

Geometric Optics

Background

This experiment deals with image formation with lenses. We will use what are referred to as *thin lenses*. Thin lenses are ordinary lenses like eyeglasses and magnifiers, but are called thin because their thicknesses are quite small compared to the optical distances involved. The position and characteristics of the image formed by a thin lens can be worked out by either ray tracing or using the thin-lens equation. Ray tracing develops physical intuition about how lenses form images while the thin lens equation provides a quantitative method for characterizing the properties of objects and their images. Questions typically associated with image formation are:

- Where is the image?
- Is it real or virtual?
- Is it upright or inverted?
- Is it magnified or reduced?

Both surfaces of any thin-lens are spherical in shape. **Converging** lenses are thicker in the middle, while **diverging** lenses are thicker at the rim (see Fig. 1). The principle axis of any lens is the symmetry axis through its center. As the lens names imply, converging lenses bend light toward the principle axis while diverging lenses bend light away.

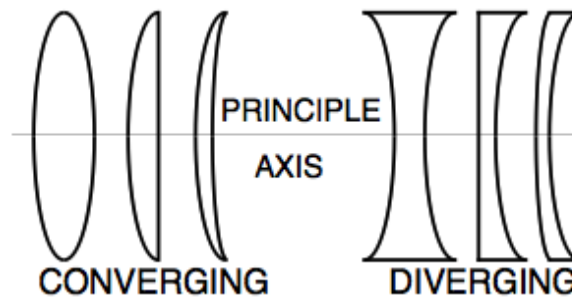


Figure 1:
Converging & Diverging Lenses

Every thin lens has two principle focus points F and F' , equidistant from the center of the lens along its principle axis. The distance from the center of the lens to either focus is the focal length, f , of the lens. Fig. 2 illustrates the significance of the principle focus points. Light rays parallel to the principle axis entering a lens from the left are converged toward and through the far focus point F' on the right side for a converging lens (Fig. 2a), or are diverged as if to come from the near focus point F on the left for a diverging lens (Fig. 2b). Similarly, for a converging lens, light rays approaching the lens through the near focus (F), will exit the lens parallel to the principle axis (Fig. 2c). For a diverging lens, rays approaching the lens from the left aligned with the far focus (F') will also exit the lens parallel to the principle axis (Fig. 2d). Note that in all cases, a converging lens bends light toward the principle axis, while a diverging lens bends it away. It does not matter which direction light passes through a thin lens; the pattern of convergence or divergence of rays is unchanged if the lens is reversed.

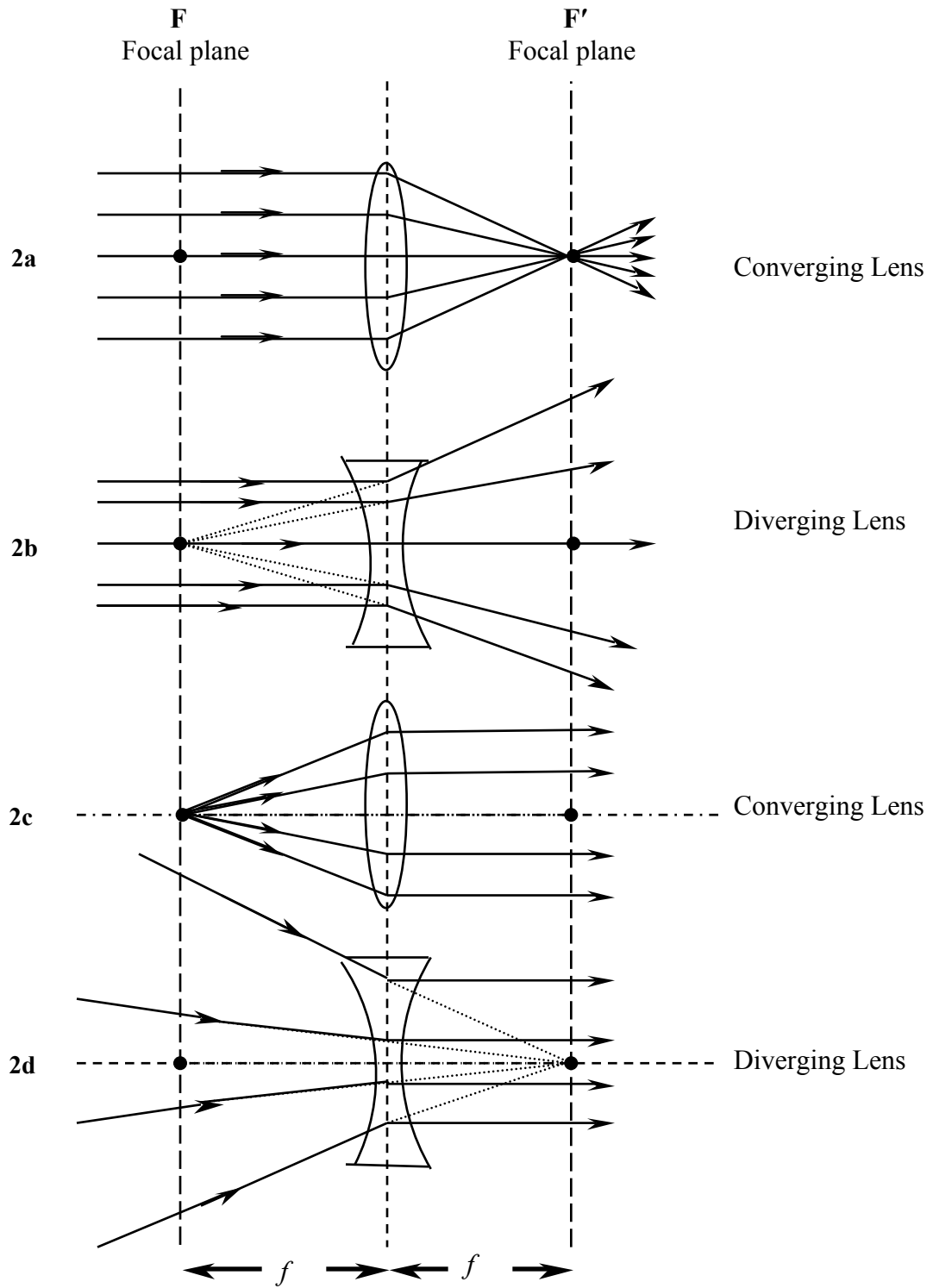


Figure 2:
Examples of how lenses bend light.
Examples of Focal Points

How are things different when parallel rays enter a thin lens, but in a direction NOT parallel to the principle axis? The answer for both converging and diverging lenses is presented in Fig. 3.

The lens converges the rays to (or diverges them from) a point which is OFF the principle axis but still at the same distance f from the lens as focal points F and F' .

One speaks of the *focal planes* of a lens, which are imaginary planes through the focal points and which are perpendicular to the principle axis. Every lens has two focal planes, one on either side. In both drawings in Fig. 3, the ray intersections caused by the lens for parallel-ray incident light occur in one of the focal planes. The point where this intersection occurs in the focal plane depends upon the direction of the incident rays: the greater their angle relative to the principle axis, the farther off the principle axis the intersection occurs.

We note that the ray passing through the center of the lens is not bent (see Figures 3a and 3b); this ray allows an easy determination of the location of the ray intersection on the focal plane.

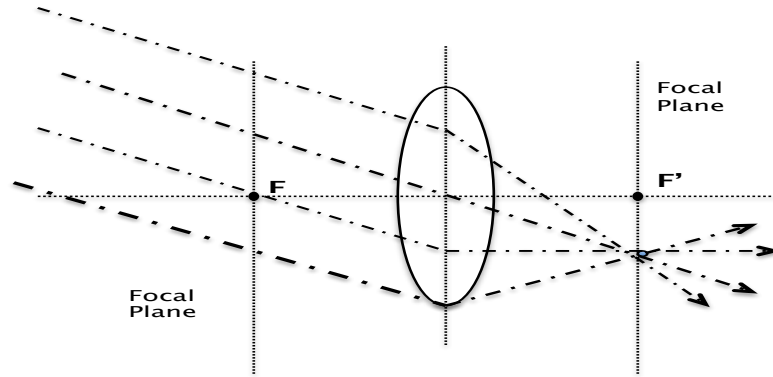


Fig. 3a

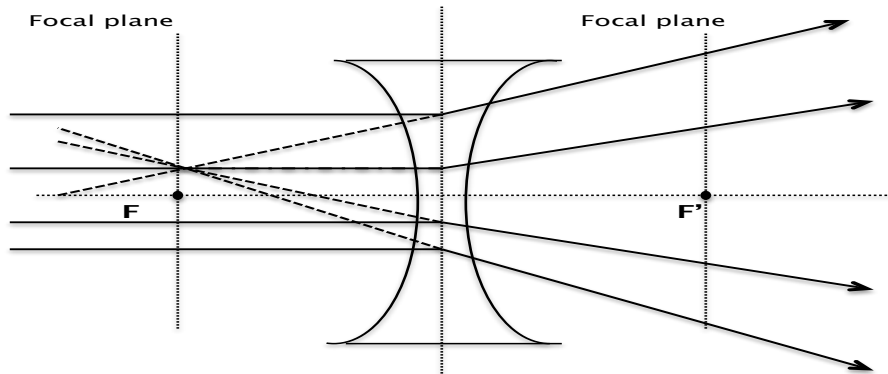


Fig. 3b

Figure 3:

When parallel rays enter the lens at an angle relative to the principle axis, they converge to (for a converging lens-figure 3a) or diverge from (for a diverging lens-figure 3b) a point in the focal plane.

Finally, consider what happens when the light appears to come from a single point, rather than from parallel rays. The answer is presented pictorially in Figs. 4a and 4b: the lens bends the rays so that, after they emerge from the lens, they either converge toward some new point P' and pass through it (Fig. 4a), or they appear as if they come from some new point P' (Fig. 4b). Know that Figs. 4a and 4b are not the only possibilities - you will encounter other scenarios in this lab.

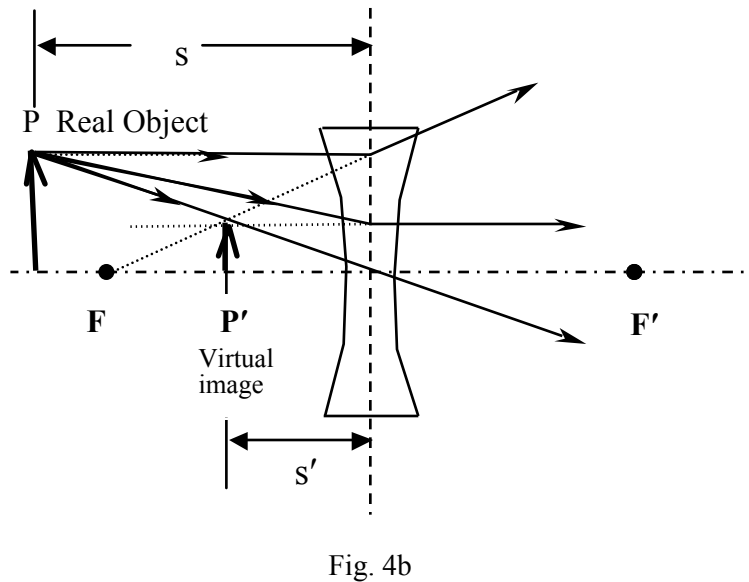
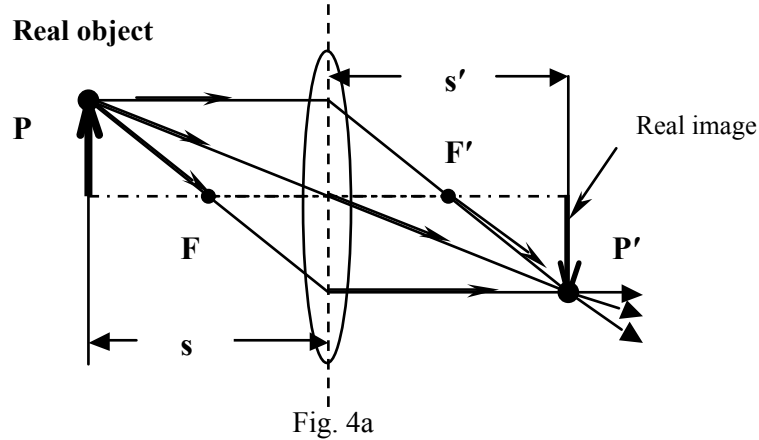


Figure 4
Image formation with converging and diverging lenses

A real object P as in Fig.4a is one from which light rays actually emanate (either by reflecting or transmitting light). The distance of the object from the center of the lens is represented by the symbol s . The object distance is positive for real objects and negative for virtual objects (more on that later). The lens forms an image of P at a point P' .

We define the distance of P' from the lens as the image distance, s' . In those cases where the emerging rays actually go through point P' , we speak of the formation of a *real* image. When the emerging rays merely *appear* to be coming from a point P' which is back behind the lens, we speak instead of the formation of a *virtual* image. An observer whose eye receives the emerging light well beyond P' (and to the right of the lens) will conclude that the source of the light is at P' , whereas, in fact, it originates from the object point, P .

Locating Images through Ray- Tracing

Figures 5 and 6 illustrate how to locate the position of image point P' that corresponds to any object point P , simply by drawing three rays, that start from P , pass through the lens, and finally intersect at P' . In principle, we could compute the path of any ray, but the following three can be drawn easily:

- 1) A ray from P , which approaches the lens parallel to its principle axis. A converging lens bends this ray through the farther focal point F' [see Fig. 5(a)]. A diverging lens bends this ray to appear as if it came from the nearer focal point F [see Fig. 5(b)].
- 2) A ray from P , which heads for the center of the lens. For both converging and diverging lenses, this ray passes directly through the center of the lens without being bent.
- 3) A ray from P , which passes through the nearer focal point F of a converging lens, or, a ray from P which appears to be headed for the farther focal point F' of a diverging lens:
In either case, the lens bends such a ray so that it emerges parallel to the principle axis.

In each drawing, two object points are shown: one on the principle axis (Q) and the other off-axis but at the same distance from the lens (P). We can think of P as being at the tip of an object arrow, while Q is at its base (the image points are P' and Q' respectively). In each drawing, the simple rays have been drawn for finding only the tip of the arrow.

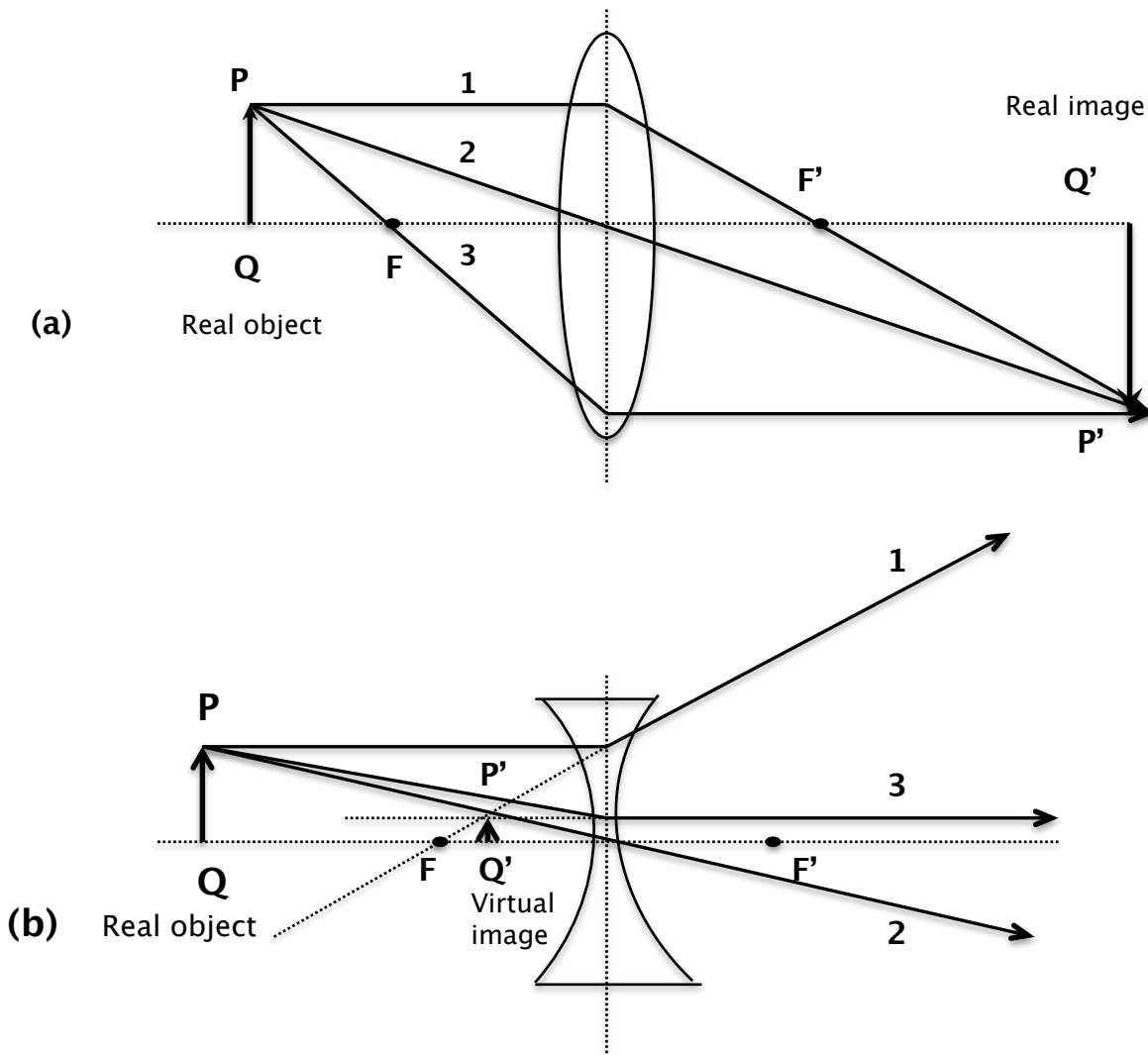


Figure 5: Various Real & Virtual Images

Carefully review the drawings in Fig. 5 and make sure you understand how the three principle rays are drawn in each case.

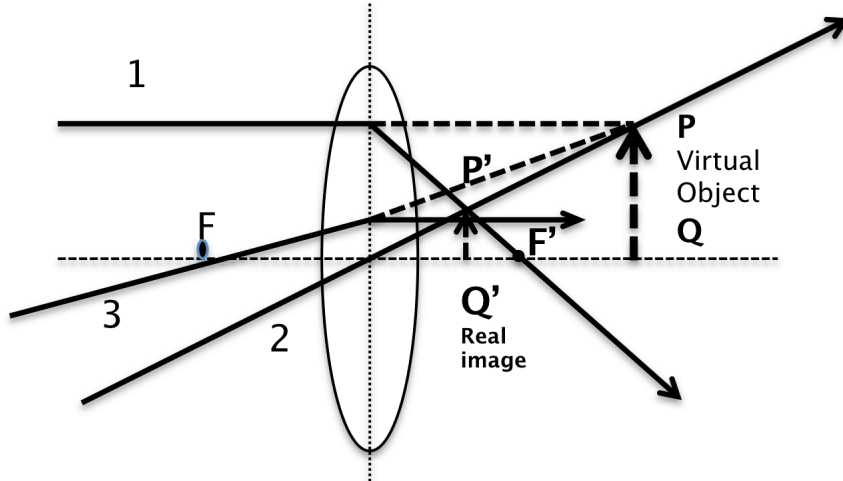


Fig.6a

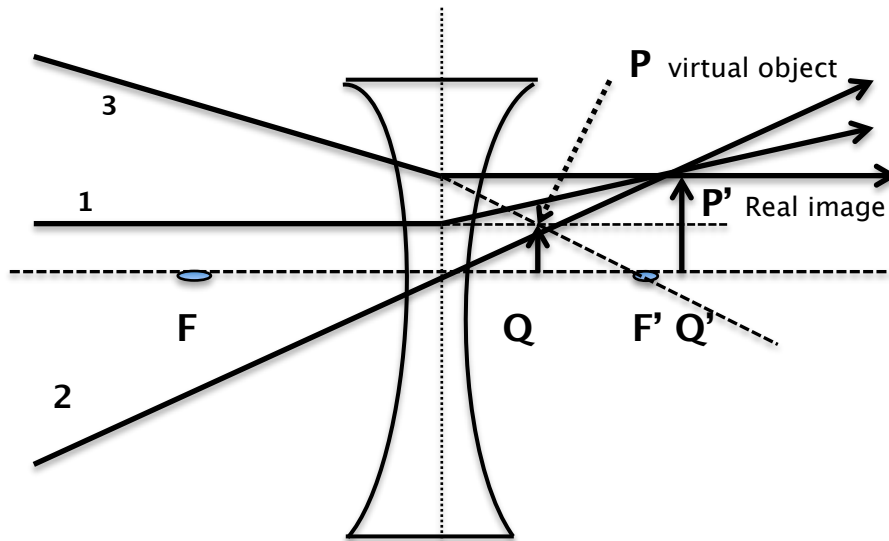


Fig. 6b

Fig. 6

Figure 6: Example Images

A virtual object occurs [Figs. 6(a) and 6(b)] when the incident light rays are converging toward a point P on the farther side of the lens. That is, the input light in Fig. 6(a) would converge at P , where it would form a real image if the lens was not there. This real image becomes a virtual object for a lens placed in the path of the converging rays. Virtual objects are on the opposite side of the lens as the incident light and are only created in multiple lens systems. Virtual images, on the other hand, are formed on the same side of the lens as the incident light.

Whenever the image arrow points in the same direction as the object arrow the image is said to be *erect*. When the image arrow points in the opposite direction from the object arrow, the image is said to be *inverted*.

Lateral Magnification

Lateral magnification is defined as the ratio of the image height to the object height. It can be less than, equal to, or greater than one. The lateral magnification, m , and the erect or inverted character of the image can both be determined from the following equation:

$$m = -s'/s$$

Both s' and s must be substituted with proper regard to their signs. Both s and s' are positive for real objects and images, respectively, and negative for virtual objects and images, respectively. The resulting sign of m is positive if the image is erect and negative if the image is inverted with respect to the object. The magnitude of m determines the height of the image relative to that of the object.

Locating Images by the Thin-Lens Equation

The location, type, orientation, and size of the image and object can be quantitatively determined with the thin-lens equation:

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$$

The symbols s , s' , and f have the same definitions as in the preceding discussion of ray-tracing:

- s = object distance from lens
- s' = image distance from lens
- f = focal length of lens

All three quantities must be treated as signed numbers when using the thin-lens equation. The sign rules are simple:

- f is positive for converging lenses and negative for diverging lenses.
- s is positive for real objects and negative for virtual objects. Real objects occur only in front of the lens, that is, on the side from which light approaches the lens. Virtual objects occur only beyond the lens (opposite the incoming light).
- s' is positive for real images and negative for virtual images. Real images occur only beyond the lens (opposite side of the lens from incoming light). Virtual images only occur in front of the lens (same side of lens as the incident light) and can only be viewed by an eye that is located on the opposite side of the lens as the incident light.

The thin-lens equation has not been proven here, but could be from the geometry of image formation in Figs. 5 and 6. Similar right triangles in drawing 5(a) give the relationships

$$\frac{PQ}{s-f} = \frac{P'Q'}{f}$$

$$\frac{PQ}{f} = \frac{P'Q'}{s'}$$

When the ratio $(PQ)/(P'Q')$ is eliminated between these two relationships, the thin-lens equation is

revealed.

Numerical Example Using the Thin Lens Equation

Consider an image formation problem somewhat like the situation pictured in Figure 5(a).

- The lens is converging (positive) with focal length 12 cm.
- The object is real, and is 21 cm in front of the lens.

Find the location of image P' , and establish whether it is real or virtual? Upright or inverted? What is its lateral magnification?

From the given information: $f = +12$ cm, $s = +21$ cm, $s' = ?$, $m = -s'/s = ?$

Substituting into the thin-lens equation:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$\frac{1}{+21\text{cm}} + \frac{1}{s'} = \frac{1}{+12\text{cm}}$$

$$\frac{1}{s'} = \frac{+3}{84\text{cm}} = \frac{+1}{28\text{cm}}$$

so $s' = +28$ cm, and the magnification, m is:

$$m = -\frac{s'}{s} = \frac{+28\text{cm}}{+21\text{cm}} = -\frac{4}{3}$$

From these results, we conclude:

- the image is real, because s' is positive
- the image is 28 cm beyond the lens
- the image height is 33% larger than the object height
- the image is inverted, because m is negative

Angular Magnification

Images are formed on the retina by the eye's ability to change the shape of its own lens and thus its focal length. The closest object that the eye can accommodate is about 7 cm for 10-year-olds, increasing with age to about 200 cm at age 60. For young adults, the minimum comfortable viewing distance for an object is about 25 cm. We will use the symbol, D , to represent the comfortable viewing distance in these studies.

The size of an object to you depends on both its height and its distance from you. Because our perception of size depends on more than its height, we introduce the notion of the angular size of an object, which is just the angle subtended at the eye by the object. When viewing an object of height y at your comfortable viewing distance D from your eye it has angular size, measured in radians

$$\theta_0 \approx y/D$$

Thus, θ_0 is the angle subtended by the object at the eye. When viewed through a lens or combination of lenses comprising a magnifying instrument, the light appears to emanate from a virtual image. This image will also be located at your comfortable viewing distance D , because you will have reflexively adjusted the position of the object with respect to the lens until that happens. (This is basically just "getting it into focus".) The height of the image, y' , will be greater than the object viewed directly, and the angular size will be:

$$\theta = \frac{y'}{D}$$

The angular magnification, M , is the ratio of the two angular sizes:

$$M = \frac{\theta}{\theta_0}$$

The reason for introducing this concept is that the lateral magnification, $m = -s'/s$, does not encapsulate how well a lens or system of lenses works to produce the desired effect, i.e., magnification.

For example, a converging lens might produce an image many times the size of the object, but located so far from your eye that it appears no larger than the object; on the other hand, a simple telescope produces an image that is very tiny compared to the object but which is much closer to your eye and so the image appears larger. You will encounter both of these situations later in the experiment.

Equipment

- Sargent-Welch Telemicroscope Kit
 - Small Optical Bench on Ring Stand
 - Six lens mounts [A, B (2), D, E, and F]
 - Screen Mount (S): A piece of tape acts as a frosted translucent screen, for viewing real images.
 - One Reticle Mount (C). A millimeter scale on a thin glass plate serves as an object of known size.
- Mounted light bulb
- Two clear plastic rulers
- Graph paper

Procedure: Part 1 (*to be done before lab*)

For the following exercise, locate the image of a 4 cm tall real object by drawing the three principle rays for ray tracing described earlier. ***In order for the ray tracing technique to work, the drawings must be made to scale, and care must be taken to draw straight lines.***

- Draw the following ray tracing diagram on graph paper using a ruler, and make a note of the scale you are using (ex., one square = 1 cm).

Converging lens with a focal length of $f = 4$ cm, Real object at $3.0f$.

- Use your ruler to measure (in cm) the position of the image and the magnification of the image. Also note whether or not the image is inverted compared to the object.
- After you have drawn ray tracing diagram for the above scenario, use the thin lens and lateral

magnification equations to calculate the position of the image, the magnification of the image, and whether or not the image is inverted compared to the object. Compare these results with those found in the ray tracing diagrams.

When more than one lens is used to form an image, ray tracing and the thin lens equation can still be used to predict the characteristics of the final image that an observer sees. For the two lens systems we will investigate in these experiments, each lens is treated one at a time, beginning with the lens closest to the original object. The image resulting from the first lens becomes the object for the second lens. Under some circumstances, the object for the second lens will be virtual like those in Fig. 6(a) and 6(b).

Complete the following exercise to determine the image characteristics of a 4 cm tall real object 12 cm from a converging lens with a 6 cm focal length followed by a diverging lens with the same focal length; the two lenses are separated by 10 cm.

- Use the three principle rays for ray tracing to locate the image of the real object after it passes through only the converging lens. Determine the position of this image (in cm) relative to the converging lens, its magnification, and whether or not it is inverted compared to the object.
- Add the diverging lens to the ray tracing diagram. The image from the first lens is now the object for the second lens. Determine if the new object is real or virtual and its distance from the second lens. Explain your reasoning.
- Use a different color writing implement than used for the converging lens, and draw the three principle rays for the image that results after the light passes through the second lens.
- Use a ruler to measure the position of this image (in cm) relative to the converging lens, its magnification compared to both the original object and the object for the second lens, and whether or not it is inverted compared to the original object and the object for the second lens.
- Use the thin lens equation to first predict the location of the image after passing through just the converging lens.
- Use this information in the thin lens equation a second time to predict the location of the final image after passing through the diverging lens.
- Compare your results with those found in the ray-tracing diagram.

----- **End of Part I** -----

Procedure: Part 2 (*to be done during lab*)

In this part of the exercise, you will utilize the Sargent-Welch Telemicroscope, a kit that includes various optical pieces, to investigate properties of lenses and build optical devices. A couple of things to note:

- The position of a lens or the reticle on the optical bench should be taken as the location of the middle of the lens reticle.
- The open side of the mounts should be on the scale side of the bench so that the edge of the optical element can be seen.
- The mounts are held to the bench by small magnets in their bases and may be moved easily to a new position.

Focal Length

Determining the focal length of a converging lens directly:

- Place the converging lens at a known position.
- Move the screen mount to find a sharply defined real image of a distant object.
- The distance between the lens and the screen is the focal length.

The focal length of a diverging lens must be determined indirectly because it does not produce a real image on a screen; its focal length can be measured by combining it with a converging lens. This creates a situation similar to Fig. 6b, which shows light rays converging before they encounter a diverging lens.

- The converging lens (lens B) is first placed by itself on the bench and the screen is used to find the real image.
- The diverging lens is then dropped in between the converging lens and its real image, which now becomes a virtual object for the diverging lens, which, in turn, forms a real image that can be located with the screen.
- Think about how to define the virtual object distance: which lens would you measure from? Each partner should carry out at least one measurement of the focal length of each lens. Average your results and create a table in your notebook like the one below to record your results.

Mount	Focal Length
A	
B	
C	reticle
D	
E	
F	

Simple Microscope

The ray-tracing diagram for a simple magnifier is shown in Fig. 7a. The object is positioned somewhat nearer the lens than the focal point, and the virtual image is formed beyond the focal point. In Fig. 7b, some of the rays have been omitted so that the angle subtended by the virtual image is shown more clearly.

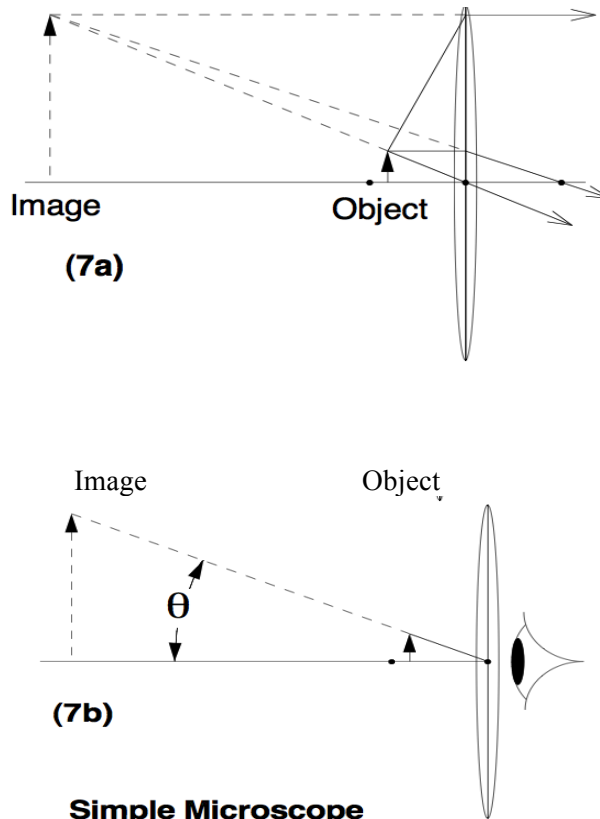


Figure 7: Ray Tracings for a Simple Microscope (a & b)

Set up the reticle on the bench and determine how close you can bring it to your eye and still focus on it. This is clearly your comfortable viewing distance, D . Check this for both lab partners. Now add converging lens B to the end of the bench where you can get it right next to your eye. Then bring the reticle as close to the lens as possible while still getting a sharp image of it. The lens should be very close to the eye. Use the thin-lens equation to find the virtual image distance. The distance between the reticle and the lens is the object distance, and the focal length was previously measured. The virtual image distance that you calculate should be the same as the comfortable viewing distance that you measured.

Fig. 8 illustrates a technique for measuring angular magnification by placing a millimeter ruler at a distance, D , alongside the image viewed through the lens of your microscope. View the ruler directly with one eye while looking through the lens at the virtual image of the reticle with the other eye. With patience and practice, you can see both to compare the millimeter rulings on the image of the reticle with the ruler viewed directly. Your two eyes will see something like Figure 8.

Both the ruler and the image are at the same distance, D , from your eyes. Since both the ruler and reticle have mm markings, the angular magnification can be found by comparing the number of mm marks on

the ruler that fit in one mm mark of the magnified reticle. Explain why this yields the angular magnification of the microscope.

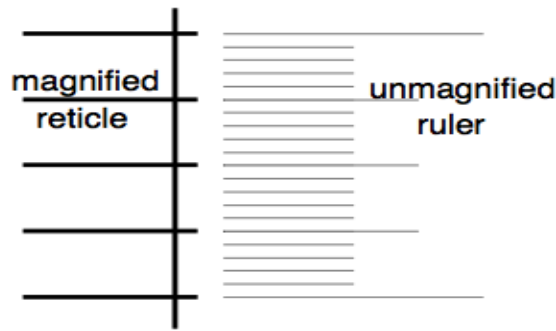
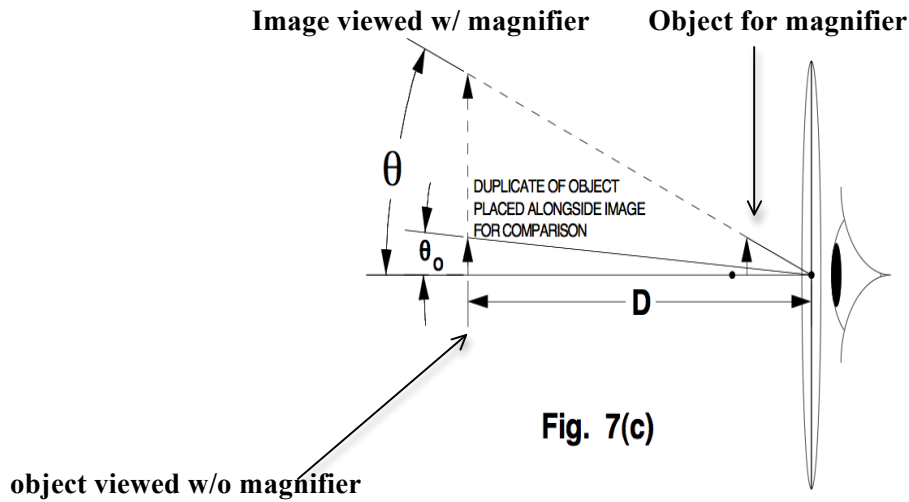


Fig. 8

Figure 8:
Determining angular magnification of a simple microscope

Manipulating the thin-lens equation gives the angular magnification as $M = 1 + D/f$; a lens of focal length 2.5 cm would yield an angular magnification of 11 using 25 cm for D . (Some authors derive the angular magnification of a magnifier more simply as D/f , which is true when the object is positioned right at the focal point of the lens.)

Real Images

Set up a converging lens (A or B) near the middle of your bench. Place the reticle (C) and the screen on opposite sides of the lens so that a sharp real image of the reticle is formed on the screen. The image is easier to see if you point the end of the bench with the reticle towards a light and use other lens B as a simple magnifier to view the screen. Place it between your eye and the screen mount in such a position as to provide a clear magnified view of the screen

(see Fig. 9).

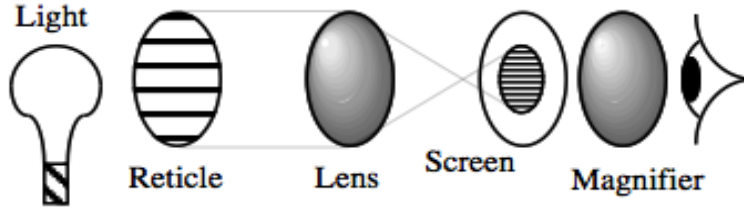


Figure 9:
Arrangement for Real Image Studies

Question: Would there be an image if the screen was not there? Discuss it with your partner and after you have formed an opinion try it out experimentally. Once you have a clear view of the image on the screen take the screen away without disturbing anything else. Is there still an image? If so, where is it? If not, where did it go? What you have just seen is relevant to the operation of the telescope.

Astronomical Telescope

An astronomical telescope in its simplest form consists of a converging objective lens of long focal length f_o and a converging eyepiece lens of short focal length f_e separated by a distance approximately the sum $f_o + f_e$ (see Figs. 10a & b).

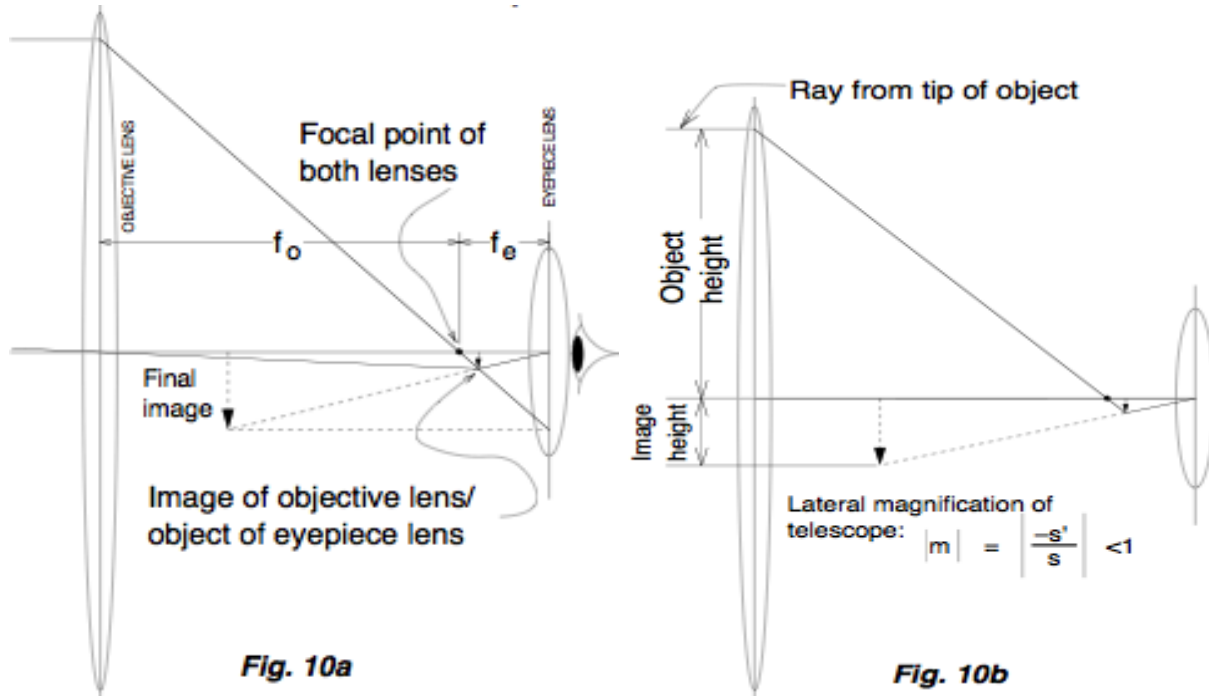


Figure 10:
Astronomical Telescope

When aimed at an object far off to the left, the objective lens forms a small, real, inverted image, which is located near its focal point. The eyepiece lens is then used as a simple magnifier with which to examine this real image.

The lateral magnification for the telescope will be less than unity. In Fig. 10b, the ray from the tip of the

object and parallel to the axis shows the height of the object, which is clearly greater than the height of the final image.

The angular magnification, however, will be much larger than unity because the final image is very close to the eye, as illustrated in Fig. 10c.

You can measure the angular magnification by adapting the two-eye technique you used earlier with the simple microscope. That is, use one eye to view directly a distant object at the same time that you view its image through the telescope with the other eye and estimate the ratio of their sizes.

You can derive from the ray diagram and/or the lens equation for this system that the angular magnification should be given by $M = f_o/f_e$.

- Set up a simple astronomical telescope on your bench, using lens D as the objective and lens E as the eyepiece.
- Separate them by a distance that is the sum of their focal lengths.
- Make final adjustments by focusing on marks spaced 5 cm apart on the blackboard.
- Determine the angular magnification by looking through the telescope with one eye and directly at the object, i.e., the lines on the blackboard, with the other eye.
- Calculate the value of angular magnification and compare it with your measured value.

Concluding Questions

When responding to the questions/exercises below, your responses need to be complete and coherent. Full credit will only be awarded for correct answers that are accompanied by an explanation and/or justification. Include enough of the question/exercise in your response that it is clear to your teaching assistant to which problem you are responding.

1. If you are nearsighted, images are formed in your eye in front of the retina. What type of corrective lenses should be placed in front of your eye to get the images to form on the retina in order for you to see clearly? Explain your reasoning.
2. Describe in words the difference between a virtual and a real image.
3. As the distance between the object and the lens gets very large ($s \rightarrow \infty$), where does the resulting image form? For a real object located at a distance greater than the focal distance of the lens, will the image ever be formed inside the focal distance of the lens? Explain your reasoning. If the object is placed at the focal distance of the lens, where is the resulting image formed? Explain. Hint: What does the thin lens equation predict?
4. When you constructed the simple microscope, you placed the object inside the focal length of a converging lens. Explain the range over which the object could be placed between the lens and infinity so that the lens can still be used as a magnifier. Justify your answer by drawing a ray-tracing diagram with the object both inside and outside the focal length of the lens. Include your eye in the diagrams.
5. When you constructed the astronomical telescope, you used two converging lenses. If the eyepiece of your telescope was removed, how is the image size and orientation changed? Explain your reasoning.