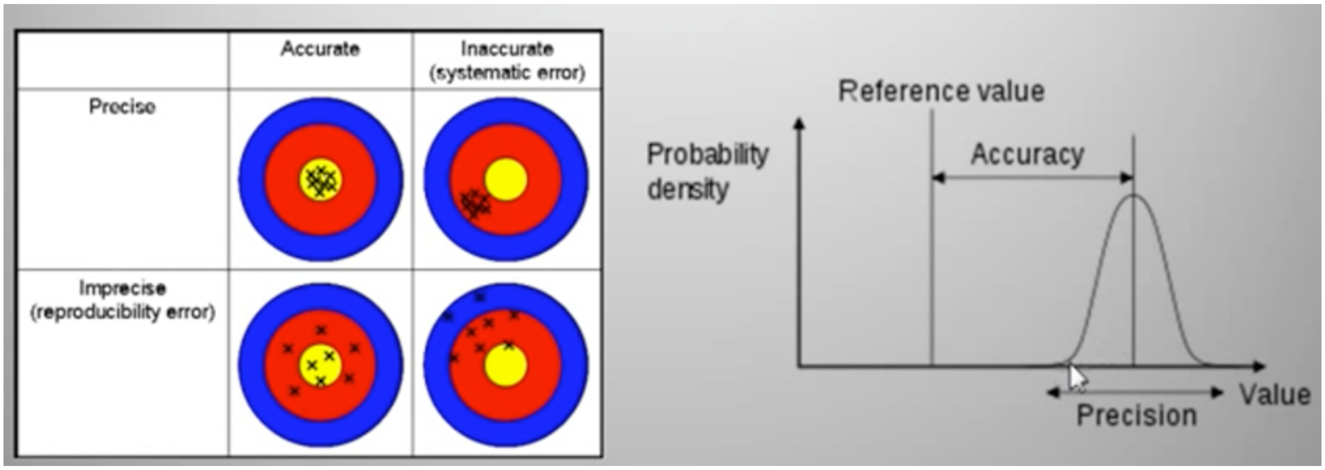
Key Terms



1) Precision

   a measure of the certainty (±), that is how close each measurement is to each other
   E.g., height = 182.23 ± 0.01 cm

2) Accuracy

   how close the measured value is to the true or accepted value

3) Random Error

   an error in measurement caused by factors that vary from one measurement to another, usually due to limitations of equipment - can be minimized by averaging multiple trials

4) Systematic Error

   an error having a nonzero mean, so that its effect is not reduced when observations are averaged, usually due to instrument or method error

Measuring Device Uncertainty

Always report uncertainty to 1 or 2 significant figures

1) Analog Measuring Device

$$\text{for each end of a ruler} = \pm\frac{1}{2}\left(\text{the smallest scale division}\right)$$

2) Digital Device

   1) Look in the equipment manual to determine the device resolution for a given setting range
   2) If you do not have a manual use

      $\pm$ 1 of the smallest unit displayed

3) Repeated Measurements

   1) If repeated measurements yield different values for each measurement then

      mean $\pm$ 1/2(Max - Min) or mean $\pm$ std dev (for roughly symmetric data sets)

   2) If repeated measurements all yield the same value then

      mean $\pm$ resolution of the measuring device

Uncertainty Propagation

1) Addition and Subtraction

   $\text{Length} = L_1 + L_2$ where δL_1 and δL_2 are the uncertainties

   1) Worst Case Uncertainty

      Add the absolute uncertainties

      $$\text{Worst Case Uncertainty in Length} = \delta L = \left|\delta L_1\right| + \left|\delta L_2\right|$$

   2) Statistical Uncertainty

      $$\text{Uncertainty in Length} = \delta L = \sqrt{\left(\delta L_1\right)^2 + \left(\delta L_2\right)^2}$$

2) Multiplication and Division

   $\text{Area} = L*W$ where δL and δW are the uncertainties

   1) Worst Case Uncertainty

      Add absolute fractional or percent fractional uncertainties

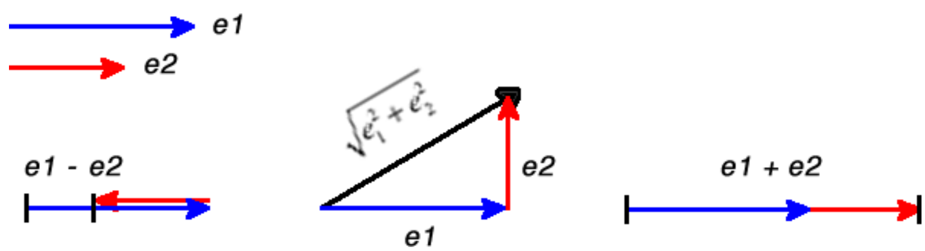
      $$\text{Worst Case Fractional Uncertainty in Area} = \frac{\delta A}{A} = \left|\frac{\delta L}{L}\right| + \left|\frac{\delta W}{W}\right|$$

   2) Statistical Uncertainty

      $$\text{Fractional Uncertainty in Area} = \frac{\delta A}{A} = \sqrt{\left(\frac{\delta L}{L}\right)^2 + \left(\frac{\delta W}{W}\right)^2}$$

Uncertainty Propagation in Quadrature (statistical best estimate)

$$\text{Length} = L_1 + L_2 \qquad \text{where} \quad \pm e_1 \quad \text{and} \quad \pm e_2 \quad \text{are the uncertainties respectively}$$



$$\begin{aligned}\sqrt{e_1^2+e_2^2} &< e_1+e_2 \\ &> e_1 \\ &> e_2\end{aligned}$$

1) Uncertainty in a Function of Several Variables

   All measurement quantities must be independent and subject to only random uncertainties

$$\delta f(x,y,z) = \sqrt{\left(\frac{\partial f}{\partial x}\delta x\right)^2 + \left(\frac{\partial f}{\partial y}\delta y\right)^2 + \left(\frac{\partial f}{\partial z}\delta z\right)^2} \qquad \textbf{Statistical Uncertainty}$$

$$\delta f(x,y,z) \le \left|\frac{\partial f}{\partial x}\delta x\right| + \left|\frac{\partial f}{\partial y}\delta y\right| + \left|\frac{\partial f}{\partial z}\delta z\right| \qquad \textbf{Worst Case Uncertainty}$$

2) Tricks and Shortcuts

   1) Multiplication by a Constant

$$f(x) = Cx \quad \text{where C is a constant} \quad \Rightarrow \quad \delta f = \frac{\partial f}{\partial x}\delta x = C\delta x$$

When multiplying your measurement by a constant, multiply its uncertainty by the same constant

$$\begin{aligned}&\text{If} \quad \text{measured } x = 5.614 \pm 0.006 \\ &\text{then} \quad 10 \text{ times } x = 56.14 \pm 0.06\end{aligned}$$

   2) Addition and Subtraction

$$f(x,y) = \pm x \pm y \Rightarrow \delta f = \sqrt{\left(\frac{\partial f}{\partial x}\delta x\right)^2 + \left(\frac{\partial f}{\partial y}\delta y\right)^2} = \sqrt{(\delta x)^2 + (\delta y)^2}$$

   3) Multiplication and Division

$$f(x,y) = xy \Rightarrow \delta f = \sqrt{\left(\frac{\partial f}{\partial x}\delta x\right)^2 + \left(\frac{\partial f}{\partial y}\delta y\right)^2} = \sqrt{(y\delta x)^2 + (x\delta y)^2} \Rightarrow \frac{\delta f}{f} = \sqrt{\left(\frac{\delta x}{x}\right)^2 + \left(\frac{\delta y}{y}\right)^2}$$

The relative or fractional uncertainties add in quadrature.

4) Powers

$$f(x) = x^n \Rightarrow \delta f = \sqrt{\left(\frac{\partial f}{\partial x}\delta x\right)^2} = \sqrt{\left(nx^{n-1}\delta x\right)^2} \Rightarrow \frac{\delta f}{f} = \sqrt{\left(\frac{n\delta x}{x}\right)^2} = \left|\frac{n\delta x}{x}\right|$$

3) Example: Kinetic Energy

$$\text{mass} = 10.12 \pm 0.11\text{kg} \quad \text{and} \quad \text{velocity} = 2.34 \pm 0.04\text{m/s}$$

$$K(v) = \frac{1}{2}mv^2 \;\Rightarrow\; \delta K = \sqrt{\left(\frac{\partial K}{\partial m}\delta m\right)^2 + \left(\frac{\partial K}{\partial v}\delta v\right)^2} = \sqrt{\left(\frac{1}{2}v^2\delta m\right)^2 + \left(mv\delta v\right)^2}$$

$$\delta K = \sqrt{\left(\frac{1}{2}\left(2.34\text{m/s}\right)^2\left(0.11\text{kg}\right)\right)^2 + \left(\left(10.12\text{kg}\right)\left(2.34\text{m/s}\right)\left(0.04\text{m/s}\right)\right)^2} = 0.9939540\text{J} \Rightarrow 0.99\text{J}$$

Scatter Plot with Error Bars

1) Best Fit Straight Line (Linear Regression)

   1) Max and Min gradient (slope) lines

      Use endpoints and error on each end to fit line for max and min slope of line