Simple Machines & Work

Introduction

Simple machines are devices like pulleys, levers, hydraulic lifts, ramps, gear sets, etc. These devices do not generate mechanical energy (they have no engines or motors). Instead, they are configured to multiply an applied force or an applied displacement to assist us in accomplishing tasks more easily or to make possible otherwise impossible tasks.

Equipment

Hydraulic Lift System

- Two-Pistoned Apparatus
- Mineral Oil (lubricant)
- Ruler
- Slotted Mass Set

Bicycle System

- Bicycle
- Spring Balance
- Meter Stick
- Various Hanging Weights

Pulley System

- Triple Pulley System

Background

Simple machines are mechanical, non-motorized devices that permit the user to exert more force or travel more distance at the output of the machine than was initially put into it. It may sound like simple machines are too good to be true, but, in fact, energy conservation is very premise upon which they operate. In lab today, we will study how simple machines work and how they can be used to accommmplish otherwise daunting tasks. Some examples of simple machines include:

- **Pulleys**: A block-and-tackle (pulley) set can be used to lift a 1000 pound piano to the third floor with a worker applying only 100 pounds of force.
- **Bicycles**: When riding a bicycle, the gears act as a simple machine. Each of your feet travel about 3.5 feet in one revolution of the pedals. With the bike in a high gear, this modest input displacement by your feet generates a significant output displacement that propels the entire bike forward approximately 24 feet.
- **Levers**: A crowbar can be used as a lever to pull a nail from a board. Without the crowbar (just grabbing the nail with pliers), you can't generate sufficient force to pull it out.

If we were to analyze how a bicycle works, we might think about the force exerted on the pedal by our foot, convert this to a torque that causes the pedal to rotate, determine the tension in the chain...and eventually calculate the force forward on the rear tire of the bike by the ground. This is a perfectly reasonable way to assess how bicycles work, but there is an easier and more elegant way to analyze simple machines that appeals to conversation of energy and work.

We all have a good sense of what it means to do work in our every day lives, but in physics work has a very specific meaning. Work is done when a force is exerted on an object over some distance. In fact, the definition is even a bit more specific than that. It is only the component of the force that is along the displacement direction that contributes to work. In a mathematical form, the most general definition of work is

$$W = \int_a^b \vec{F} \cdot d\vec{s} = \int_a^b \left(F\cos\theta\right) ds$$

where W is the work done on an object by the force, F. The integral says that the force applied to the object needs to be summed up as the object is moved from point a to point b. The dot product of the force and displacement ($d\vec{s}$) ensures that only the component of the force along the displacement direction contributes to the work. The angle θ is measured between the force and displacement vectors when the tails of these two vectors are touching one another.

If the force and angle are constant over the entire displacement of the object, the work done by the force can be written as

$$W = Fs\cos\theta$$

where s is the distance over which the force was exerted.

In the MKS (or SI) system, work has units of Newton-meters (N-m). In English units, work is measured in foot-pounds (ft-lbs). Note that the units of work are equivalent to energy units (1 N-m = 1 kg-m^2/s^2 = 1 J). If a constant force and the displacement are in the same direction ($\theta = 0$), the work done by the force is simply the product of the force and displacement magnitudes, since cos $0^o = 1$. For example, if a weight lifter hoists a 1000 N weight (225 lbs.) straight up to a height of 2.5 m, he does 2500 N-m of work.

Notice that when our weight lifter did work on the weight by lifting it up, the total energy of the system increased. Initially, our system, which includes the weight and the Earth, had no energy, but after the weightlifter exerted a force on the weight, the system had 2500 J stored as gravitational potential energy. The work done by the weight lifter corresponds to the increase in potential energy of the weight-Earth system. Whenever an external force (in our example, the weight lifter) acts on a system (the weight and the Earth), it either adds energy to or removes energy from the system. In general, when the external force does positive work, it adds energy to the system, and when it does negative work, it removes energy from the system.

Let's consider a favorite simple machine of the ancient pyramid builders, the incline. Figure 1 shows a shallow incline or ramp of length, l, with the top end 1.5 meters higher than the bottom. Pyramid builders used shallow inclines to move large stones weighing tens of tons from ground level up to where they were placing stones into the pyramid walls. What is the advantage of sliding heavy objects up an incline rather than lifting them straight up?

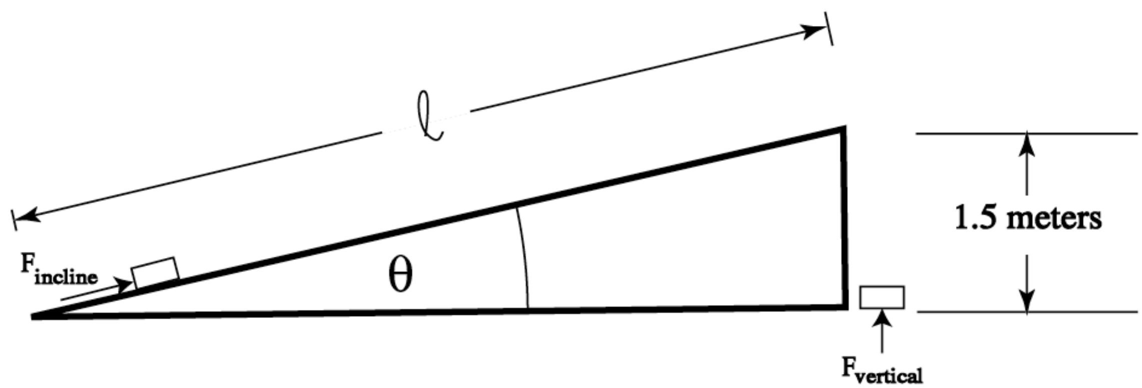

Figure 1: Example of an Incline Plane

If the pyramid builders were to lift a 10 ton stone straight up 1.5 meters, the work they would do would correspond to an increase in the gravitational potential energy of the stone-Earth system of

$$[(10 \text{ tons}) \times (2000 \text{ lbs/1 ton})] \times [(1.5 \text{ m}) \times (3.28 \text{ ft/m})] = 98400 \text{ ft-lbs}$$

Now consider the alternative approach of sliding the same stone up the ramp to a height of 1.5 m. The gravitational potential energy of the stone-Earth system would increase by the same amount because the stone still ends up 1.5 m above the ground. In both cases, the workers do the same amount of work on the stone. Again we ask, why use the incline rather than lift the stone straight up? Since the amount of work is the same in both cases,

$$W_{incline} = W_{vertical}$$
$$F_{incline} \times l = F_{vertical} \times h$$

where $F_{incline}$ is the force exerted on the stone when it is slid along the ramp, $F_{vertical}$ is the force exerted on the stone if it is lifted straight up, and h = 1.5 m is the vertical distance the stone is lifted (see Figure 1). Solving for the force required to push the stone up the incline we find that

$$F_{incline} = \left(\frac{h}{l}\right)F_{vertical} = \left[\frac{h}{h/\sin\theta}\right]F_{vertical} = \sin\theta \times F_{vertical}$$

The angle θ could range from 0 to 90 degrees, so $\sin\theta$ is a number between 0 and 1. Thus, the force required to push a stone up a ramp is always less than the force required to lift the same stone straight up. This is what the pyramid builders figured out when they were moving heavy stones day after day.

Consider the case of the bicycle again. As the rider pushes down on the pedals, we can calculate the work done by the rider on the bike as the product of the force exerted and the displacement (distance) traveled by the feet. This is the work supplied to the machine. We will refer to this as the work input to the simple machine, and thus we use the subscript '*i*' to denote it. The rear tire exerts a force backwards on the ground. By Newton's Third Law, the ground exerts an equal force forward on the bike's rear tire. This force multiplied by the forward displacement of the bike is the work done by the machine. We refer to this work as the work output by the simple machine, employing the subscript '*o*' for its label.

In an ideal or perfect simple machine, the output work is equal to the input work. No energy is dissipated by friction in the simple machine. However, for real simple machines

Output Work = Input Work - Energy Lost by Friction

The energy dissipated by friction appears as heat at the site of the friction. Experience and common sense tell us that energy dissipated by friction is greater than or equal to zero. Thus, we have as a general rule for simple machines that says ***Output Work ≤ Input Work***. For the special case of negligible friction losses, ***Output Work ≅ Input Work***.

Bicycles have quite modest friction losses in the bearings, chain, etc. so we will analyze this simple machine assuming that loss due to friction is negligible,

$$Work_o = Work_i$$
$$Force_o \times Displacement_o = Force_i \times Displacement_i$$

where $Force_o$ (output force) is the forward force on the rear wheel and $Force_i$ (input force) is the force applied on the pedal by the rider. The above equation can be rewritten

$$\frac{Force_o}{Force_i} = \frac{Displacement_i}{Displacement_o} = \textbf{Mechanical Advantage}$$

Mechanical advantage is a term that expresses the force multiplication of the simple machine. If the mechanical advantage > 1, the output force exceeds the input force. However, the output displacement is correspondingly smaller than the input displacement. Thus, if you select a very low gear on your bike, it is easy to pedal uphill (F_i is small), but you will have to crank the pedals lots of times (input displacement is large). Likewise, if a frictionless pulley set allows an input force of 100 pounds to support a 1000 pound piano, the worker will have to pull his rope 10 feet to lift the piano by 1 foot.

Procedure

Pulleys

Briefly operate the pulley with the mass attached to get a sense of the force multiplication attainable with this ancient device. You do not have to take any data, but take note of the comparative ease with which the weight can be lifted and the distance over which the input force must be applied. The triple pulley is operated with lab partners pulling on opposite ends. It has a spring balance on each side to measure the forces.

It is worth taking note of the force ratios and the displacement ratios of the pulleys compared to those of the bicycles. These devices are both used to enhance the capabilities of the human body, but in different ways. One multiplies the force output but requires a greater displacement at the input. These ratios are reversed in the other device, that is, a greater displacement is achieved at the expense of a reduction in the force obtained.

Be sure that you understand how the pulley system works; there are concluding questions at the end of this experiment that are based on your observations.

Bicycles and Hydraulic Lifts

There are several bicycles and small hydraulic lifts with which to investigate the mechanical advantages of simple machines. You can perform the experiments in any order, but there are not enough setups for everyone to do the same experiment simultaneously.

Determining Mechanical Advantage of a Bicycle From Force Measurements

The bikes have been modified so that you can hang weights where the pedals are normally attached (See Figure 2). A loop of cord on the rear wheel provides an attachment point for a spring balance.

In this first part of the experiment, you will determine the mechanical advantage of the bicycle from force measurements.

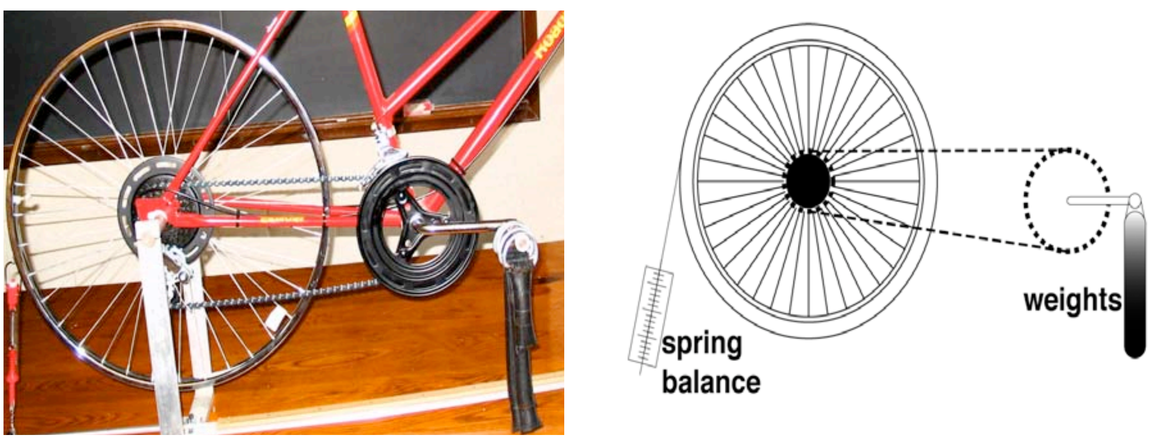

Figure 2: Picture and Diagram of the Bicycle System

- There are five 1.0 kg masses and one 0.5 kg mass to place on the pedal to create an input force, F_i.
- The reading of the spring balance shows the output force, F_o.
- The bikes are locked in different gears, so they will all have different mechanical advantages.
- Since gravity acts straight downward, you must keep the pedal crank level (horizontal) to have the full turning force in effect. At any other angle, only part of the applied force is in a direction to turn the pedal, going to zero if the pedal crank was vertical.
- Connect the spring balance to the rear wheel, and rotate the wheel slightly to take the slack out of the loop that connects the balance to the wheel.
- While still holding the wheel, rotate the pedal crank backwards into the horizontal forward position as shown in Figure 2, and place a 0.5 kg mass on it.
- Read the balance to obtain the force exerted on the rear wheel.
- Proceed to take a series of balance readings, incrementing the pedal load in 0.5 kg steps, until the pedal load reaches 5.0 kg.
- Plot the wheel force as a function of the pedal force, and determine the mechanical advantage of the bicycle from your graph.

Determining Mechanical Advantage of a Bicycle From Displacement Measurements

In the next part of this experiment, you will determine the mechanical advantage of the bicycle from measurements of the input and output displacement. Note that in one complete rotation of the pedals, your foot travels $displacement_i = 2\pi R_i$ where R_i is the radius of the pedal crank. The output displacement is correspondingly $displacement_o = 2\pi N R_o$ where R_o is the radius of the rear wheel, and N is the number of revolutions of the rear wheel for one complete turn of the pedals.

- Remove the weights and disconnect the spring balance from the wheel.
- While holding your hand so that it rubs against the rear wheel to prevent it from free-wheeling, turn the pedal crank forward one complete revolution, and determine the number of revolutions of the rear wheel. You may find it helpful to count spokes of the wheel to determine fractions of a revolution (36 spokes = one revolution).
- Compare the mechanical advantage of the bike determined from displacement measurements to the value obtained from force measurements.

Determining Mechanical Advantage of a Hydraulic Lift From Force Measurements

Hydraulic systems are widely used because of the convenience they offer. For example, in your automobile the brake pedal pushes a piston, which is connected by a hydraulic line to another piston that operates the brakes. Since the wheels bounce up and down and rotate (to turn left and right), it is easier to run a rubber hose to the brakes on each wheel than it would be to use complicated mechanical linkages. On large aircraft, the control surfaces (elevator, rudder, ailerons) are hydraulically actuated.

The simple hydraulic lift provided for this experiment is sketched in Figure 3. An input piston of cross-sectional area, A_i, has a force, F_i, applied to it. A second piston of cross-sectional area, A_o, supports the load, a weight, F_o. Pushing on one piston exerts a force on the incompressible fluid, which transmits that force to the other piston.

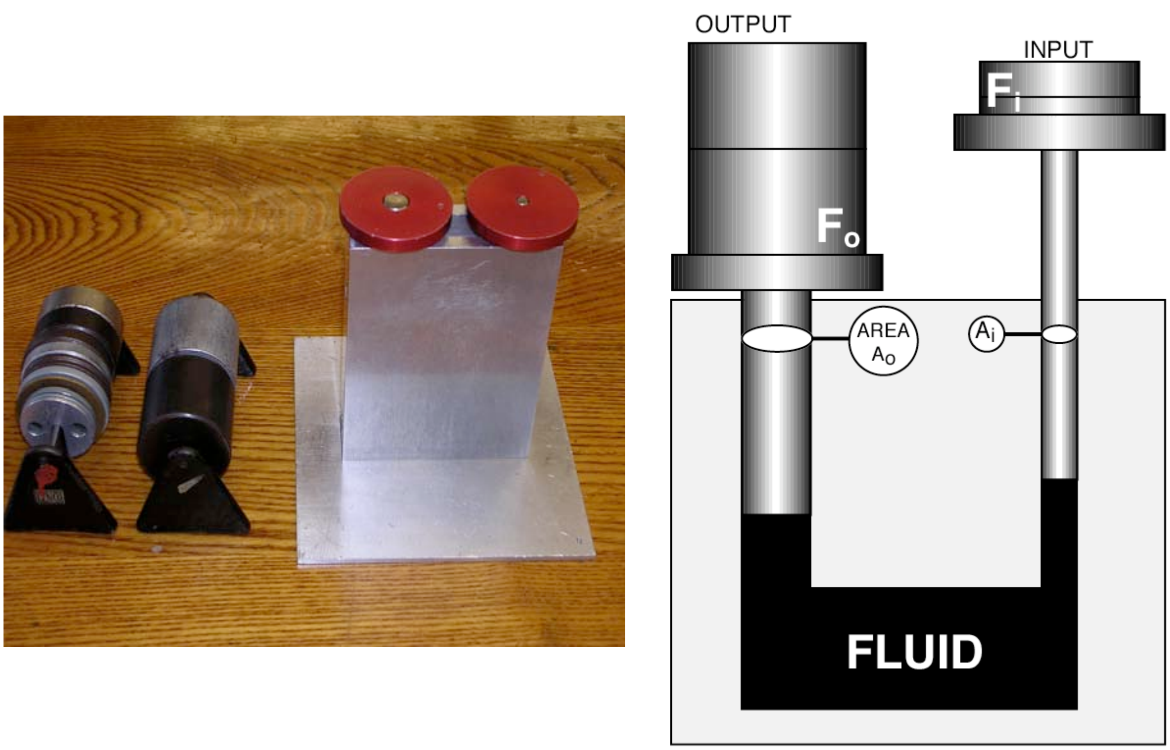

Figure 3: Picture vs. Diagram of the Hydraulic Lift (Piston) System

Equipment Note:
The fluid in this hydraulic lift is mineral oil - it serves both as a medium through which the applied force is transmitted to the output piston and as a lubricant between the pistons and the surrounding walls of the container, reducing the friction in the simple machine. If oil begins to seep out of the device, the input force will no longer be effectively transmitted to the output piston due to the presence of air in the hydraulic lift. When this happens, the large piston will begin to sink over several minutes. To remedy the problem, carefully withdraw the large piston completely by taking hold of the platform and pulling the piston straight up, then re-insert the piston and continue with your measurements.

In this part of this experiment, you will determine the mechanical advantage of the hydraulic lift from measurements of input and output force.

- Place 500 grams on the platform of the larger diameter piston and 75 grams on the platform of the small piston.
- Adjust the mass on the small piston until equilibrium is reached and neither piston is moving.
- When the system appears to be close to equilibrium, lightly push, tap or spin the smaller piston to ensure that it really is stationary. If it moves, continue to adjust the mass on the small piston until true equilibrium is achieved.
- Continue this process for output loads of 600, 700, 800, 900 and 1000 grams on the large piston.
- Create a linear plot from your output and input force measurements, and determine the mechanical advantage of the hydraulic lift from the slope of the best-fit line to your data.

Determining Mechanical Advantage of a Hydraulic Lift From Displacement Measurements

In the second part of this experiment, you will determine the mechanical advantage of the hydraulic lift from displacement measurements.

- Remove all the masses from the platforms.
- Measure the amount of upward displacement of the small piston when you push the large piston down 1 cm.
- Calculate the mechanical advantage of the hydraulic lift from your displacement measurements.
- When the input piston is displaced, it moves a cylindrical volume of the fluid, V_i, inside the hydraulic lift. The volume of fluid moved by the output piston, V_o, is the same because the fluid is incompressible:

$$V_i = V_o$$
$$A_i \times Displacement_i = A_o \times Displacement_o$$

- Since $A=\pi R^2$

$$\frac{Displacement_i}{Displacement_o} = \frac{A_o}{A_i} = \frac{R_o^{\ 2}}{R_i^{\ 2}}$$

- In this experiment, the radii of the pistons are 1/4" and 1/8". Use this information to compare the square of the radii ratio to your measurements of the mechanical advantage from both the force and displacement measurements.

Concluding Questions

When responding to the questions/exercises below, your responses need to be complete and coherent. Full credit will only be awarded for correct answers that are accompanied by an explanation and/or justification. Include enough of the question/exercise in your response that it is clear to your teaching assistant to which problem you are responding.

1. Is the bicycle a force multiplier or a distance multiplier? Justify your response by considering F_o/F_i and D_i/D_o.
2. Explain in words why you might push a heavy object up an inclined plane to raise it 1 meter off the ground rather than lifting it straight up.
3. Is the inclined plane a force multiplier or a distance multiplier? Justify your response by considering F_o/F_i and D_i/D_o.
4. Determine the minimum and maximum values for the mechanical advantage of simple machines that are designed to be a) force multipliers, and b) distance multipliers.
5. Based on your observations of the two pulley systems in the lab, explain whether these pulleys are force multipliers or distance multipliers. Keep in mind that pulleys are usually used to make it easier to lift heavy objects.