ALIASING ON THE DIGITAL OSCILLOSCOPE

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A typical digital oscilloscope samples the input waveform at fixed time intervals, and then displays the digitized samples on the oscilloscope screen. These samples are normally connected by straight line segments in order to give, at least roughly, the appearance of a smooth waveform. Figure 1A shows a 2200 Hz sinusoidal waveform and, for comparison, the sampled approximation. The samples are shown as

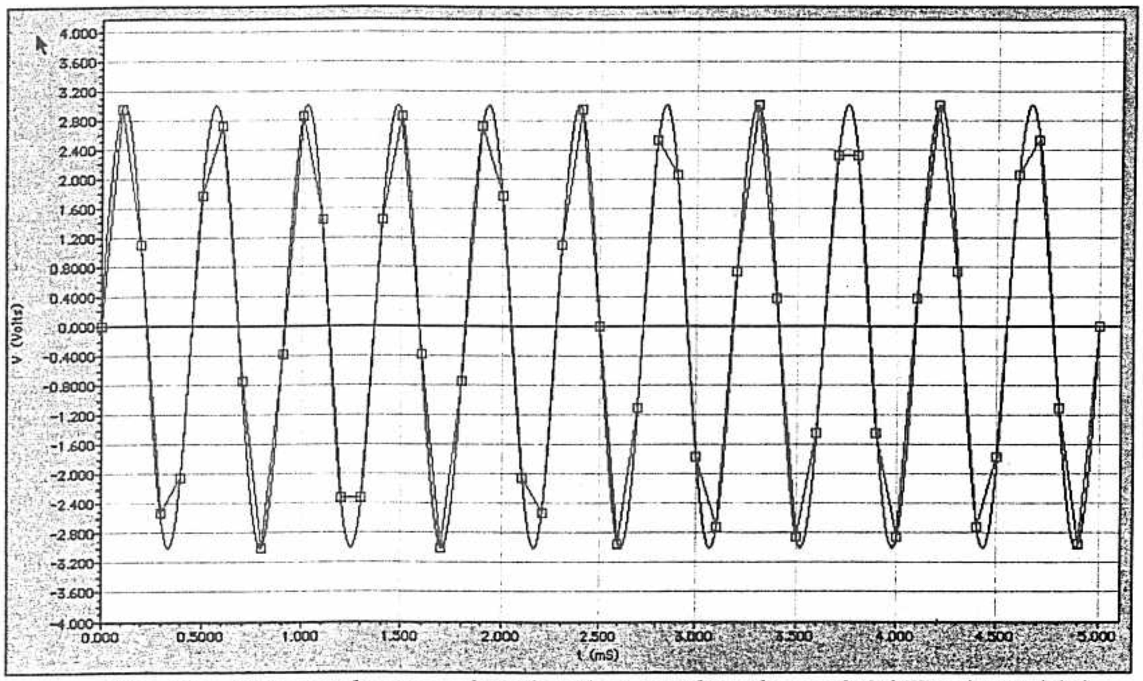

Fig. B1. The rough approximation that results when a 2.2 kHz sinusoidal waveform is sampled at 10 kHz

squares, and the squares are shown connected by straight line segments. In constructing Fig. B1 the time between samples was assumed to be 0.1 ms, which corresponds to a sampling frequency of 10 kHz, which is only 4.5 times the frequency of the input signal. Thus, on average, each cycle of the input signal is approximated by only about 5 points. Furthermore, the points are joined by straight line segments rather than a smooth curve. The distortion is obvious. Some of the cycles appear to have "missing peaks" since the oscilloscope digitizer did not happen to sample the input waveform when it was at a maximum.

A similar graph is shown in Fig. B2, but in this case the sampling frequency has been increased to 20 kHz, which is almost ten times the frequency of the input waveform. The digitized approximation now appears to have about the same shape as the input waveform. Now each cycle is approximated by ten straight line segments, rather than just five as before. However, there are still some obvious distortions. Note, for example, the 4^{th} peak from the left, which occurs near time $t = 1.5$ ms.

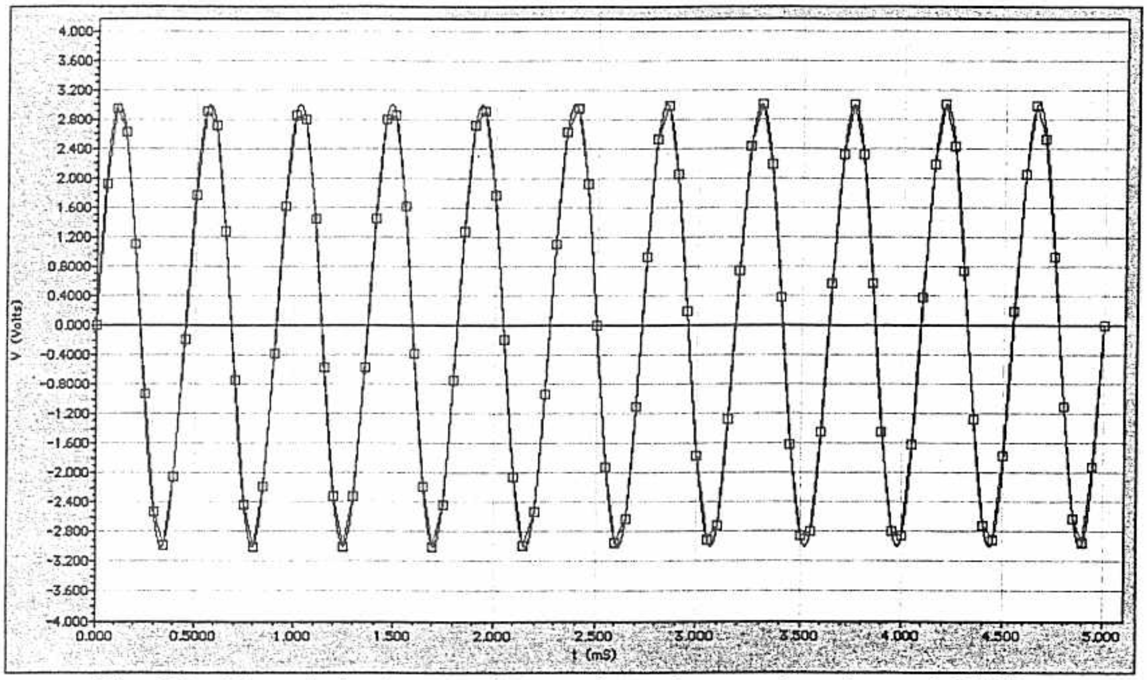

Fig. B2. The relatively smooth waveform that results when a 2.2 kHz sinusoidal waveform sampled at 20 kHz

The oscilloscope happened to sample the waveform on either side of the peak, but not at the peak itself. Therefore, when viewed on the oscilloscope screen, the peak will appear to be "flattened off" and slightly reduced in size relative to other peaks of the waveform. Thus, in order to obtain a waveform that is displayed smoothly on the screen, it is necessary to have an oscilloscope sampling frequency that is *more* than ten times the frequency of the signal.

At low sampling frequencies it is easy to be completely misled by the digitized waveform displayed on the oscilloscope screen. Figure B3 shows the same 2200 Hz signal as before, but the sampling frequency has been reduced to 2 kHz. Note that the input waveform is sampled only about once each cycle, and the digitized waveform

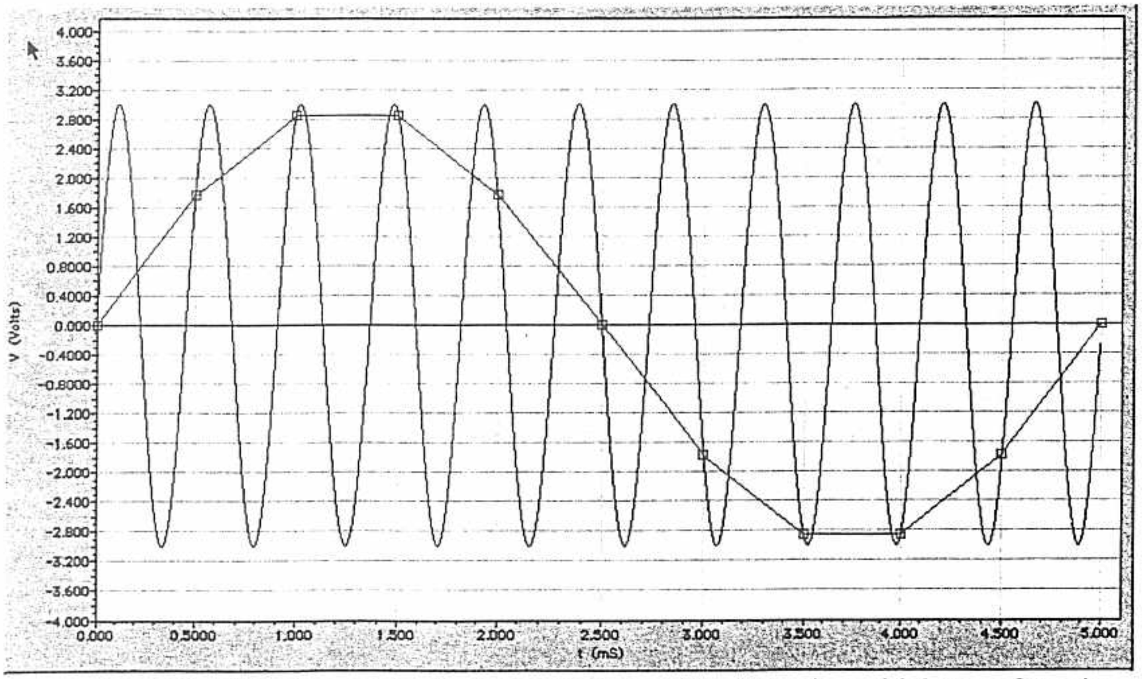

Fig. B3. Missing cycles that result when a 2.2 kHz sinusoidal waveform is sampled at 2 kHz, which does not meet the Nyquist criterion.

appears to be, at least approximately, a sinusoidal signal of much lower frequency than the input waveform! To avoid these kinds of errors, the sampling frequency must meet the requirement of the Nyquist theorem, which states that the sampling frequency must be at least twice the signal frequency. For comparison, Fig. B4 shows the 2200 Hz input waveform sampled at 5 kHz (time between samples is 0.2 ms). This sampling frequency meets the Nyquist criterion, which effectively means that the sampled waveform has no "missing cycles." However, it still looks greatly distorted. Based on the appearance of the sampled waveform in Fig. B4 one might be tempted to conclude, quite incorrectly, that the input waveform exhibits beats. (For an example of apparent beats in a free precession signal, refer to Fig. 18 on p. 22.)

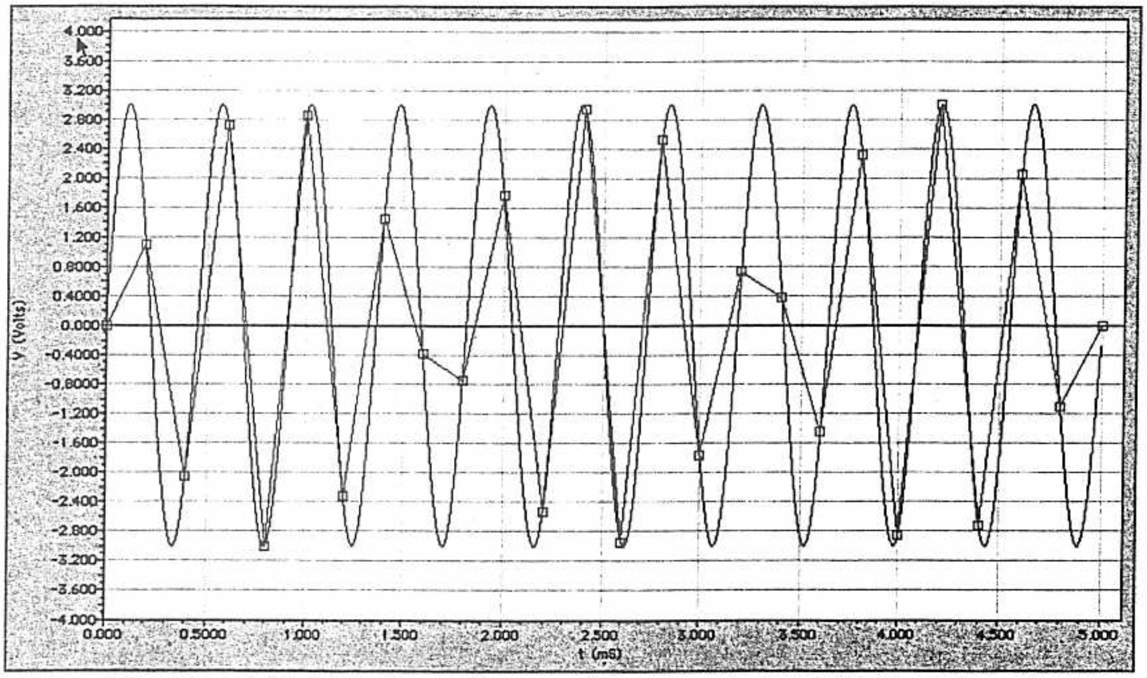

Fig. B4. A 2.2 kHz sinusoidal waveform sampled at 5.0 kHz. There is significant distortion even though the sampling frequency meets the Nyquist criterion.

So, what does all this mean with regard to using digital storage oscilloscopes for capturing free precession signals? First of all, it is obviously desirable to have a sampling frequency that is at least ten times the frequency of the free precession signal. For signals at 2.1 kHz, corresponding to an earth's field of 0.05 mT, the sample frequency should be of the order of 20 kHz or more. Specifications for digital oscilloscopes may claim maximum sampling frequencies of hundreds of thousands, or even millions, of samples per second. But those specifications may be misleading. Digital oscilloscopes vary, but, for most commonly-used oscilloscopes, the oscilloscope takes only 500 or 1000 samples during one horizontal sweep. Assuming the screen is 10 divisions wide, that corresponds to as few as 50 samples per division. At a sweep speed of 0.2 ms/Div, 50 samples per division corresponds to a time interval of 0.004 ms between each sample, which is equivalent to a sample frequency of only 25 kHz, barely sufficient to produce a smooth digitized waveform. At 50 samples per division and a sweep speed of 20 ms/Div, the time between samples is 0.4 ms. The corresponding sample frequency is only 2.5 kHz, which does not even meet the Nyquist criterion. Therefore, severe sampling errors like that shown in Fig. B3 can be expected when the oscilloscope sweep speed is 20 ms/Div or slower. The following examples will illustrate this principle.

Figure B5 shows a photograph of an oscilloscope waveform obtained from a 125-ml sample of water. The frequency of the free precession signal was 2.088 kHz. The oscilloscope sweep speed was 50 ms/Div. The waveform appears to be relatively

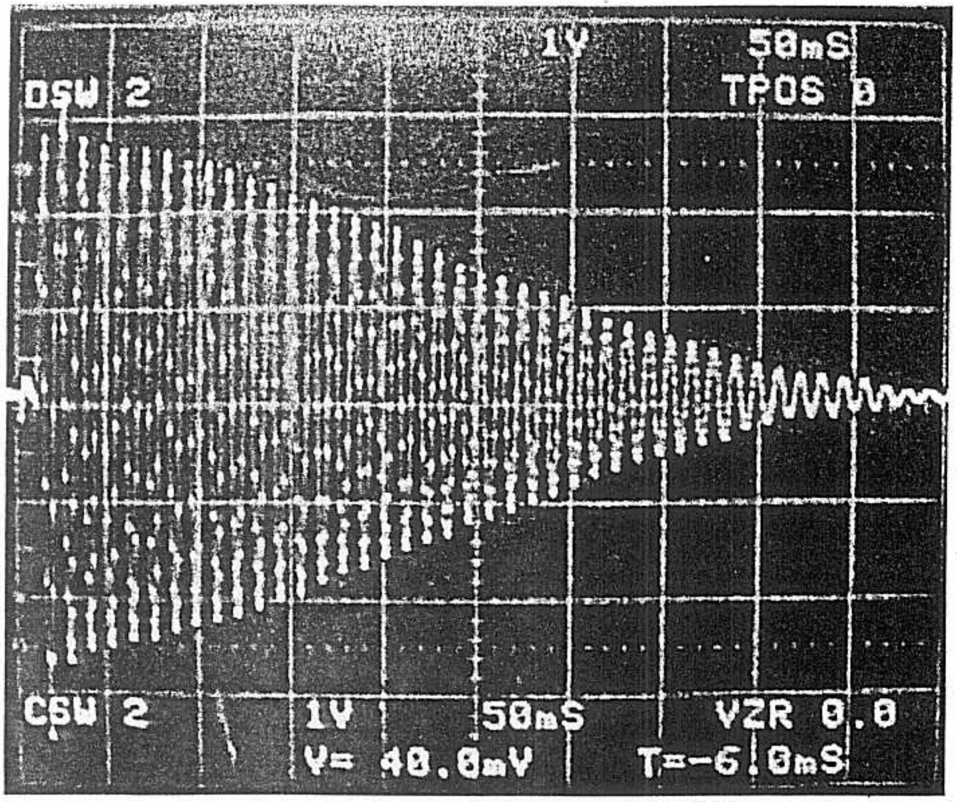

Fig. B5. Apparently smooth free precession signal at a frequency of 2.088 kHz. The sample frequency was 2 kHz.

smooth, with multiple samples per cycle. However, that is not the case. The oscilloscope was sampling at 100 samples/division, which corresponds to a time interval between samples of 0.5 ms and a sample frequency of 2 kHz. Thus, the sample frequency is just slightly less than the frequency of the signal. The situation is similar to that in Fig. B3. What appears in Fig. B5 to be many closely-spaced samples on the same cycle are actually single samples taken on many successive cycles. For a signal frequency of 2.088 kHz and a sample frequency of 2.000 kHz, the difference in frequencies (or beat frequency) is 88 Hz, which is the apparent frequency of the digitized waveform in Fig. B5.

The waveform in Fig. B6 shows an even more extreme example of aliasing The 2.007 kHz signal was obtained from fluorine nuclei in a 25-gram sample of

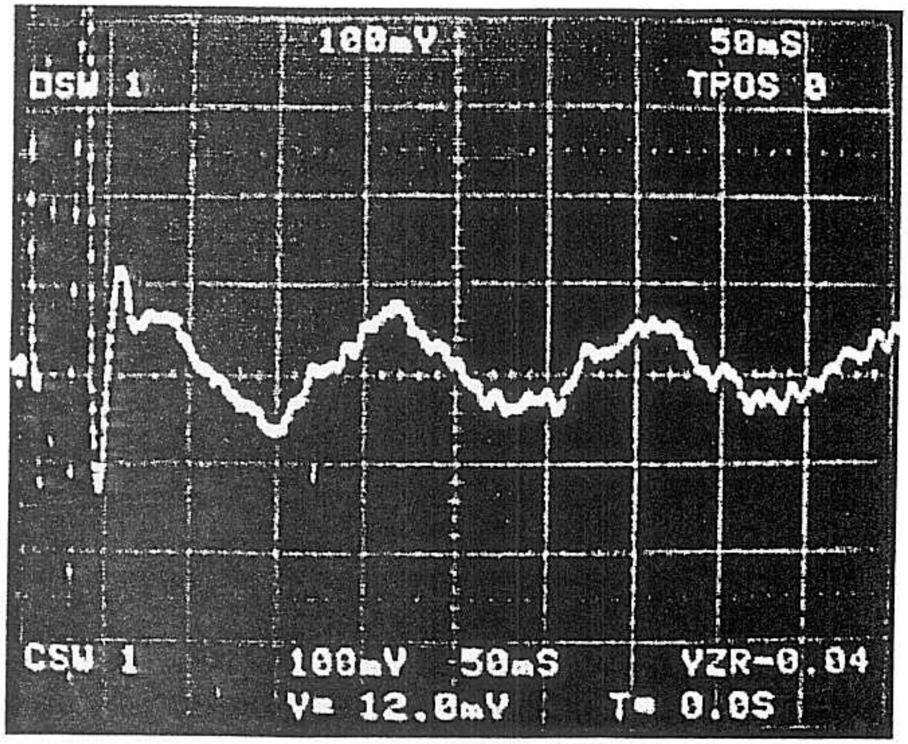

Fig. B6. 2.007 kHz fluorine signal from a 25-gram sample of C_6F_6. The oscilloscope sample frequency was 2.000 kHz. The digitized waveform appears to have a frequency equal to the difference, 7 Hz.

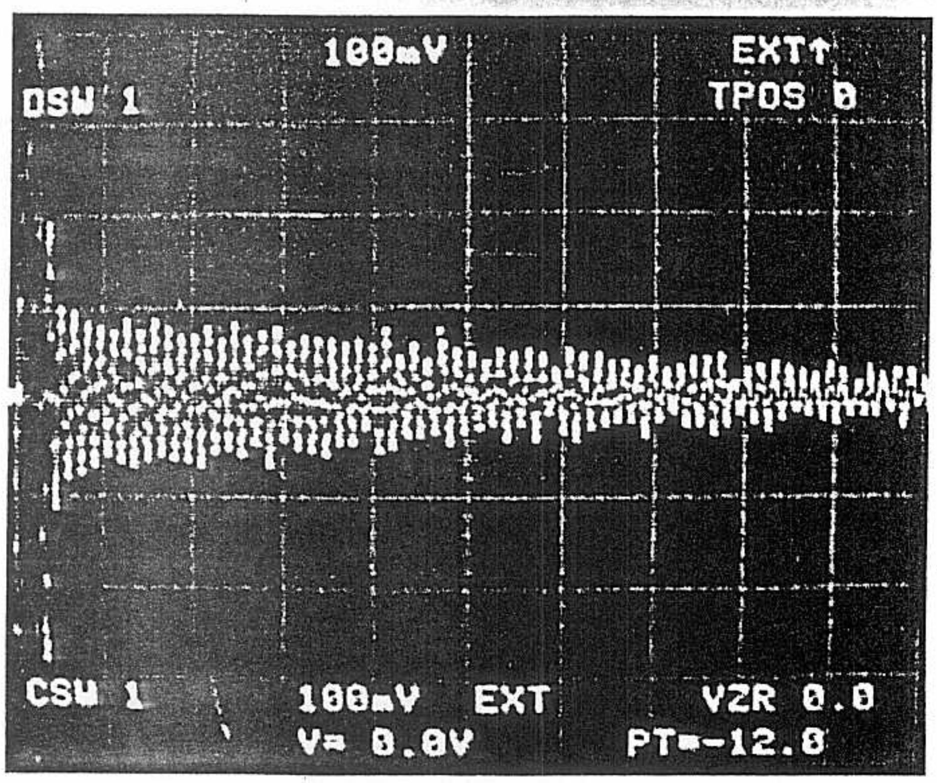

Fig. B7. Same as Fig. B6, except the oscilloscope sample frequency was reduced to 1.040 kHz. Since the oscilloscope displays 100 samples per division, the equivalent sweep speed is 96 ms/Div.

hexafluorobenzene, C_6F_6. The oscilloscope sweep speed was 50 ms/Div, and the oscilloscope sample frequency was 2 kHz. Here, the signal and sample frequencies differ by only 7 Hz, which is identical to the apparent frequency of the sampled waveform observed on the oscilloscope screen. The same fluorine signal is shown in Fig. B7, but there the sample frequency was reduced to 1.040 kHz. The time between samples was 0.96 ms, which is almost twice the period. The situation is similar to that shown in Fig. B3, except the sample frequency was so low that the sampling process skipped whole cycles. Yet, the sampled waveform appears surprisingly smooth.