Conservation of Momentum

Introduction

Collisions occur all around us and on many size scales. We observe them in our everyday world as car accidents, batters hitting a baseball out of the ballpark, raindrops pelting the hood of your car during a downpour, and butterflies landing gently on flower petals. Collision experiments at the atomic and sub-atomic levels have also permitted scientists to probe the structure of the atom and nucleus.

Background

Collisions are simply interactions of two or more objects. The interaction may be characterized by objects coming into direct contact with one another, like two billiard balls colliding in a pool game, or they may not ever physically touch, as with Earth-moon interactions that arise from their gravitational attraction. In both cases, forces are exerted on the objects during the interaction, and, as a result, the motion of the objects is altered. Newton's 2nd law expressed in the form:

$$\vec{F} = m\frac{d^2\vec{x}}{dt^2} = m\vec{a}$$

describes how the motion of an object changes when a force is applied to it. This specific formulation of the 2nd law assumes that the mass of the object is constant. Newton's 2nd law can be written more generally as:

$$\vec{F} = \frac{d\vec{p}}{dt}$$

where *p* is a vector quantity called the linear momentum, which is equal to the product of mass and velocity. For the special case of constant mass, this reduces to the familiar $\vec{F} = m\vec{a}$.

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}\left(m\vec{v}\right) = m\frac{d\vec{v}}{dt} = m\vec{a}$$

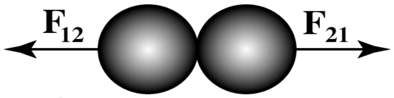
In the general form, we can interpret the 2nd law in a new way - forces exerted on an object over some time interval cause changes in the momentum of the object.

Let’s consider an example where two billiard balls collide (Figure 1). During the brief time interval when the collision occurs, Ball 1 exerts a force on Ball 2, and Ball 2 exerts a force on Ball 1.

These two forces are an action-reaction pair. Ball 1 pushes on Ball 2 just as much as Ball 2 pushes on Ball 1. Newton’s 3rd law tells us that these forces are equal in magnitude but opposite in direction

$$\vec{F}_{12} + \vec{F}_{21} = \vec{F}_{12} + (-\vec{F}_{12}) = 0$$

Figure 1: Forces at work during a collision



$$\vec{\mathbf{F}}_{12} = \frac{d\vec{\mathbf{p}}_1}{dt} \quad \text{Force on 1 exerted by 2.}$$

$$\vec{\mathbf{F}}_{21} = \frac{d\vec{\mathbf{p}}_2}{dt} \quad \text{Force on 2 exerted by 1.}$$

Re-writing the forces in terms of changes in momentum, we find that

$$\frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = \frac{d}{dt}(\vec{p}_1 + \vec{p}_2) = 0$$

If we treat the two balls as a system, the total momentum is constant. This is true so long as the system is “isolated”. By “isolated” we mean that no net forces are applied to any object in the system by objects that are not a part of the system.

Forces exerted by objects outside a system are referred to as ***external*** forces while forces exerted on objects in a system by other objects in the system are called ***internal*** forces. If we were taking into consideration the gravitational pull that the Earth exerts on both billiard balls in Figure 1, the gravitational force would be an external because the Earth was not included in the definition of our system. The forces illustrated in Figure 1 are internal forces, because they were generated by objects (i.e., the billiard balls) that belong to our system under study.

The idea that the total momentum of an isolated system is constant is one of the most fundamental laws of physics known as conservation of momentum. Like conservation of energy, no isolated systems have been found that violate conservation of momentum.

Conservation of momentum is a very powerful tool in analyzing collisions. If the objects participating in the collision experience no net external forces, the total momentum of the system is the same before and after the collision. For our colliding billiard balls in Figure 1, momentum conservation requires the following:

Total Momentum Before Collision = Total Momentum After Collision

$$\underbrace{m_1\vec{v}_1}_{\substack{\textit{Ball 1}\\ \textit{Before}}} + \underbrace{m_2\vec{v}_2}_{\substack{\textit{Ball 2}\\ \textit{Before}}} = \underbrace{m_1\vec{v}_1'}_{\substack{\textit{Ball 1}\\ \textit{After}}} + \underbrace{m_2\vec{v}_2'}_{\substack{\textit{Ball 2}\\ \textit{After}}}$$

where the subscript '1' refers to properties of Ball 1, the subscript '2' refers to properties of Ball 2, the unprimed velocities correspond to values before the collision takes place and the primed velocities to those after the collision occurs. Careful attention must be paid to both the magnitude **and** direction of each velocity in momentum conservation because, like velocity, momentum is also a vector quantity.

Collisions can be categorized based on the degree to which kinetic energy is conserved. If all the kinetic energy present prior to the collision goes into the kinetic energy of the objects present after the collision, kinetic energy is conserved, and the collision is deemed ***elastic***. On the other hand, if kinetic energy is lost during the collision (it may be converted to heat and dissipated, for example), the collision is ***inelastic***. A perfectly inelastic collision is one in which colliding objects move together as a single entity after the collision.

The experiments performed in this lab are designed to investigate both momentum and energy conservation. You will devise a series of collisions, both elastic and inelastic, for two carts rolling on a track. From measurements of their masses, initial velocities, and final velocities, you will compute the momentum and kinetic energy of the two cart system both before and after the collisions to assess whether momentum and kinetic energy are conserved in your experiments.

Momentum, Energy and Collisions

The collision of two carts on a track can be described in terms of momentum conservation and, in some cases, energy conservation. If there is no net external force experienced by the system of two carts, then we expect the total momentum of the system to be conserved. This is true regardless of the force acting between the carts. In contrast, energy is only conserved when certain types of forces are exerted between the carts.

Collisions are classified as *elastic* (kinetic energy is conserved), *inelastic* (kinetic energy is lost) or *completely inelastic* (the objects stick together after collision). Sometimes collisions are described as *super-elastic*, if kinetic energy is gained. In this experiment you can observe most of these types of collisions and test for the conservation of momentum and energy in each case.

OBJECTIVES

- Observe collisions between two carts, testing for the conservation of momentum.
- Measure energy changes during different types of collisions.
- Classify collisions as elastic, inelastic, or completely inelastic.

MATERIALS

dynamics cart track
Vernier computer interface
Logger *Pro*
two Motion Detectors
two low-friction dynamics carts with
magnetic and Velcro bumpers
2 - 250 gram weights

PRELIMINARY QUESTIONS

1. Consider a head-on collision between two billiard balls. One is initially at rest and the other moves toward it. Sketch a position *vs*. time graph for each ball, starting with time before the collision and ending a short time afterward.

2. As you have drawn the graph, is momentum conserved in this collision? Is kinetic energy conserved?

PROCEDURE

1. Measure the masses of your carts and record them in your data table. Label the carts as cart 1 and cart 2.

2. Set up the track so that it is horizontal. Test this by releasing a cart on the track from rest. The cart should not move.

3. Practice creating gentle collisions by placing cart 2 at rest in the middle of the track, and release cart 1 so it rolls toward the first cart, magnetic bumper toward magnetic bumper. The carts should smoothly repel one another without physically touching.

4. Place a Motion Detector at each end of the track, allowing for the 0.15 m minimum distance between detector and cart. Connect the Motion Detectors to the DIG/SONIC 1 and DIG/SONIC 2 channels of the interface. If the Motion Detectors have switches, set them to Track.

5. Open the file “18 Momentum Energy Coll” from the *Physics with Vernier* folder.

6. Click *Collect* to begin taking data. Repeat the collision you practiced above and use the position graphs to verify that the Motion Detectors can track each cart properly throughout the entire range of motion. You may need to adjust the position of one or both of the Motion Detectors.

7. Place the two carts at rest in the middle of the track, with their Velcro bumpers toward one another and in contact. Keep your hands clear of the carts and click *ZERO*. Select both sensors and click *OK* . This procedure will establish the same coordinate system for both Motion Detectors. Verify that the zeroing was successful by clicking *Collect,* and allowing the still-linked carts to roll slowly across the track. The graphs for each Motion Detector should be nearly the same. If not, repeat the zeroing process.

Part I: Magnetic Bumpers

8. Reposition the carts so the magnetic bumpers are facing one another. Click *Collect* to begin taking data and repeat the collision you practiced in Step 3. Make sure you keep your hands out of the way of the Motion Detectors after you push the cart.

9. From the velocity graphs you can determine an average velocity before and after the collision for each cart. To measure the average velocity during a time interval, drag the cursor across the interval. Click the Statistics button to read the average value. Measure the average velocity for each cart, before and after collision, and enter the four values in the data table. Delete the statistics box.

10. Repeat Step 9 as a second run with the magnetic bumpers, recording the velocities in the data table.

Part II: Velcro Bumpers

11. Change the collision by turning the carts so the Velcro bumpers face one another. The carts should stick together after collision. Practice making the new collision, again starting with cart 2 at rest.

12. Click *Collect* to begin taking data and repeat the new collision. Using the procedure in Step 9, measure and record the cart velocities in your data table.

13. Repeat the previous step as a second run with the Velcro bumpers.

Part III: Velcro to Magnetic Bumpers

14. Face the Velcro bumper on one cart to the magnetic bumper on the other. The carts will not stick, but they will not smoothly bounce apart either. Practice this collision, again starting with cart 2 at rest.

15. Click *Collect* to begin data collection and repeat the new collision. Using the procedure in Step 9, measure and record the cart velocities in your data table.

16. Repeat the previous step as a second run with the Velcro to magnetic bumpers.

Momentum, Energy and Collisions

DATA TABLE

Mass of cart 1 (kg)	Mass of cart 2 (kg)

Run number	Velocity of cart 1 before collision (m/s)	Velocity of cart 2 before collision (m/s)	Velocity of cart 1 after collision (m/s)	Velocity of cart 2 after collision (m/s)
1				
2				
3				
4				
5				
6				

Run number	Momentum of cart 1 before collision (kg•m/s)	Momentum of cart 2 before collision (kg•m/s)	Momentum of cart 1 after collision (kg•m/s)	Momentum of cart 2 after collision (kg•m/s)	Total momentum before collision (kg•m/s)	Total momentum after collision (kg•m/s)	Ratio of total momentum after/before (kg·m/s)
1							
2							
3							
4							
5							
6							

Run number	KE of cart 1 before collision (J)	KE of cart 2 before collision (J)	KE of cart 1 after collision (J)	KE of cart 2 after collision (J)	Total KE before collision (J)	Total KE after collision (J)	Ratio of total KE after/before
1							
2							
3							
4							
5							
6							

ANALYSIS

1. Determine the momentum (*mv*) of each cart before the collision, after the collision, and the total momentum before and after the collision. Calculate the ratio of the total momentum after the collision to the total momentum before the collision. Enter the values in your data table.

2. Determine the kinetic energy (½ mv^2) for each cart before and after the collision. Calculate the ratio of the total kinetic energy after the collision to the total kinetic energy before the collision. Enter the values in your data table.

3. If the total momentum for a system is the same before and after the collision, we say that momentum is *conserved*. If momentum were conserved, what would be the ratio of the total momentum after the collision to the total momentum before the collision?

4. If the total kinetic energy for a system is the same before and after the collision, we say that kinetic energy is *conserved*. If kinetic were conserved, what would be the ratio of the total kinetic energy after the collision to the total kinetic energy before the collision?

5. Inspect the momentum ratios. Even if momentum is conserved for a given collision, the measured values may not be exactly the same before and after due to measurement uncertainty. The ratio should be close to one, however. Is momentum conserved?
6. Repeat the preceding question for the case of kinetic energy. Is kinetic energy conserved in the magnetic bumper collisions? How about the Velcro collisions? Is kinetic energy consumed in the third type of collision studies? Classify the three collision types as elastic, inelastic, or completely inelastic.

EXTENSIONS

1. Using a collision cart with a spring plunger, create a super-elastic collision; that is, a collision where kinetic energy increases. The plunger spring should be compressed and locked before the collision, but then released during the collision. Measure momentum before and after the collision. Is momentum conserved in this case? Is energy conserved?

2. Using the magnetic bumpers, consider other combinations of cart mass by adding weight to one cart. Are momentum or energy conserved in these collisions?

3. Using the magnetic bumpers, consider other combinations of initial velocities. Begin with having both carts moving toward one another initially. Are momentum and energy conserved in these collisions?

4. Perform the momentum and energy calculations for the data tables using a spreadsheet.

Concluding Questions

When responding to the questions/exercises below, your responses need to be complete and coherent. Full credit will only be awarded for correct answers that are accompanied by an explanation and/or justification. Include enough of the question/exercise in your response that it is clear to your teaching assistant to which problem you are responding.

1. Could the linear momentum of a hummingbird be greater than that of a tortoise? Explain your answer.
2. Consider a two cart collision on the track similar to the ones you performed in this lab. Draw a free body diagram for each cart showing all the forces acting on each cart the instant when they collide. Identify which of these forces are internal and external to the two cart system. Explain why you can treat the two carts as an isolated system when external forces act on the system.
3. An astronaut is working with some tools outside the spacecraft during a mission. Due to a malfunction in his propulsion equipment, he gets stranded some distance from the ship. How could the astronaut use the tools so that he could return safely to the ship? Explain your reasoning.
4. Two ice skaters stand face to face, and then they push each other away. If one skater weighs 75% of the other, which skater has the greater speed? Explain your reasoning.