Estimating Uncertainty

I. Reporting Measurements

A measured value of a parameter should be reported in this form:

$$X = X_{best} \pm \Delta X$$

II. Uncertainty in a single measurement

Many instruments will have an uncertainty value given by the manufacturer. For other tools, a useful rule of thumb gives that the uncertainty of a measurement tool is half of the smallest increment that tool can measure.

III. Uncertainty in multiple measurements of the same quantity

If on the other hand, the best estimate of a parameter is determined by making repeated measurements and computing the average value from the multiple trials, the uncertainty associated with each measurement can be determined from the ***standard deviation, σ***. For n measurements of a quantity x, the standard deviation can be expressed as:

$$\sigma = \sqrt{\frac{\sum(x - \overline{x})^2}{n - 1}}$$

IV. Combining uncertainties in calculations

When calculating a value from several measured quantities, the uncertainty of your calculated value can be found using the following rules:

Addition/Subtraction:

If several quantities $x, \ldots, w$ are measured with uncertainties $\Delta x, \ldots, \Delta w$, and the measured values are used to compute

$$q = x + \cdots + z - (u + \cdots + w),$$

then the uncertainty in the computed value of q is the quadratic sum,

$$\Delta q = \sqrt{(\Delta x^2) + \cdots + (\Delta z^2) + (\Delta u^2) + \cdots + (\Delta w^2)}$$

of all the original uncertainties.

Multiplication/Division:

If several quantities x, …, w are measured with uncertainties $\Delta x, \ldots, \Delta w$, and the measured values are used to compute

$$q = \frac{x \cdot \ldots \cdot z}{u \cdot \ldots \cdot w}$$

then the *fractional uncertainty* in the computed value of q is the quadratic sum,

$$\frac{\Delta q}{q} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \cdots + \left(\frac{\Delta z}{z}\right)^2 + \left(\frac{\Delta u}{u}\right)^2 + \cdots + \left(\frac{\Delta w}{w}\right)^2}$$

of the *fractional uncertainties* in x, …, w.

If both addition/subtraction and multiplication/division are included, calculate the uncertainty in several steps. Exact numbers like 2, π, g, etc. do not contribute to the uncertainty of calculations that contain them.

Power:

If a quantity x *is* measured with uncertainties Δx and the measured value is used to compute

$$q = x^n$$

then the uncertainty in the computed value of q is

$$\frac{\Delta q}{q} = n\frac{\Delta x}{x}$$

V. Comparing values quantitatively

Percentage error is used when the theoretical value of a physical quantity is known. However, you are asked to perform an experiment to find the experimental value. Then, you want to know how accurate your experimental value is. One example can be the free fall acceleration g=9.81 $\frac{m}{s^2}$. The formula used to calculate the precentage error is below.

$$percentage\ error = \left|\frac{experimental\ value - theoretical\ value}{theoretical\ value}\right| \times 100\%$$

Percentage difference is used when the quantity measured does not have a theoretical value. At this case, you are comparing two measured values. The formula is below. Terms x_1 and x_2 represent any two measured values of that physical quantity.

$$percentage\ difference = \frac{|x_1 - x_2|}{\left(\frac{x_1 + x_2}{2}\right)} \times 100\%$$