_{\gamma\delta} G^{(0)}(\alpha,\gamma;E) \times\ i^2 \sum_{\epsilon\theta} \sum_{\lambda\zeta} \int_{C\uparrow} \frac{dE'}{2\pi}$$

$$\times\ \langle\gamma\epsilon|\, V\, |\delta\theta\rangle\, G^{(0)}(\lambda,\epsilon;E')G^{(0)}(\theta,\zeta;E')$$

$$\times\ \sum_{\mu\xi} \int_{C\uparrow} \frac{dE''}{2\pi} \langle\zeta\xi|\, V\, |\lambda\mu\rangle\, G^{(0)}(\mu,\xi;E'')$$

$$\times\ G^{(0)}(\delta,\beta;E)$$



$$\Rightarrow\ \sum_{\gamma\delta} G^{(0)}(\alpha,\gamma;E)$$

$$\times\ (-1)i^2 \tfrac{1}{2} \int \frac{dE_1}{2\pi} \int \frac{dE_2}{2\pi} \sum_{\lambda,\epsilon,\theta} \sum_{\zeta,\xi,\mu} \langle\gamma\lambda|\, V\, |\epsilon\theta\rangle$$

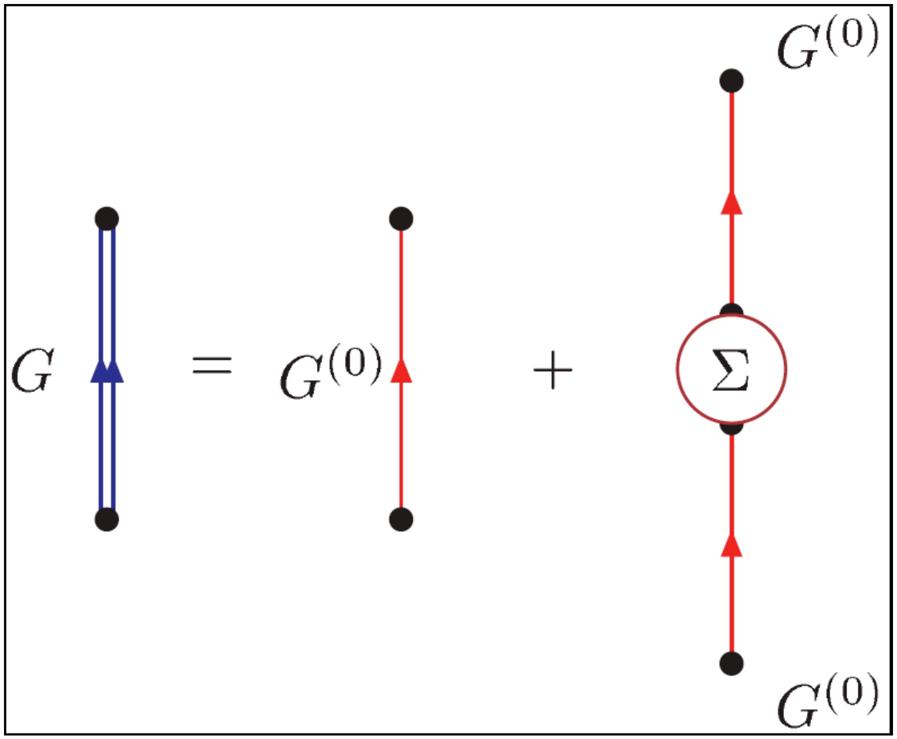
$$\times\ G^{(0)}(\epsilon,\zeta;E_1)G^{(0)}(\mu,\lambda;E_1+E_2-E)$$

$$\times\ G^{(0)}(\theta,\xi;E_2)\, \langle\zeta\xi|\, V\, |\delta\mu\rangle$$

$$\times\ G^{(0)}(\delta,\beta;E)$$

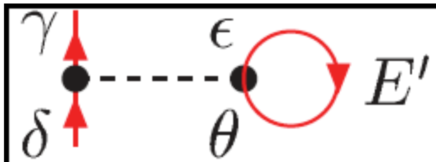
Diagram organization

Sum of all diagrams can be written as



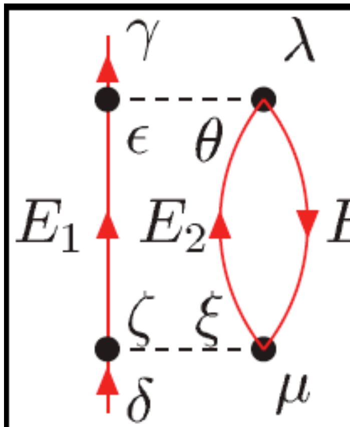
Introducing some self-energy diagrams

First order



$$\Rightarrow -i \sum_{\epsilon\theta} \langle \gamma\epsilon | V | \delta\theta \rangle \int_{C\uparrow} \frac{dE'}{2\pi} G^{(0)}(\theta, \epsilon; E')$$

One of the second order diagrams



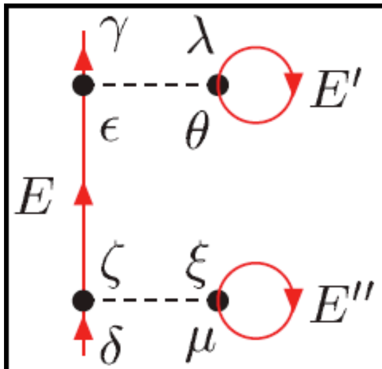
$$\Rightarrow (-1) i^2 \frac{1}{2} \int \frac{dE_1}{2\pi} \int \frac{dE_2}{2\pi} \sum_{\lambda,\epsilon,\theta} \sum_{\zeta,\xi,\mu} \langle \gamma\lambda | V | \epsilon\theta \rangle$$

$$\times G^{(0)}(\epsilon, \zeta; E_1) G^{(0)}(\mu, \lambda; E_1 + E_2 - E)$$

$$\times G^{(0)}(\theta, \xi; E_2) \langle \zeta\xi | V | \delta\mu \rangle$$

The irreducible self-energy

The following self-energy diagram is reducible (previous two were irreducible), *i.e.* can be obtained from lower order self-energy terms by iterating with $G^{(0)}$



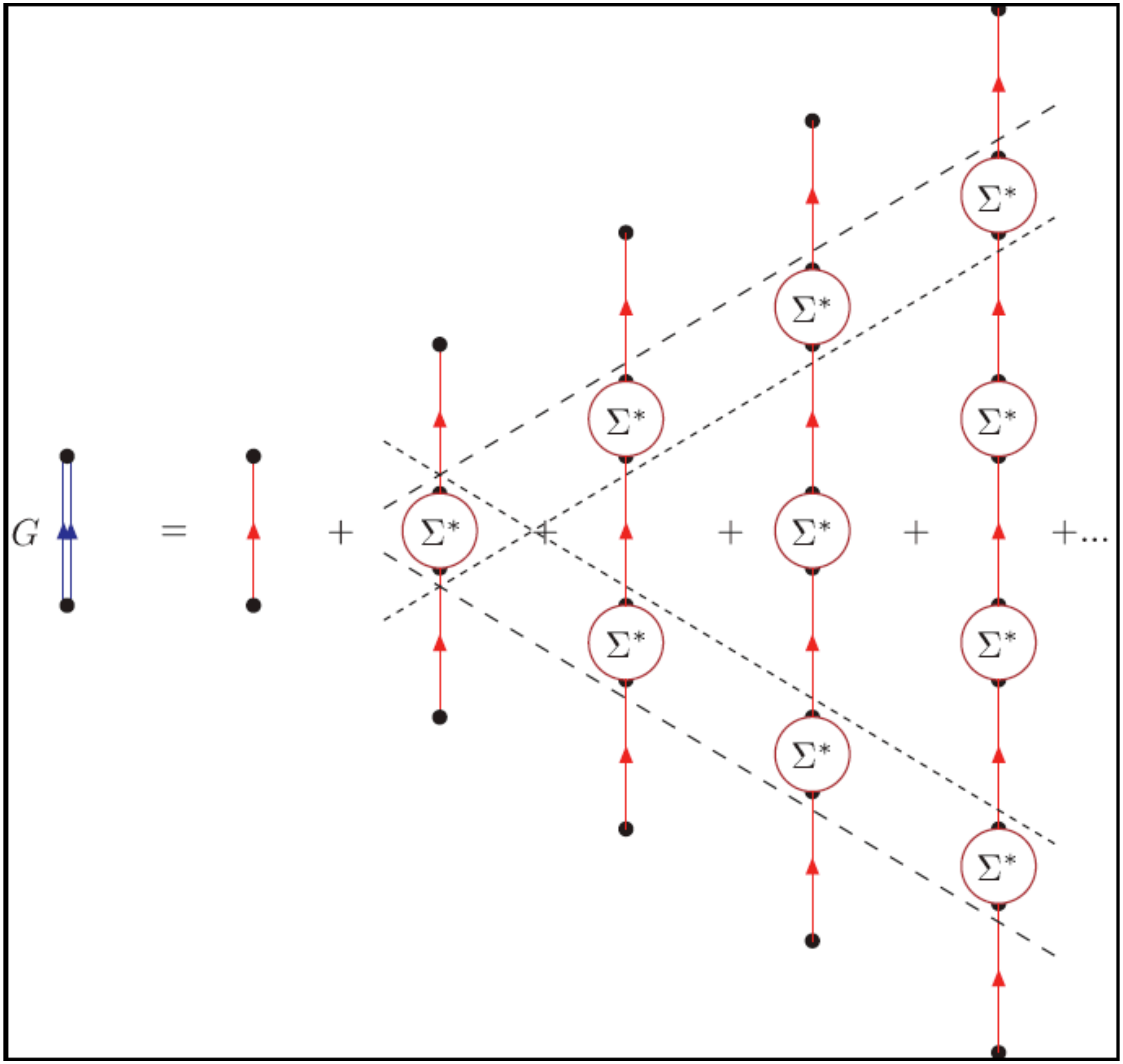
$$\Rightarrow (-1)^2 i^2 \sum_{\epsilon,\zeta} \sum_{\lambda,\theta} \int_{C\uparrow} \frac{dE'}{2\pi} \langle \gamma\lambda | V | \epsilon\theta \rangle G^{(0)}(\theta,\lambda;E')$$
$$\times G^{(0)}(\epsilon,\zeta;E) \sum_{\xi,\mu} \int_{C\uparrow} \frac{dE''}{2\pi} \langle \zeta\xi | V | \delta\mu \rangle G^{(0)}(\mu,\xi;E'')$$

Sum of all irreducible diagrams is denoted by Σ^*.
All diagrams can then be obtained by summing

$$G(\alpha,\beta;E) = G^{(0)}(\alpha,\beta;E) + \sum_{\gamma,\delta} G^{(0)}(\alpha,\gamma;E)\Sigma^*(\gamma,\delta;E)G^{(0)}(\delta,\beta;E) + \cdots$$

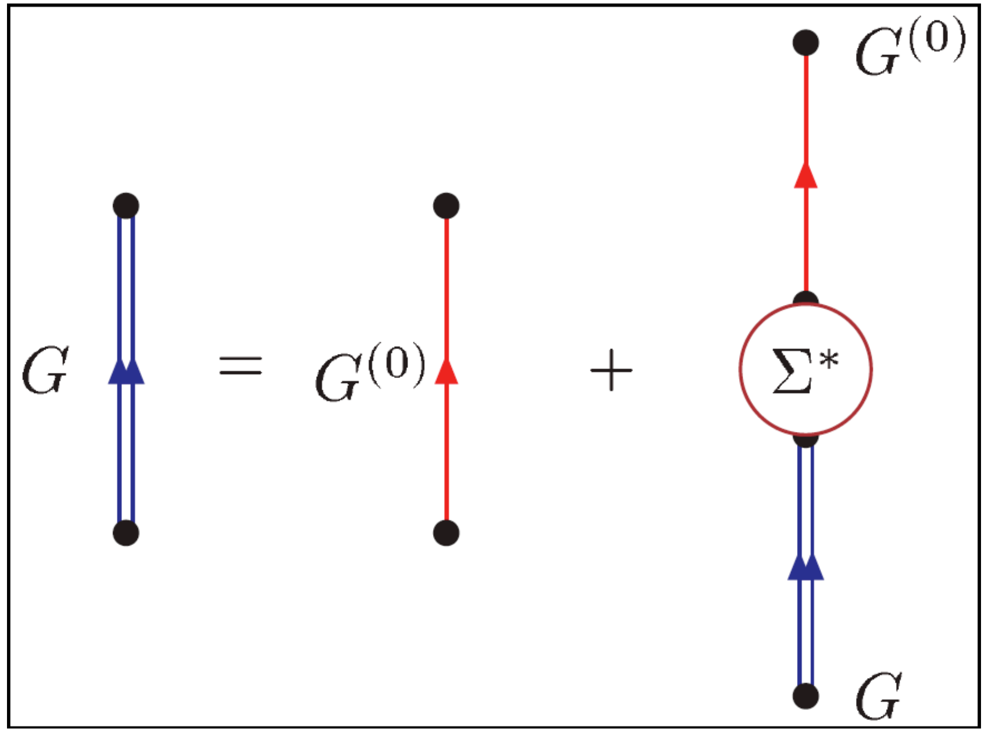
diagrammatically …

Towards the Dyson equation



Can be summed by

Dyson equation



Looks like the propagator equation for a single particle

$$G(\alpha,\beta;E) = G^{(0)}(\alpha,\beta;E) + \sum_{\gamma,\delta} G^{(0)}(\alpha,\gamma;E)\Sigma^*(\gamma,\delta;E)G(\delta,\beta;E)$$

with the irreducible self-energy acting as the in-medium (complex) potential.

Homework

Recover the time-independent Schrödinger equation for bound states from

$$G(\alpha,\beta;E) = G^{(0)}(\alpha,\beta;E) + \sum_{\gamma\delta} G^{(0)}(\alpha,\gamma;E)\langle\gamma|V|\delta\rangle G(\delta,\beta;E)$$

in momentum space for a particle without spin

$$G^{(0)}(\alpha,\beta;E) = \langle\alpha|\frac{1}{E - H_0 + i\eta}|\beta\rangle$$ can then be written as

$$G^{(0)}(\vec{p},\vec{p}\,';E) = \langle\vec{p}|\frac{1}{E - \frac{\vec{p}_{op}^{\,2}}{2m} + i\eta}|\vec{p}\,'\rangle = \delta(\vec{p}-\vec{p}\,')\frac{1}{E - \frac{\vec{p}^{\,2}}{2m} + i\eta}$$

Strategy:

- Introduce complete set of eigenstates of *H* in *G*
- Calculate $\lim_{E\to\varepsilon_n}(E-\varepsilon_n)\left[G = G^{(0)} + G^{(0)}VG\right]$

  with $H|n\rangle = \varepsilon_n|n\rangle$ and $\varepsilon_n < 0$