f_B for Various Actions : Approaching the Continuum Limit with Dynamical Fermions

C. Bernard[a], S. Datta[b*], C. DeTar[c], Steven Gottlieb[b], U.M. Heller[d], J. Hetrick[e], C. McNeile[f], K. Orginos[g], R. Sugar[h] and D. Toussaint[g]

[a]Department of Physics, Washington University, St. Louis, MO 63130, USA

[b]Department of Physics, Indiana University, Bloomington, IN 47401, USA

[c]Physics Department, University of Utah, Salt Lake City, UT 84112, USA

[d]CSIT, Florida State University, Tallahassee, FL 32306-4120, USA

[e]Department of Physics, University of the Pacific, Stockton, CA 95211, USA

[f]Dept. of Math. Sci., University of Liverpool, Liverpool, L69 3BX, UK

[g]Department of Physics, University of Arizona, Tucson, AZ 85721, USA

[h]Department of Physics, University of California, Santa Barbara, CA 93106, USA

We present results for pseudoscalar decay constants of heavy-light mesons using both quenched and $N_f = 2$ dynamical fermion configurations. A variety of fermion actions is investigated: Wilson, nonperturbative clover, and fat-link clover. For heavy quarks the Fermilab formalism is applied. In the quenched approximation, results with the nonperturbatively improved clover action of the Alpha collaboration allow us to study the systematic error of the continuum extrapolation from the Wilson action. In addition, we use quenched configurations to explore the effects of fattening. The lessons from the quenched analyses are then applied to data with dynamical fermions, where both Wilson and fat-link clover actions have been used. This allows us to attempt a continuum extrapolation of the dynamical results.

Decay constants of the B and B_s mesons are essential for accurate determination of parameters of the CKM matrix. We present here lattice studies of decay constants of heavy-light mesons. A MILC collaboration computation of the quenched decay constants has appeared previously [1]; in that work, dynamical simulations were used only to estimate the quenching error. Our first continuum-limit results for dynamical lattices with $N_f = 2$ staggered quarks were presented at Lattice '99 [2]. Here the dynamical lattices are studied in more detail, with both Wilson and clover valence fermions. We also update the quenched lattice results, based on some new runs and revised analysis. Some of these new runs use clover fermions with different levels of fattening (including no fattening, the standard "thin" case), thereby enabling us to understand better both the fat-link clover dynamical results and the quenched discretization errors.

Table 1 gives the lattice parameters. The sets J and CP are new additions to the quenched lattices previously analyzed in Ref. [1]. Unimproved Wilson valence fermion propagators were generated for all these sets except J. On J, and a 200-lattice subset "CP1" of CP, we used thin link clover fermion propagators. For a 100-lattice subset "CP2" of CP1, we generated clover propagators with four different levels of fattening (see below). For the dynamical lattices, valence Wilson fermions were studied on all the lattices; fat link clover fermions were also studied on a 98-lattice subset "RF" of set R, with 10 levels of fattening

*presented by S. Datta

and coefficient $c = 0.45$.

For our central values, the dynamical quark configurations are treated as fixed backgrounds and chiral extrapolation is performed in the valence quark mass only. To estimate the systematic error due to this "partial quenching," we also extrapolate with $m_{\rm valence} = m_{\rm dynamical}$, the equality being defined by parameters corresponding to the same m_π.

Table 1
Lattice parameters. The upper group corresponds to quenched lattices; the lower group, to dynamical lattices with $N_f = 2$ staggered quarks. The set G was generated by HEMCGC.

name	$\beta(am_q)$	size	# configs.
A	5.7	$8^3 \times 48$	200
B	5.7	$16^3 \times 48$	100
E	5.85	$12^3 \times 48$	100
C	6.0	$16^3 \times 48$	100
CP	6.0	$16^3 \times 48$	305
J	6.15	$16^3 \times 48$	200
D	6.3	$24^3 \times 80$	100
H	6.52	$32^3 \times 100$	60
L	5.445 (0.025)	$16^3 \times 48$	100
N	5.5 (0.1)	$24^3 \times 64$	100
O	5.5 (0.05)	$24^3 \times 64$	100
M	5.5 (0.025)	$20^3 \times 64$	199
P	5.5 (0.0125)	$20^3 \times 64$	199
G	5.6 (0.01)	$16^3 \times 32$	200
R	5.6 (0.01)	$24^3 \times 64$	200
S	5.6 (0.02)	$24^3 \times 64$	202
T	5.6 (0.04)	$24^3 \times 64$	201
U	5.6 (0.08)	$24^3 \times 64$	202

For clover fermions, both thin and fat, the full Fermilab formalism [3] is applied, including use of the EKM norm $\sqrt{1-6\kappa}$, the 3-d rotation of the fields, and the identification of kinetic mass m_2 (rather than the pole mass m_1) as the physical mass. The shift $m_1 \to m_2$ is performed at tadpole improved tree level. In the thin-link case, we take the clover coefficient $c_{\rm SW}$ and the light-light renormalization/improvement constants from the nonperturbative determinations by the Alpha collaboration [4]. For the heavy-light renormalizations around the B meson mass, we use either the one-loop perturbative calculations [5] or a version of tree-level tadpole improvement designed to smoothly join on to the nonperturbative results as the quark mass gets small. The former approach is designated "NP-IOY;" the latter, "NP-tad." For more details on NP-tad, see Ref. [6]. In both cases, the renormalization of the 3-d rotation ("d_1") term is performed perturbatively using [5]. For the D meson, only NP-tad is used since the approximations in [5] are not applicable for smaller masses.

In the fat-link clover cases, $c_{\rm SW}$ is set equal to the tadpole improved tree level value $1/u_0^3$. The light-light renormalization coefficients are taken from the perturbative calculations of Ref. [7]. The heavy-lights (for which perturbative calculations do not exist) are normalized using the *static-light* results of [7]. This should be roughly correct for the large values of aM at the B meson.

For Wilson valence quarks, we again use the EKM norm and identify m_2 as the physical mass. However since the magnetic and kinetic masses are not equal in this case [3], there is little point in including the 3-dimensional rotations, and we do not. The associated systematic errors are estimated as in [1]. We normalize the heavy-light axial current as a function of mass perturbatively [8]. For the central values, the static-light value of the scale, $q^*_{\rm SL} \approx 1.43/a$ [7] is used. (This scale is different from that quoted in Ref. [9] and used by us previously [1]; for further discussion see [6].)

For the light quarks, 3 – 5 κ values are used, in the range 0.7 – 2.0 m_s. To get to physical light quark masses, we extrapolate m_π^2 vs. m_2 quadratically and f_π, m_{Qq} and f_{Qq} vs. m_2 linearly. The systematic error of the chiral extrapolation is estimated by comparing with results from quadratic extrapolations for all the above. For heavy clover quarks, we use 3 – 5 κ values (giving meson masses in the range 1.8 – 5 GeV); in the Wilson case we have ~ 10 heavy quark masses as well as static heavy quarks. Continuum extrapolation is done using both constant and linear fits, and the spread is taken as an estimate of the systematic error.

Figure 1 shows the continuum extrapolations of our full set of data for f_B, for both quenched and dynamical lattices. Both the NP-IOY and NP-tad quenched calculations, which should have

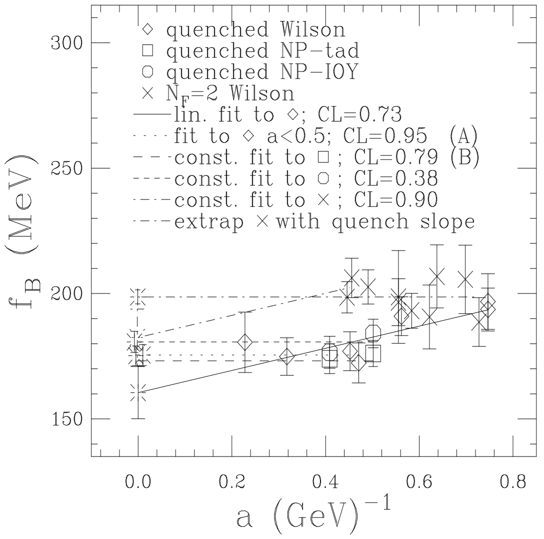

Figure 1. The complete set of f_B data, for both quenched and dynamical lattices, showing the different continuum extrapolations. The line for fit (B), which is very close to the line for fit (A), has been shifted downward slightly for clarity.

small discretization errors, are extrapolated with constants. The results are consistent with both the linear and constant quenched Wilson extrapolations, although they favor the constant extrapolation. All four extrapolations are averaged for our quenched central values; the spread determines our discretization error.

The dynamical lattice data clearly favor a constant fit; the best linear fit has a tiny slope. However, to be conservative, we also make a linear extrapolation from the data for the smallest a, with the same slope as the linear fit for the quenched set. Other systematic errors, including the extrapolation in $1/M_{Qq}$, finite size errors, effects of excited states, higher order perturbative corrections, *etc.*, are estimated in basically the same way as in Refs. [1,2]. However the range of q^* values considered for the perturbative error has changed along with the change in the central value of q^*. We now consider $1/a \leq q^* \leq 2q^*_{SL}$, where q^*_{SL} is the value calculated in [7].

As reported in Ref. [2], the fat-link clover results for decay constants f_{D,D_s,B,B_s} on dynamical lattices were found to be much smaller than the apparent continuum limit results of the Wilson quarks, thus raising the possibility that at least one of these results had a large uncontrolled systematic error. To study this issue, we have computed with clover fermions on quenched lattices (set CP2) with four different levels of fattening: links fattened 2, 6 and 10 times with a fattening coefficient c=0.45, and links fattened 7 times with c = 0.25. A comparison with the thin-link clover computations (from sets CP1 and F) is shown in Fig. 2a. The fat-link results are considerably suppressed compared to those from the thin links. As mentioned above, the latter are consistent with the results for continuum-extrapolated quenched Wilson fermions. Furthermore, in an unpublished direct investigation of the static $q\bar{q}$ potential, we found that the form of the $q\bar{q}$ potential at short distances is distorted by fattening. Figure 2a also reveals the interesting fact that even the lowest level of fattening studied by us, *viz.*, 2 levels of smearing with a fattening coefficient of 0.45, already substantially suppresses the decay constant. In fact, there is not much difference in the values of the decay constants between the four different levels of fattening we studied.

The fat-link studies on quenched lattices, where the approach to the continuum limit is in much better control, allow us to attribute the cause of the discrepancy between Wilson and fat clover results on dynamical lattices to fattening. We note however that the fattening effects in our calculation include not only the change in the short-distance potential, but also the possible error in using the static-light rather than heavy-light renormalization. Be that as it may, the quenched studies clearly show that we should not average the dynamical fat clover and Wilson results (as we did in Ref. [2]); instead, we use the Wilson values only. Interestingly, correcting the fat-clover dynamical-lattice result by a factor obtained from the quenched lattices at comparable lattice spacing produces agreement with the Wilson fermion results (Fig. 2b), indicating that the suppression of decay constants due to fattening has similar origins for the dynamical and quenched lattices.

Our final results for heavy-light decay constants in the quenched approximation are (in MeV for the decay constants):

$$f_B = 173 \pm 6 \pm 16; \quad f_{B_s} = 199 \pm 5^{+23}_{-22};$$

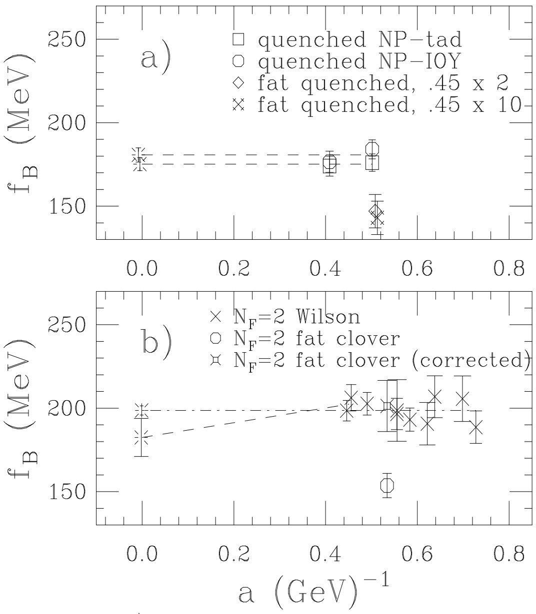

Figure 2. a) f_B for clover fermions on quenched lattices. b) f_B on dynamical lattices, for both Wilson and fat-link clover fermions. Also shown is the "corrected" fat-link result.

$$f_D = 200 \pm 6^{+12}_{-11}; \quad f_{D_s} = 223 \pm 5^{+19}_{-17};$$
$$\frac{f_{B_s}}{f_B} = 1.16 \pm 0.01 \pm 0.02;$$
$$\frac{f_{D_s}}{f_D} = 1.13 \pm 0.01 \pm 0.02.$$

The errors are statistical and systematic (within the quenched approximation), respectively. The results differ from those in Ref. [1] due to:

1) The inclusion of new data from sets CP, CP1 and F.

2) Setting the central value of the heavy-light scale as q^*_{SL} from [7], rather than from [9].

3) Other changes in analysis, motivated by the new runs. These include the error estimate for the chiral extrapolation (some alternative fits used previously are excluded by the new data) and the central value of the continuum extrapolation (averaging our four possible versions rather than taking only the linear Wilson fit).

Our results for the dynamical lattices are (in MeV for decay constants):

$$f_B = 191 \pm 6^{+24+11}_{-18-0}; \quad f_{B_s} = 217 \pm 5^{+33+9}_{-29-0};$$
$$f_D = 215 \pm 5^{+17+8}_{-15-0}; \quad f_{D_s} = 241 \pm 4^{+32+9}_{-31-0};$$
$$\frac{f_{B_s}}{f_B} = 1.16 \pm 0.01 \pm 0.02 \pm 0.02;$$
$$\frac{f_{D_s}}{f_D} = 1.14 \pm 0.01^{+0.02}_{-0.03} \pm 0.02.$$

Here the errors are statistical, systematic within $N_f = 2$, and systematic (due to partial quenching and missing the virtual strange quark), respectively. The last error is taken to be the largest of four components : 1) half the difference of the quenched and the $N_f = 2$ results (see Ref. [2]), 2) the spread obtained from setting the scale by m_ρ instead of by f_π, 3) for quantities involving the strange quark, the change in fixing κ_s from m_ϕ instead of the pseudoscalars, 4) the difference from the "full unquenching" result with $m_{\text{valence}} = m_{\text{dynamical}}$. For the individual decay constants, errors 1) and 2) are the largest; for the ratios, however, errors 3) and 4) dominate.

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