

Physics 217

Midterm Exam

- (a) Compton scattered X-rays off of electrons.

(b) Compton observed that the X-rays scattered off of the target had wavelength peaks in two locations, one at the wavelength of the initial X-ray and a second at some shifted wavelength (λ').

(c) Compton's experiments were surprising because of the two peaks which occurred. Physicists of that time thought light behaved only wavelike and this could not explain the two peaks.

(d) The Compton effect tells us that light demonstrates particle like properties.
- (a) Inside of the well the Hamiltonian is $\hat{H} = \frac{\hat{p}^2}{2m}$. Outside of the well the potential energy is infinite which gives rise to the necessary boundary conditions (there can be no wave function outside the well). The boundary conditions are $\psi(x = a) = \psi(x = 0) = 0$.

(b) $\hat{H}\psi(x) = \frac{\hat{p}^2}{2m}\psi(x) = \frac{\hbar^2 k^2}{2m}\psi(x)$. This works if the $\psi(x)$ satisfies the boundary conditions which means that $k = \frac{n\pi}{a}$ must be true.

(c) From part (b) we see that the energy eigenvalues are $\frac{\hbar^2 k^2}{2m} = \frac{n^2 \pi^2 \hbar^2}{2ma^2}$.

(d) $\hat{p}\psi(x) = \frac{-i\hbar n\pi}{a} \sqrt{\frac{2}{a}} \cos(kx) \neq \hat{p}\psi(x)$. This works because $\psi(x) = \sin(kx)$ is a linear combination of momentum eigenfunctions (e^{ikx}) meaning that $\psi(x)$ itself is not a momentum eigenfunction.

(e) Adding the normal time dependence we get $\Psi(x, t) = \frac{1}{2}\psi_1(x)e^{-iE_1 t/\hbar} - \frac{\sqrt{3}}{2}\psi_2(x)e^{-iE_2 t/\hbar}$. The time-dependent Schrödinger equation states $-i\hbar \frac{\partial}{\partial t} \Psi = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \Psi$. Putting our

wavefunction into this equation the left hand side looks like $-i\hbar\Psi = -\frac{E_1}{2}\psi_1(x)e^{-iE_1t/\hbar} + \frac{\sqrt{3}E_2}{2}\psi_2(x)e^{-iE_2t/\hbar}$. The left hand side looks like $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi = -\frac{1}{2}\frac{\hbar^2k_1^2}{2m}\psi_1(x)e^{-iE_1t/\hbar} + \frac{\sqrt{3}}{2}\frac{\hbar k_2^2}{2m}\psi_2(x)e^{-iE_2t/\hbar} = -\frac{E_1}{2}\psi_1(x)e^{-iE_1t/\hbar} + \frac{\sqrt{3}E_2}{2}\psi_2(x)e^{-iE_2t/\hbar}$. We see that the left and right sides of the equation are equivalent thus this wavefunction satisfies the time-dependent Schrödinger equation.

3. (a) $\int_{-\infty}^{\infty} |\Psi|^2 dx = 2$ thus this wavefunction is not normalized. In order to normalize we must multiply it by $1/\sqrt{2}$ obtaining the wavefunction $\Psi(x, t = 0) = \sqrt{q}e^{-q|x|}$ which is normalized. The units of q are inverse length.

(b) $\tilde{\psi}(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \sqrt{q}e^{-q|x|}e^{ikx}dx$. Making use of the hint provided you obtain $\tilde{\psi}(k) = q^{3/2} \sqrt{\frac{2}{\pi}} \left[\frac{1}{q^2+k^2} \right]$.

(c) $\langle \hat{p} \rangle = \int_{-\infty}^{\infty} \tilde{\psi}^*(k)(\hbar k)\tilde{\psi}(k)dk$. Computing this integral you get an expectation value of $\langle \hat{p} \rangle = 0$.

(d) For free particles the energy eigenvalues are $\frac{\hbar^2k^2}{2m}$. With this you find $\Psi(x, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{\psi}(k)e^{-ikx + \frac{\hbar^2k^2}{2m}t} dk$