Heavy-light decay constants from Wilson and static quarks

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MILC collaboration results for f_B, f_{B_s}, f_D, f_{D_s} and their ratios are presented. These results are still preliminary, but the analysis is close to being completed. Sources of systematic error, both within the quenched approximation and from quenching itself, are estimated. We find, for example, $f_B = 153 \pm 10\ {}^{+36}_{-13}\ {}^{+13}_{-0}$ MeV, and $f_{B_s}/f_B = 1.10 \pm 0.02\ {}^{+0.05}_{-0.03}\ {}^{+0.03}_{-0.02}$, where the errors are statistical, systematic (within the quenched approximation), and systematic (of quenching), respectively. The extrapolation to the continuum and the chiral extrapolation are the largest sources of error. Present central values are based on linear chiral extrapolations; a shift to quadratic extrapolations would raise f_B by $\approx$20 MeV and make the error within the quenched approximation more symmetric.

The MILC collaboration is continuing its program [1–4] of calculating the decay constants of heavy-light pseudoscalar mesons. The computations use Wilson light quarks and Wilson and static heavy quarks. We work on both quenched lattices, with a wide range of lattice spacings, and $N_F = 2$ dynamical staggered lattices. Table 1 gives the lattice parameters.

A major improvement in the past year has been the completion of dedicated runs to determine the static-light decay constants on lattices A,B,Q,E,G,L,N,O,M,P. These runs use a multi-source technique, with relative wavefunctions taken from [5]. On lattices C,D,H,F,G — generally the smaller physical volumes — we get acceptable static-light data as a simple by-product of the Henty-Kenway hopping parameter expansion [6] used for the heavy Wilson quarks. On lattice G, where both methods are available, the results are consistent.

A second improvement has been the calculation, following the approach of [7], of the scale q^* of the coupling in the perturbative renormalization constant Z_A of the axial current. For propagating Wilson quarks, the result is, after tadpole improvement, $q^* = 2.32/a$ [8]. Mass dependent effects are not included at this point. We estimate the systematic error of the renormalization by changing q^* by a factor of 2 and reanalyzing. The error is rather small ($\lesssim$3%).

Currently, we find (in MeV):

$$f_B = 153(10)({}^{+36}_{-13})({}^{+13}_{-0});\ \ f_{B_s} = 164(9)({}^{+47}_{-13})({}^{+16}_{-0})$$
$$f_D = 186(10)({}^{+27}_{-18})({}^{+9}_{-0});\ \ f_{D_s} = 199(8)({}^{+40}_{-11})({}^{+10}_{-0}).$$

The errors are statistical (plus "fitting"), systematic (within the quenched approximation), and

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Table 1
Lattice parameters. Lattices F, G, and L–P use variable-mass Wilson valence quarks and two flavors of fixed-mass staggered dynamical fermions; all other runs use quenched Wilson quarks. Lattice G was generated by HEMCGC; lattice F, by the Columbia group.

name	β (am_q)	size	# configs.
A	5.7	$8^3 \times 48$	200
B	5.7	$16^3 \times 48$	100
E	5.85	$12^3 \times 48$	100
Q	6.0	$12^3 \times 48$	235
C	6.0	$16^3 \times 48$	100
D	6.3	$24^3 \times 80$	100
H	6.52	$32^3 \times 100$	60
F	5.7 (0.01)	$16^3 \times 32$	49
G	5.6 (0.01)	$16^3 \times 32$	200
L	5.445 (0.025)	$16^3 \times 48$	100
N	5.5 (0.1)	$24^3 \times 64$	100
O	5.5 (0.05)	$24^3 \times 64$	100
M	5.5 (0.025)	$20^3 \times 64$	100(200)
P	5.5 (0.0125)	$20^3 \times 64$	199

systematic (of quenching), respectively. For the ratios, we get:

$$
\begin{aligned}
f_{B_s}/f_B &= 1.10(2)(^{+5}_{-3})(^{+3}_{-2}) \\
f_{D_s}/f_D &= 1.09(2)(^{+5}_{-1})(^{+2}_{-0}) \\
f_B/f_{D_s} &= 0.76(3)(^{+7}_{-4})(^{+2}_{-0}).
\end{aligned}
$$

A discussion of the most important sources of systematic errors follows.

• Chiral Extrapolation. The proper functional form to use in extrapolating physical quantities to the chiral limit is not clear. For example, although lowest order chiral perturbation theory predicts that m_π^2 is a linear function of quark mass, we observe [2–4] small but significant deviations from linearity.

These deviations could be due to unphysical effects such as the finite lattice spacing or volume. In addition, the curvature can be changed significantly, and sometimes made negligible, by shifting the fit range (in t) on the individual propagators. Even the more "physical" cause (chiral logs or higher order analytic terms) are a source of possible spurious effects because quenched chiral logs are in general different from those in the full theory [9].

For these reasons, we presently fit quantities like m_π^2 to their lowest order chiral form, despite the poor confidence levels. The systematic error is estimated by repeating the analysis with quadratic (constrained) fits. This error is $\leq 10\%$ for decay constants on all quenched data sets used to extrapolate to the continuum; usually it is $\lesssim 5\%$. (After extrapolation to the continuum, the error is larger: 7% to 15%.)

Our reasons for choosing linear chiral fits for the central values are somewhat subjective, and it is possible that we will switch to quadratic fits in the final version of this work. To help us make the choice, we are studying a large sample of quenched lattices at $\beta = 5.7$, with volumes up to 24^3 [10]. On this sample we have six light quark masses (as opposed to three for each of the lattices used for the heavy-light computation) and have light-light mesons with nondegenerate as well as degenerate quarks.

If a switch to quadratic fits were to be made now, it would raise the central values of f_B, f_{B_s}, f_D and f_{D_s} by 23, 19, 13, and 14 MeV, respectively. The systematic error within the quenched approximation would then become much more symmetric, with the continuum extrapolation the dominant positive error and the chiral extrapolation the dominant negative one.

• Heavy-Quark Interpolation. Having static-light results on all lattices allows us to find decay constants for physical B mesons by interpolation between heavy-light and static-light data, rather than extrapolation from the former. The interpolating fits have good confidence levels on all our data sets, reassuring us that the procedure [11] we use for the heavy-light data is reasonable.

One estimate of the systematic error of this approach is obtained by comparing decay constants computed with two different mass ranges of propagating quarks: "lighter heavies," (mesons 1.25 to 2 GeV) and "heavier heavies," (mesons 2 to 4 GeV). The difference is less than 1% at the three weakest quenched couplings ($\beta = 6.0, 6.3, 6.52$), less than 5% over all lattices, and less than 4% after linear extrapolation of all quenched lattices to $a=0$.

For quenched lattices A,B,E, the new static-light results produce only small changes from that reported previously [2]. However, on the large $N_F=2$ lattices, including the static point raises f_B by about one old (statistical) standard deviation, and reduces the statistical error (and the difference between using heavier-heavies and lighter-heavies) by about 50%.

• Extrapolation to the Continuum. For any physical quantity Q computed here, we expect $Q(a) = Q_{a=0}(1 + a\mathcal{M}_1 + \cdots)$. In practice, we find the slope to be large for the decay constants ($\mathcal{M}_1 \sim$ 300–650 MeV), with f_{B_s} the worst offender. This leads to large extrapolation errors ($\sim$ 12–27%). The ratios of decay constant are much better behaved, with $\mathcal{M}_1 \sim 100$ MeV and an error of $\sim$4–5%.

Figure 1 shows several fits of f_B *vs.* a used to estimate the two largest sources of systematic error. The central value is obtained from a linear fit to all the diamonds, which in turn use linear chiral fits, a lattice scale set by f_π, and the "EKM" corrections [11]. The error of the continuum extrapolation is estimated by comparing the central value with the results of a constant fit to the three diamonds with smallest values of a, a linear fit to the octagons (which use "lighter heavies" and no EKM corrections), and two other similar types of fits (not shown). The continuum extrapolation error is defined as the largest of these four differences and is in practice almost always given by the first difference. The difference of the extrapolation of the squares (which have a quadratic chiral extrapolation) and the central value determines the chiral extrapolation error.

• Quenching Effects. We have repeated our computations on several N_F=2 dynamical fermion lattices (crosses in Fig. 1). Such computations are not yet "full QCD" because (1) the virtual quark mass is fixed and not extrapolated to the chiral limit, (2) the $N_F = 2$ data is not yet good enough to extrapolate to $a=0$, and (3) we have two light flavors, not three. Thus the $N_F=2$ simulations are used at this point only for systematic error estimation. See [4] for details.

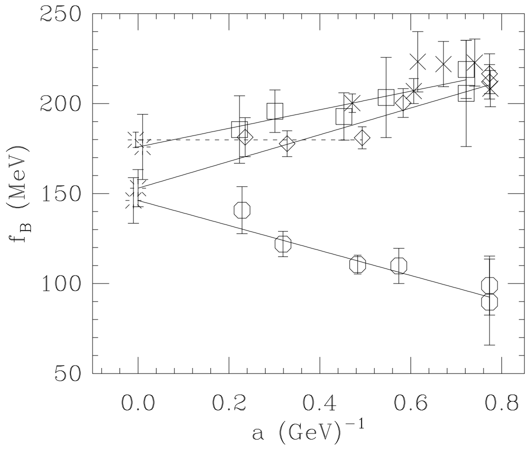

Figure 1. Results for f_B as a function of lattice spacing. The diamonds are quenched approximation points. Octagons and squares are used for estimating systematic errors — see text. Crosses are from $N_F=2$ lattices but are otherwise computed like the diamonds.

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