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SU(3) Flavor Breaking in Hadronic Matrix Elements for $B-\bar{B}$ Oscillations

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Abstract

Results in the quenched approximation for SU(3) breaking ratios of the heavy-light decay constants and the $\Delta F=2$ mixing matrix elements are reported. Using lattice simulations at $6/g^2 = 5.7$, 5.85, 6.0, and 6.3, we directly compute the mixing matrix element $M_{hl} = \langle \bar{P}_{hl}|\bar{h}\gamma_\mu(1-\gamma_5)l\bar{h}\gamma_\mu(1-\gamma_5)l|P_{hl}\rangle$. Extrapolating to the physical B meson states, B^0 and B^0_s, we obtain $M_{bs}/M_{bd} = 1.76(10)^{+57}_{-42}$ in the continuum limit. The systematic error includes the errors within the quenched approximation but not the errors of quenching. We also obtain the ratio of decay constants, $f_{bs}/f_{bd} = 1.17(2)^{+12}_{-6}$. For the B parameters we find $B_{bs}(2GeV) = B_{bd}(2GeV) = 1.02(13)$; we cannot resolve the SU(3) breaking effects in this case.

Using lattice methods, one can calculate the $\Delta F = 2$ heavy-light mixing matrix element,

$$M_{hl}(\mu) \equiv \langle \bar{P}_{hl} | \bar{h}\gamma_\rho(1-\gamma_5) l \bar{h}\gamma_\rho(1-\gamma_5) l | P_{hl} \rangle \ . \tag{1}$$

As is well known, these matrix elements govern $B^0 - \bar{B}^0$ and $B_s^0 - \bar{B}_s^0$ oscillations [1–3]. In the above h and l denote heavy and light quark fields, P_{hl} the corresponding pseudoscalar meson, and μ is the energy scale appropriate to the calculation. Here we compute directly the SU(3) flavor breaking ratio [4],

$$r_{sd} = M_{bs}(\mu)/M_{bd}(\mu) \ . \tag{2}$$

Our central result is that $r_{sd} = 1.76(10)^{+57}_{-42}$ in the quenched approximation, where the first error is statistical and the second systematic. The importance of this ratio is that, in conjunction with the eventual experimental measurement of $B_s^0 - \bar{B}_s^0$ oscillations, it should allow the cleanest extraction of the crucial CKM parameter V_{td}.

Since the CKM matrix elements are fundamental parameters of the Standard Model, it is clearly important to determine them precisely. V_{td} is especially significant because low energy manifestations of CP violation, which enter through virtual t–$\bar{t}$ loops, invariably involve V_{td}. At present, V_{td} is deduced from $B^0 - \bar{B}^0$ oscillations via the mixing parameter $x_{bd} = \Delta M_{bd}/\Gamma_{bd}$ [5].

$$x_{bd} = \tau_{bd} \frac{G_F^2}{6\pi^2} m_{bd} b(\mu) B_{bd}(\mu) f_{bd}^2 M_W^2 \eta_{QCD} S(x_t) |V_{td}|^2, \tag{3}$$

where m_{bd}, $\tau_{bd} \equiv \Gamma_{bd}^{-1}$, and f_{bd} are the mass, life time, and decay constant of the B^0 meson, and $(\Delta M)_{bd}$ is the mass difference of the two mass eigenstates of the $B^0 - \bar{B}^0$ system. x_{bd} is the mixing parameter characterizing the oscillations and has been determined experimentally, $x_{bd} = 0.73(5)$ [6]. B_{bd} is the so called bag parameter, and $b(\mu)$ and $S(x_t)$ are perturbatively calculated short distance quantities [5]. To extract V_{td} from Eq. (3) requires knowledge of two hadronic matrix elements, f_{bd} and B_{bd}. These are being calculated using lattice and other methods. f_{bd} may eventually be measured experimentally through, for example, the decay $B \to \tau\nu_\tau$. However, B_{bd} is a purely theoretical construct which is inaccessible to experiment. Thus determination of V_{td} from experiment will ultimately be limited by the precision of the nonperturbative quantity $f_{bd}^2 B_{bd}$. These parameters are related to the matrix element Eq. (1) via

$$M_{bd}(\mu) = \frac{8}{3} f_{bd}^2 m_{bd}^2 B_{bd}, \tag{4}$$

and often one writes $b(\mu)M_{bd}(\mu) = \hat{M}_{bd}$, a renormalization group invariant (RGI) quantity.

Making the replacement $d \to s$ in Eq. (3) and taking the ratio with Eq. (3), we arrive at an alternate way to extract V_{td},

$$\frac{|V_{td}|^2}{|V_{ts}|^2} = r_{sd} \frac{m_{bd}}{m_{bs}} \frac{\tau_{bs}}{\tau_{bd}} \frac{x_{bd}}{x_{bs}} \tag{5}$$

Thus, in contrast to the above method for determining V_{td} via use of Eq. (3), once the $B_s^0 - \bar{B}_s^0$ oscillation parameter, $x_{bs} \equiv \frac{(\Delta M)_{bs}}{\Gamma_{bs}}$, is experimentally measured, we can use Eq. (5) to determine V_{td}. The right hand side of Eq. (5) involves three SU(3) breaking ratios, only one of which, namely r_{sd}, needs to be calculated non-perturbatively. The remaining two can be measured experimentally, at least in principle. Indeed, since the spectator approximation is expected to hold to a very high degree of accuracy [7], it is also reasonable to expect that $\tau_{bs}/\tau_{bd} = 1$ within a few percent. Of course, the measurement of x_{bs} is very challenging. A variety of experimental efforts are underway at both e^+e^- and hadronic machines towards that goal [8]. Note also that V_{ts} in Eq. (5) is related by three generation unitarity to V_{cb} and is therefore already quite well determined, $|V_{ts}| \approx |V_{cb}| = 0.041 \pm 0.003 \pm .002$ [6]. The important distinction between using Eq. (5) instead of Eq. (3) is that the former requires only knowledge of *corrections* to SU(3) flavor symmetry while the latter requires the *absolute* value of the matrix element M_{bd}. It is also important to realize that since r_{sd} is a ratio of two very similar hadronic matrix elements, it is less susceptible to common systematic errors in lattice calculations, among which are scale dependence, matching of continuum and lattice operators, and heavy quark mass dependence. Indeed, the ratio r_{sd} is, to an excellent approximation, RG invariant, even though the individual matrix elements M_{bs} and M_{bd} are scale dependent.

In passing, we recall that flavor symmetries have also played a crucial role in determining other CKM matrix elements. In particular, SU(3) flavor symmetry has been important in precisely determining $V_{us} \equiv \sin\theta_c$. More recently, heavy quark symmetry (HQS) [9] has been used to improve systematically the determination of V_{cb}.

The lattice methodology for calculating these matrix elements (*i.e.* Eq. (1)) is, by now, well known [10]. The amplitudes for $B_0 - \bar{B}_0$ mixing, usually called "box" amplitudes, occur at second order in the weak interaction. After integrating out the W boson, the operator product expansion (OPE) allows one to write the corresponding amplitude as a short distance expansion. In this case there is only one operator in the expansion, $\mathcal{O}_{LL} = \bar{b}\gamma_\mu(1-\gamma_5)d\bar{b}\gamma_\mu(1-\gamma_5)d$. Its (Wilson) coefficient, $C_{LL}(\mu)$, is calculated most easily in continuum perturbation theory. The matrix element of $\mathcal{O}_{LL}$ must be calculated non-perturbatively on the lattice since it contains the long distance QCD information of the physical process in question. The product of the two yields the scale invariant amplitude, which is obtained by translating either result from one regularization scheme to the other. We accomplish this in the usual way by matching the lattice operator to the continuum operator in a particular scheme at some low energy scale. For convenience we choose the scale $\mu = 2$ GeV. Using the renormalization group equations, $C_{LL}(M_W)$ is then run down to this scale, which yields $C_{LL}(\mu)$.

For Wilson quarks the continuum-lattice matching for $\mathcal{O}_{LL}$ has been carried out to one loop in perturbation theory [11–13].

$$\mathcal{O}_{LL}^{cont} = 4\tilde{\kappa}_h\tilde{\kappa}_l \Bigg(\mathcal{O}_{LL}^{latt} + \frac{g^2}{16\pi^2}\big(\ Z_+(a\mu)\mathcal{O}_{LL}^{latt}$$
$$+ \frac{Z^*}{48}(2\mathcal{O}_{SS} + 6\mathcal{O}_{PP} - 11\mathcal{O}_{VV} + 11\mathcal{O}_{AA} + 2\mathcal{O}_{TT})\big)\Bigg) \qquad (6)$$

where $\mathcal{O}_{ii}$ corresponds to $\gamma_\mu(1-\gamma_5) \to 1$, γ_5, γ_μ, $\gamma_5\gamma_\mu$, and $\sigma_{\mu\nu}$ in the expression for $\mathcal{O}_{LL}$. The Wilson quark action explicitly breaks chiral symmetry, so these new operators arise to cancel the chiral symmetry breaking terms in $\mathcal{O}_{LL}^{latt}$. We use the naive dimensional regularization (NDR) scheme with "tadpole improvement," so $Z_+ = (-50.841 - 4\ln(a\mu) + 34.28)$ [13,11,12] where a is the lattice spacing. $Z^* = 9.6431$ and depends only on the Wilson r parameter

TABLE I. Summary of simulation parameters. κ_h and κ_l are the heavy and light Wilson quark hopping parameters.

$6/g^2$	conf.	size	κ_{light}	κ_{heavy}
5.7	83	$16^3 \times 33$	0.160 0.164 0.166	0.095 0.105 0.115 0.125 0.135 0.145
5.85	100	$20^3 \times 61$	0.157 0.158 0.159 0.160	0.092 0.107 0.122 0.130 0.138 0.143
6.0	60	$16^3 \times 39$	0.152 0.154 0.155	0.103 0.118 0.130 0.135 0.142
6.0	100	$24^3 \times 39$	0.152 0.154 0.155	0.103 0.118 0.130 0.135 0.142
6.3	100	$24^3 \times 61$	0.148 0.149 0.150 0.1507	0.100 0.110 0.125 0.133 0.140
6.5	40	$32^3 \times 75$	0.146 0.147 0.148 0.1486	0.100 0.110 0.120 0.132 0.137 0.142

which we set to 1. The last term in Z_+ comes from mean field improved perturbation theory [14], which removes tadpole terms. (Z^* is an off-diagonal correction which does not have tadpole contributions at this order.) The scale at which the coupling g in Eq. 6 is to be evaluated is not fixed at one loop, however. It has been estimated for the decay constant using the methods of Ref. [14] as $q^* = 2.316/a$ [15]. We use this scale to find the central values; the variation with two choices for the scale, $1/a$ and π/a, is used to determine the associated systematic error. The usual naive renormalization of the fermion fields, $4\kappa_h\kappa_l$, is modified by the El-Khadra-Kronfeld–Mackenzie(EKM) norm [16] which is more suitable for the heavy quarks in our simulations.

The Wilson quark action also introduces errors proportional to (powers of) the lattice spacing in observables. We attempt to remove these by simulating at several values of the coupling $6/g^2(a)$ and extrapolating to $a = 0$.

Table (I) summarizes the lattice data used in our analysis. For each κ_l and κ_h in Table I we calculate a quark propagator using a single point source at the center of the lattice and a point sink. These are contracted to obtain two and three point meson correlation functions which are fit simultaneously to obtain the matrix element M_{hl}.

In Fig. 1 we show sample results at $6/g^2 = 6.3$ for M_{hl} *vs.* κ_l^{-1} for each value of κ_h, where the quark mass in units of the lattice spacing is $am_q = (\kappa_q^{-1} - \kappa_c^{-1})/2$. Here, κ_c is the critical hopping parameter where the pion mass vanishes. M_{hl} is extracted from the three point pseudoscalar correlation function, which is proportional to M_{hl} for large time separations of the four quark operator, and the two pseudoscalar meson interpolating operators. Results for the physical B and B_s meson systems follow from a series of fits to the lattice data, which we use to extrapolate in the two parameters κ_h and κ_l. We use covariant fits and a jackknife procedure at each step to account for the correlations in the data.

To begin the extrapolations, κ_c and κ_s (the strange quark hopping parameter) are determined from a fit to the squares of the pseudoscalar masses as a function of κ_l^{-1} and $\kappa_{l'}^{-1}$ (l and l' refer to non-degenerate light quarks). We use the following fit form, which does *not* include the logarithmic terms relevant at very small quark mass [17].

$$m_{ll'}^2 = c_0 + c_1(\kappa_l^{-1} + \kappa_{l'}^{-1}) + c_2\kappa_l^{-1}\kappa_{l'}^{-1} + c_3(\kappa_l^{-2} + \kappa_{l'}^{-2}). \tag{7}$$

A typical fit is shown in Fig. 2. The values for κ_c and κ_s and χ^2/dof for each fit are summarized in Table II. The curvature in Fig. 2 is small but certainly present: including only constant and linear terms in the fits generally yields poor χ^2 values. The linear fits shown in Table II were obtained by omitting the 3, 2, 0, 3, 8, and 6 heaviest points for $6/g^2 = 5.7$ to 6.5, respectively. The linear fits then had acceptable values of χ^2 except at $6/g^2 = 6.0(24^3)$ and 6.3 where completely constrained fits were used. The values for κ_c obtained from the linear fits are in rough agreement with the quadratic fits; they are systematically low by one to two statistical standard deviations. Values for κ_c determined from the linear fits agree with earlier calculations [18–20] at $6/g^2 = 6.0$ and [18] at $6/g^2 = 5.85$. At $6/g^2 = 5.7$, 5.85, 6.0, and 6.3, κ_c is systematically higher by several statistical standard deviations than the values found by the MILC collaboration [21], and the value at 6.0 in Ref. [22]. In this study we use point sources on lattices with modest extent in the time direction. A detailed comparison with the data from Ref. [21] indicates that this is likely to be the main cause of the discrepancy.

Since higher order chiral effects are completely different in the quenched and full theories, one might argue that the linear fits are preferable on physical grounds. For our central values, we stick with the quadratic fits, which describe our data well, but we take the difference arising from a switch to linear fits (as well as from the κ_c shift necessary to reproduce the Ref. [21] data) as an estimate of one source of systematic errors.

Finding κ_s requires the scale a, which we set from af_π, to determine the lattice value of the kaon mass am_K (a^{-1} is also tabulated in Table (II)). Our values for κ_s using linear fits agree to about one σ with Ref. [18] at $6/g^2 = 5.85$ and Refs. [18,19] at 6.0. Refs. [18,19] used m_ρ to set the lattice spacing and am_K, among others, to determine κ_s. Here we compare with values determined from am_K. At $6/g^2 = 5.7$, 5.85, 6.0, and 6.3, κ_s determined from am_K (with af_π used to set the lattice spacing) agrees well with the results from Ref. [21]. One might expect the values of κ_s, like κ_c, to disagree among the various calculations since they are determined from the same data. However, the added statistical uncertainty from the kaon mass is enough to mask the systematic error. We mention the above because the flavor breaking ratios given below are sensitive to the (relative) values of κ_c and κ_s. We also note that at $6/g^2 = 5.7$ the choice of the coupling constant scale for Z_A, the lattice axial current renormalization which appears in the determination of f_π, has a significant effect on the lattice spacing determination; Z_A differs by $\sim 7\%$ when the scale changes from $1/a$ to π/a.

Next, we linearly extrapolate M_{hl} to $\kappa_l = \kappa_c$ and κ_s. The results for M_{hl} at $6/g^2 = 6.3$ (see Fig. 1) show a smooth linear behavior. Similar results are obtained at the other couplings. Up to this step all of the covariant fits have acceptable values of χ^2, except the point at $6/g^2 = 6.0$ (24^3). Results at this point also showed significant variation with the form of the chiral extrapolation. The three point correlators here do not exhibit true plateaus but instead monotonically decrease with time, so there is undoubtedly contamination from excited states and additional uncertainty coming from the choice of fit range, which is necessarily small. Also, at $6/g^2 = 6.5$ the data were too noisy to extract M_{hl}. Finally, the

TABLE II. Inverse lattice spacing and critical and strange hopping parameters. For each value of $6/g^2$, the two rows correspond to a determination of κ_c and κ_s by quadratic and linear fits, respectively, to the pseudoscalar spectrum. For the linear fits, the 3, 2, 0, 3, 8, and 6 heaviest points are omitted for $6/g^2 = 5.7$ to 6.5, respectively. Each value of κ_c results in a corresponding lattice spacing from af_π. χ^2/dof refers to the fit used to determine the quantity immediately to the left. An entry of "cf" means a completely constrained fit.

$6/g^2$	a^{-1}(GeV)	χ^2/dof	κ_c	χ^2/dof	κ_s	χ^2/dof
5.7	1.37(10)	0.31/2	0.16973(15)	0.24/2	0.1645(7)	0.24/2
	1.35(9)	0.31/2	0.16953(9)	0.43/1	0.1640(8)	0.43/1
5.85	1.65(13)	0.01/1	0.16170(8)	0.33/3	0.1576(6)	0.33/3
	1.64(13)	0.01/1	0.16157(5)	1.30/4	0.1575(9)	1.30/4
6.0(16^3)	2.03(17)	3.16/1	0.15725(23)	0.62/1	0.1544(5)	0.62/1
	2.01(16)	3.16/1	0.15715(6)	1.08/3	0.1545(4)	1.08/3
6.0(24^3)	2.08(13)	0.67/1	0.15714(4)	2.6/1	0.1544(4)	4.0/1
	2.17(15)	0.67/1	0.15739(4)	cf	0.1548(4)	cf
6.3	3.09(21)	0.81/2	0.15199(4)	9.5/6	0.1502(2)	9.5/6
	3.10(21)	0.81/2	0.15191(4)	cf	0.1503(2)	cf
6.5	4.29(50)	0.38/2	0.14993(18)	1.04/5	0.1486(3)	1.04/5
	4.22(49)	0.38/2	0.14972(1)	3.04/1	0.1487(3)	3.04/1

point at $6/g^2 = 5.85$ is somewhat problematic. The statistical errors are large, so this point does not have a large impact on the continuum extrapolation. The difficulty arises in the three point correlators which show plateaus with a somewhat large oscillation. Three of the four light κ's happen to be below κ_s while the fourth is just above. Thus, our light κ's are closely spaced. The above considerations lead to a relatively inaccurate determination of the slope of M_{hl} *vs.* κ_l^{-1}, which essentially determines r_{sd}.

The heaviest mass points in our calculation suffer from heavy quark systematic errors; the lattices are too coarse to resolve objects with mass greater than the inverse lattice spacing.

The biggest correction of these errors comes from using the EKM norm mentioned above. An additional correction can be made by using the so-called kinetic mass [16] in place of the meson (pole) mass in the heavy mass extrapolations described below. As in Ref. [23], we use the tadpole improved tree level definition of the kinetic mass, $m_P^{kin} = m_P + \tilde{m}_2 - \tilde{m}$. $\tilde{m}_2$ and $\tilde{m}$ are the tadpole improved heavy quark kinetic and pole masses, respectively. This definition is motivated by a non-relativistic expansion of the heavy-light meson mass and reduces to the usual meson pole mass in the limit where the heavy quark becomes light. For the heaviest masses, the kinetic pseudoscalar mass is almost double the pole mass. This correction is also used in Refs. [24,25].

We fit M_{hc} to the HQET form

$$M_{hc} = c_{-1} m_P + c_0 + c_1 \frac{1}{m_P}. \tag{8}$$

Here m_P is any definition of the heavy-light pseudoscalar mass. The resulting fit is evaluated at the experimentally known B^0 meson mass to determine the physical value of the matrix element. For the heavy-strange case we first extrapolate M_{hl} to κ_s instead of κ_c. The form in Eq. 8 follows from the HQET results for the decay constants [1,2,9] and the B parameters:

$$f_P \sqrt{m_P} = d_0 + d_1/m_P + O(m_P^{-2}), \tag{9}$$

$$B_P = b_0 + b_1/m_P + O(m_P^{-2}). \tag{10}$$

Our data are consistent with these forms. We note that for each value of $6/g^2$ all of the data points are covariantly fit to the above form, and each fit has a good confidence level except the one at 6.0 (24^3). An example is shown in Fig. 3. It is noteworthy that the data fit the form in Eq. 8 over such a large range (this is true for all of the couplings we studied). At each coupling the heaviest (kinetic) mass is close to the physical B mass.

Fig. (4) shows $r_{sd} = M_{bs}/M_{bd}$ as a function of a. The ratio is greater than unity for each value of $6/g^2$. Using Eq. 6, the renormalization scale is set to $\mu = 2$ GeV and the coupling is evaluated at q^*. r_{sd} is also tabulated in Table III.

TABLE III. Summary of results for r_{sd}. The last two rows refer to constant and linear continuum extrapolations, respectively. The errors shown in parentheses are statistical. Column 1 gives central values; columns 2-8 represent systematic differences in r_{sd} relative to column 1 and are used to estimate the corresponding error (see text).

$6/g^2$	1	2	3	4	5	6	7	8
5.7	1.65(15)	1.64 (15)	1.50 (15)	1.52 (25)	1.52 (25)	1.68 (15)	1.59 (14)	1.76 (20)
5.85	1.78(25)	1.75 (24)	1.44 (30)	2.87 (2.20)	2.07 (45)	1.77 (27)	1.68 (21)	2.21 (27)
6.0(16^3)	1.80(23)	1.79 (23)	1.64 (25)	1.70 (23)	1.76 (22)	1.74 (19)	1.66 (19)	1.79 (23)
6.3	1.96(23)	1.94 (23)	1.95 (33)	1.85 (34)	1.97 (26)	1.83 (19)	1.68 (16)	1.99 (30)
∞	1.76(10)	1.75 (10)	1.58 (8)	1.67 (15)	1.77 (13)	1.74 (9)	1.64 (8)	1.90 (12)
∞	2.18(39)	2.21 (42)	2.03 (42)	2.10 (56)	2.29 (48)	1.94 (34)	1.76 (29)	2.09 (49)

As mentioned earlier, we expect the Wilson quark action to introduce discretization errors of order a in all observables. However, for the ratio of two similar quantities, we also expect a significant cancellation of these errors. A constant fit gives $M_{bs}/M_{bd} = 1.76(10)$ while a linear extrapolation in a gives 2.18(39) (column 1 in Table III). The above fits have small χ^2 values due to the large statistical errors, and we cannot rule out one fit in favor of another based on χ^2. The measured slope for the linear fit differs from zero by $\lesssim 1\,\sigma$. The linear trend may easily disappear with a one standard deviation change in either of the two end points, so we use the constant fit as our central value and the linear result as an estimate of the systematic error in the continuum extrapolation.

Next we estimate other systematic uncertainties in our analysis. The details are given in Table III. Columns 2-8 refer to separate analyses where one parameter was changed from its reference value used to obtain column 1. The difference in the new extrapolated value is then taken as an estimate of the systematic error in r_{sd}. In the following we list the uncertainties (numbers in parentheses refer to the corresponding column in Table III). (2.) Changing the coupling constant scale to a^{-1} yields $r_{sd} = 1.75(10)$ and 2.21(42) for constant and linear continuum extrapolations, respectively. (3.) Using quadratic chiral extrapolations for the matrix elements yields 1.58(8) and 2.03(42). The fits used for the chiral extrapolations were completely constrained except at $6/g^2 = 6.3$ and 5.85, each of which had one degree of freedom. (4.) Using the uncorrected pole mass yields $r_{sd} = 1.67(15)$ and 2.10(56). For this last case we note that χ^2 values were uniformly poor. The resulting fits underestimated the data at the heavy masses; the lighter masses, which had smaller statistical errors, dominated the fits. (5.) Constrained fits using only the heaviest masses yield 1.77(13) and 2.29(48), which we use to estimate the systematic error from including heavy masses that may be too light. (6.) As previously noted, we expect r_{sd} to be sensitive to small relative shifts between κ_c and κ_s. Using the linear extrapolations for κ_c and κ_s, we find 1.74(9) and 1.94(34). The main effect is to lower the value of r_{sd} at $6/g^2 = 6.3$, which primarily affects the linear continuum extrapolation. (7.) As mentioned earlier, our values of κ_c may be systematically high. Shifting κ_c by -0.0003 at each coupling gives $r_{sd} = 1.64(8)$ and 1.76(29). The shift was estimated from the difference of our κ_c values with those of Ref. [21] where Gaussian sources and longer lattices in the time direction were used. While the absolute shift is numerically small, it amounts to several statistical standard deviations and is thus not accounted for in the jackknife analysis.

(8.) Finally, we estimate the systematic error resulting from changing the fit range of the three point correlation functions. Thus far acceptable values of χ^2 were obtained using covariant fits to the three point correlation functions. However, with point sources (and lattices with modest extent in the time direction) the correlators do not exhibit long plateaus, so the allowed fit range is necessarily small. The fit ranges were shifted up or down by one or two time slices at each $6/g^2$ which generally resulted in worse χ^2 fits. The only appreciable variations were at $6/g^2 = 5.7$ and 5.85.

Columns 2,3,4,6, and 7 give lower results than the central value (column 1) for the preferred (constant) continuum extrapolation. Adding the differences linearly gives a systematic error estimate of -0.42. Combining the positive differences in columns 5 and 8 with that from the linear continuum extrapolation gives $+0.57$. The final result is then $r_{sd} = 1.76(10)^{+57}_{-42}$.

After our initial determination [4], another group calculated r_{sd} in the *static* approximation at $6/g^2 = 6.0$. They find 1.35(5) [26]. When extrapolated to the static limit, $m_P^{-1} = 0$, our data at $6/g^2 = 6.0$ yield $r_{sd} = 1.39(30)$ which agrees well with the above. In addition, our data at each value of $6/g^2$ indicates that r_{sd} is a smoothly increasing function of m_P^{-1}; thus the static result may be a lower bound for r_{sd}.

The extraction of the individual values of M_{bd} and M_{bs} is clearly expected to have larger errors. Thus, conventionally [27,1,2] these matrix elements are given in terms of the corresponding B parameter, which is better behaved. Carrying out a similar continuum extrapolation as above for $B_{bd}(\mu)$, we find a constant fit yields $B_{bd}(2\,\mathrm{GeV}) = 0.97(3)$ while linear extrapolation in a gives 1.02(13). We cannot, however, distinguish $B_{bs}(2\,\mathrm{GeV})$ from $B_{bd}(2\,\mathrm{GeV})$ since our data for B_{hl} *vs.* κ_l^{-1} are fit equally well to constant or linear fits. This was not true for M_{hl}, as is evident from Fig. 1. Using linear extrapolations in both a and κ_l^{-1}, we quote $B_{bd}(2\,\mathrm{GeV}) = B_{bs}(2\,\mathrm{GeV}) = 1.02(13)$, where the error is purely statistical (systematic errors are small in comparison).

We recall that until now [1,2], lattice results for the SU(3) breaking ratio r_{sd} have been obtained by using Eqs. (2) and (4) and the lattice determinations of $f_{bd(s)}$ and $B_{bd(s)}$. A simultaneous fit of the pseudoscalar and axial vector correlators yields the decay constant f_{hl}. Using Eq. 9 plus corrections up to $O(m_{hl}^{-2})$, we find for the ratio of B meson decay constants, $f_{bs}/f_{bd} = 1.17(2)(+2)(+5)(+1)(-2)(\pm 4)$ (see Fig. 5 and Table IV). This result is for a constant continuum extrapolation which is reasonable for the data shown in Fig. 5. The uncertainties are statistical, and the following systematic differences from: (2.) using the pole mass instead of the kinetic mass, (3.) using quadratic chiral extrapolations for the heavy-light mesons instead of the linear ones, (4.) replacing quadratic chiral extrapolations with linear extrapolations in the determination of κ_c and κ_s, and (5.) a shift in k_c as before. Again, the numbers in parentheses correspond to the columns in Table IV. Finally, we consider the overall uncertainty in the slope of the ratio versus lattice spacing. A linear continuum extrapolation using all the data has a negative slope; while omitting the point at $6/g^2 = 5.7$ yields a positive slope. Also, the results in columns 4 and 5 have positive

TABLE IV. Summary of results for the ratio f_{bs}/f_{bd}. The last two rows refer to constant and linear continuum extrapolations, respectively. The errors shown in parentheses are statistical. Columns 2-5 represent systematic differences relative to column 1 (see text). Where there is no entry, a reasonable fit was not found.

$6/g^2$	1	2	3	4	5
5.7	1.156 (26)	1.165 (97)	1.151 (66)	1.181 (30)	1.140 (25)
5.85	1.190 (80)	-	1.493 (249)	1.184 (79)	1.287 (95)
6.0(16^3)	1.187 (36)	1.180 (92)	1.204 (61)	1.182 (34)	1.172 (35)
6.3	1.159 (32)	1.180 (93)	1.266 (74)	1.177 (33)	1.149 (31)
6.5	1.190 (66)	1.289 (192)	1.408 (243)	1.145 (40)	1.100 (46)
∞	1.167 (17)	1.184 (52)	1.214 (37)	1.174 (17)	1.148 (16)
∞	1.183 (55)	1.241 (169)	1.375 (138)	1.152 (45)	1.133 (44)

slopes. Thus we include a symmetric error of ± 0.04 due to the continuum extrapolation. For f_{bs}/f_{bd}, there were no significant differences due to changing the fit range by one or two units. Adding the above systematic errors, we find $f_{bs}/f_{bd} = 1.17(2)^{+12}_{-6}$. We have omitted the larger volume at $6/g^2 = 6.0$ from the analysis for reasons similar to those described above. In addition, the data set at $6/g^2 = 5.85$ causes the same difficulties as before. For the ratio of decay constants we are able to get a statistically significant result at $6/g^2 = 6.5$ which is included in the above analysis.

Our result for f_{bs}/f_{bd} is consistent with previous estimates [1,2,23,28]. Note that while the decay constant using Wilson quarks has a perturbative correction (which does not depend on the scale μ), it cancels in the ratio (up to small quark mass corrections). As indicated above, the ratio of B parameters is consistent with unity, and the ratio of masses is 1.017 [6], so the old method leads to $r_{sd} \approx 1.42(5)^{+28}_{-15}$ which is compatible with, though somewhat lower than, $1.76(10)^{+57}_{-42}$ from our direct method. As we have emphasized, the direct method has many desirable features which may allow future lattice computations to significantly improve the precision of this method for the determination of the ratio r_{sd}.

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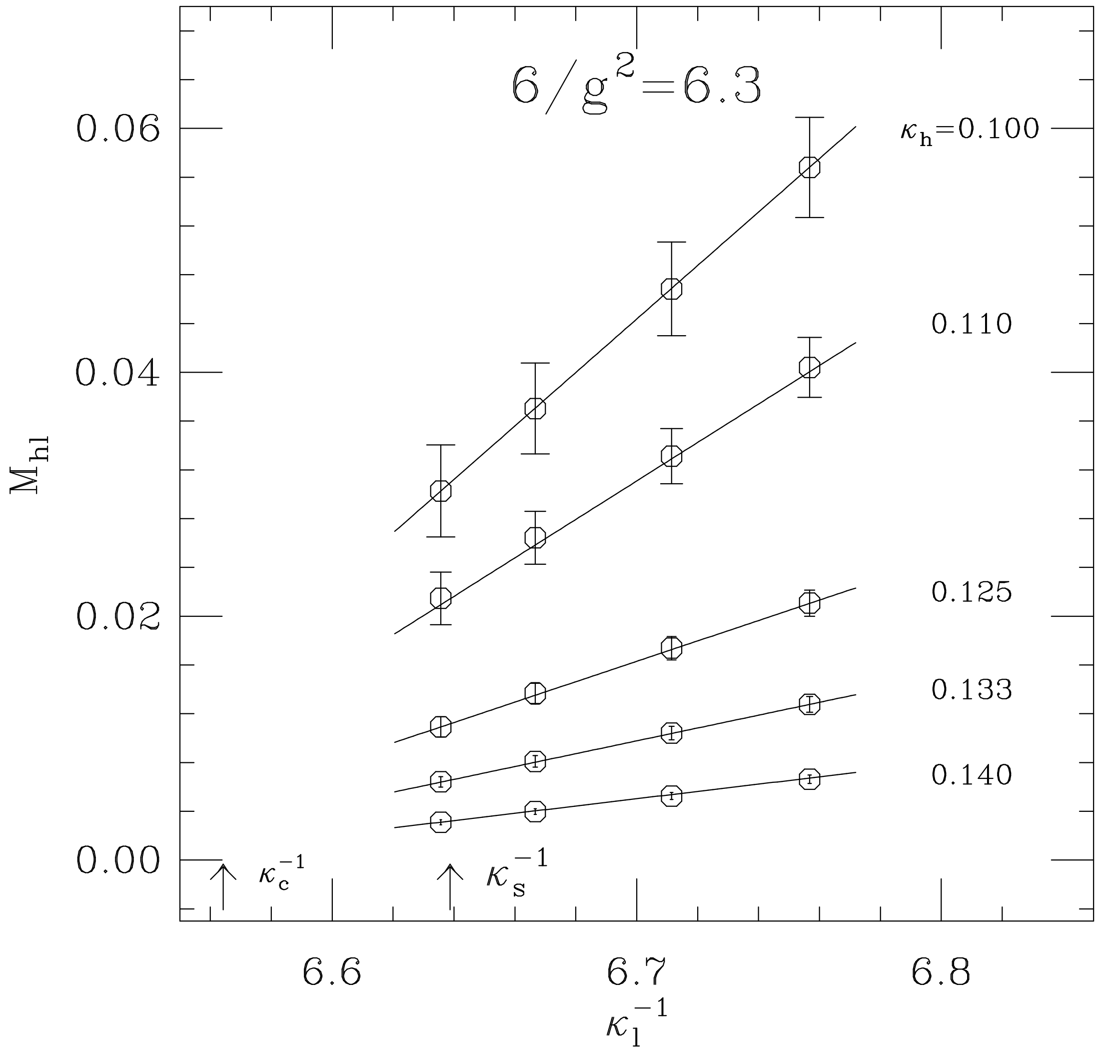

FIG. 1. The four quark matrix element M_{hl} at $6/g^2 = 6.3$ as a function of the inverse light quark hopping parameter. Results for the other values of $6/g^2$ are similar. The solid lines are covariant linear fits to the data.

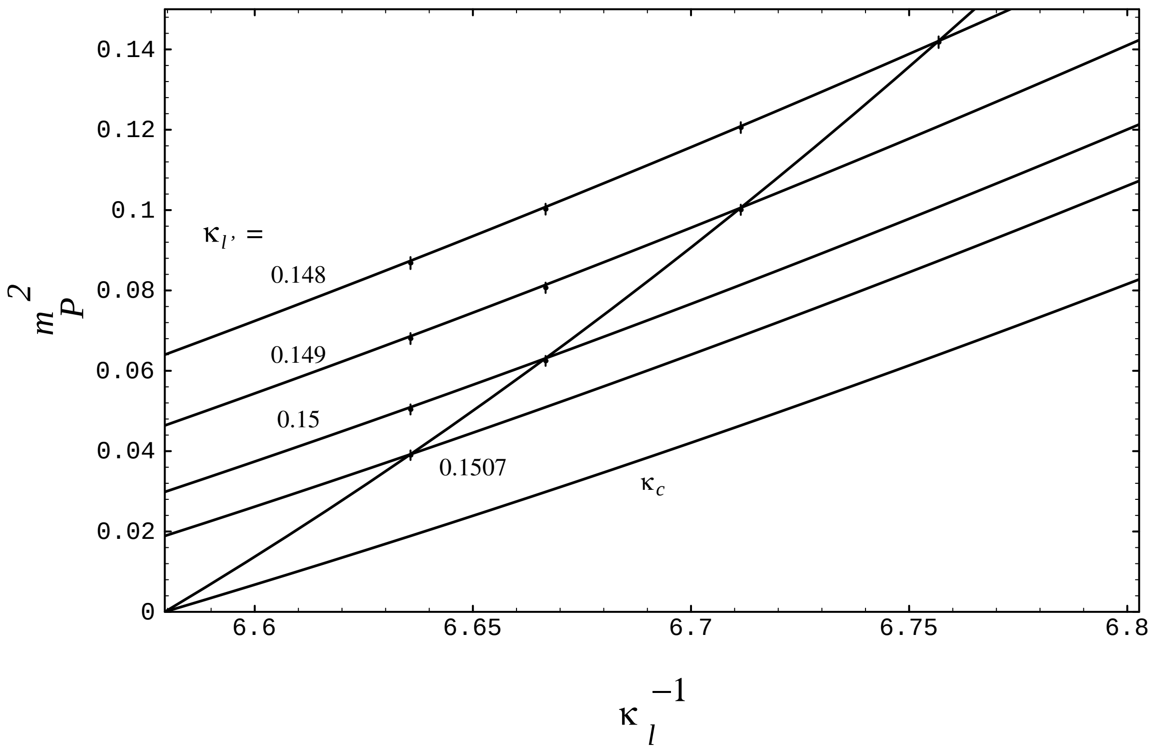

FIG. 2. The pseudoscalar mass squared as a function of the non-degenerate light quark hopping parameters. $6/g^2 = 6.3$. Results for the other values of $6/g^2$ are similar. The solid lines are from a covariant fit to the form in Eq. 7.

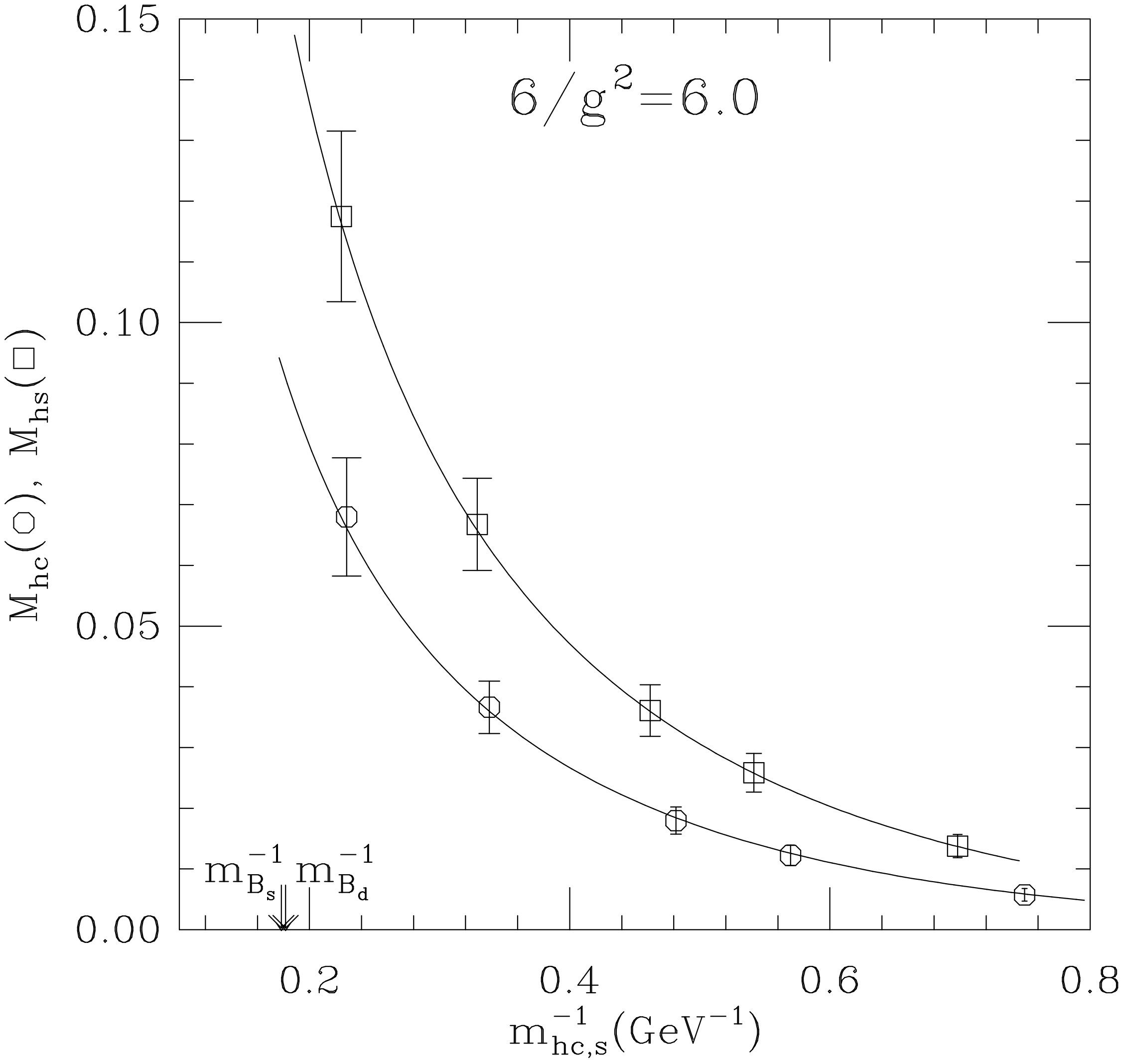

FIG. 3. The four quark mixing matrix element for down(octagons) and strange(squares) pseudoscalar mesons as a function of the inverse heavy-down(strange) meson mass at $6/g^2 = 6.0(16^3)$. The solid lines are covariant fits using Eq. 8 to all of the data points. We find very similar results at the other values of $6/g^2$.

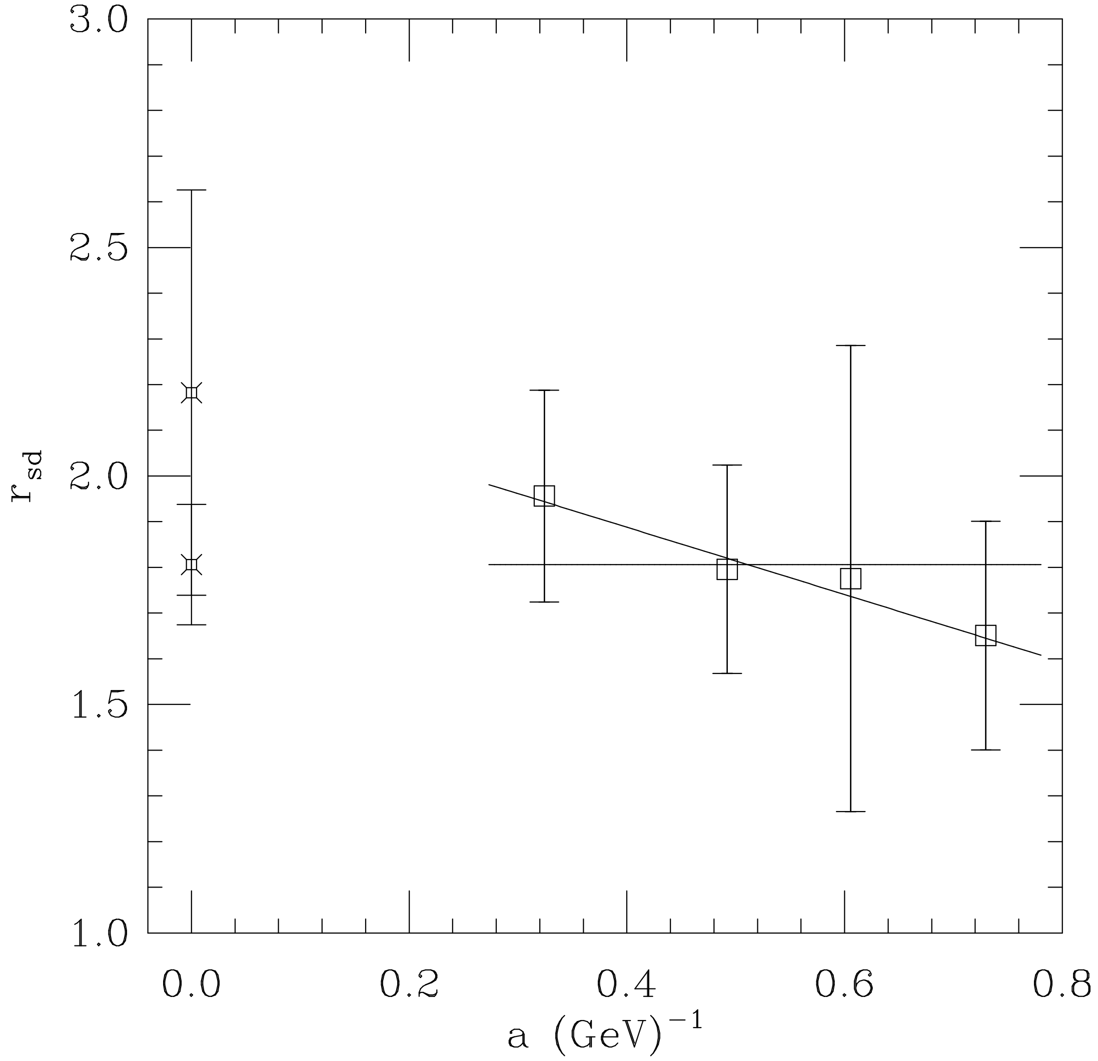

FIG. 4. The SU(3) flavor breaking ratio $r_{sd} \equiv M_{bs}/M_{bd}$ *vs.* the lattice spacing a. $\mu = 2$ GeV and the coupling in Eq. 6 has been evaluated at q^*. The lines denote constant and linear fits to the data, fancy squares the corresponding continuum extrapolations.

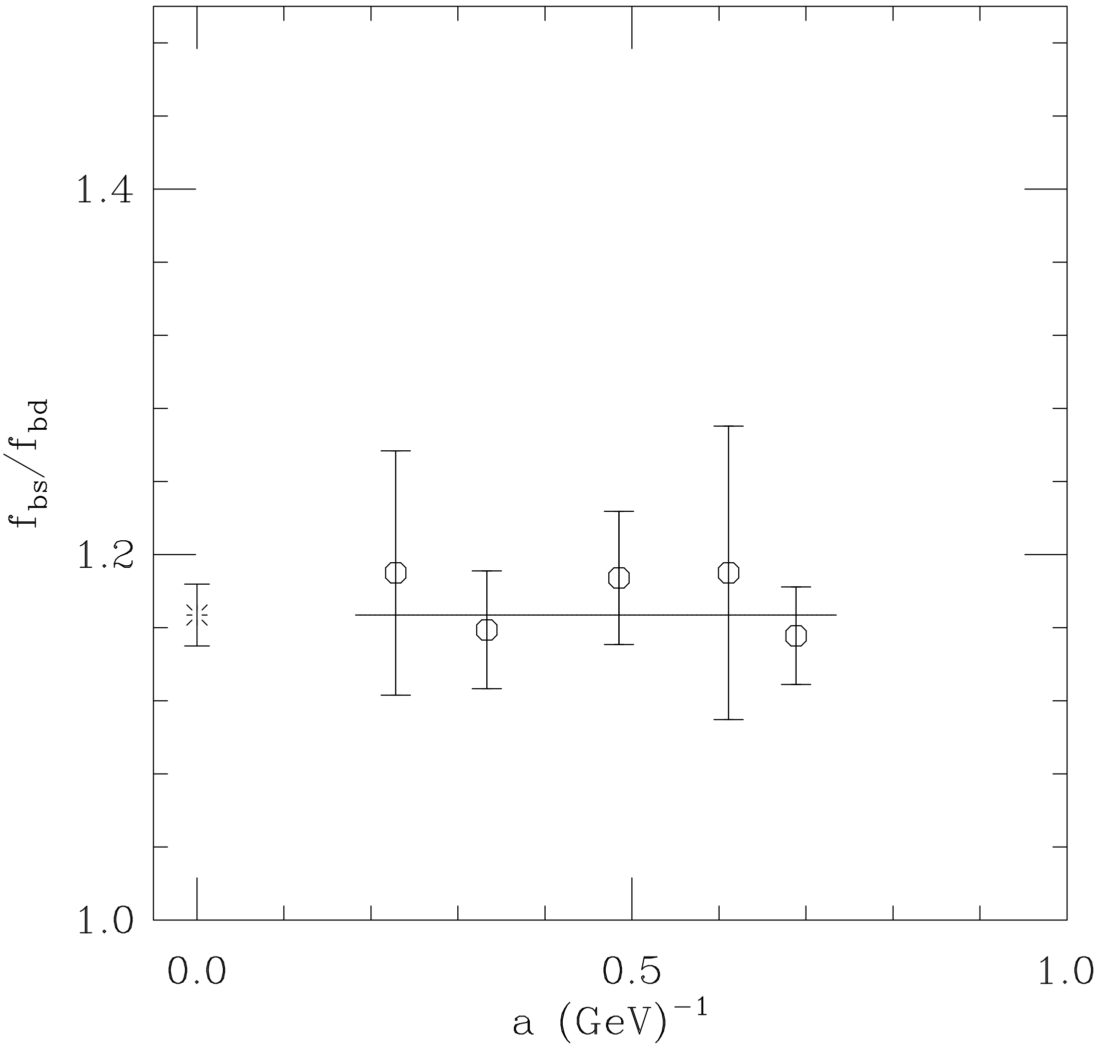

FIG. 5. The SU(3) flavor breaking ratio of decay constants f_{bs}/f_{bd} *vs.* the lattice spacing a. Results are plotted for linear chiral extrapolations. The line denotes a constant fit, and the burst is the corresponding continuum extrapolation.