Pion and kaon physics with improved staggered quarks*

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We compute pseudoscalar meson masses and decay constants using staggered quarks on lattices with three flavors of sea quarks and lattice spacings $\approx$0.12 fm and $\approx$0.09 fm. We fit partially quenched results to "staggered chiral perturbation theory" formulae, thereby taking into account the effects of taste-symmetry violations. Chiral logarithms are observed. From the fits we calculate f_π and f_K, extract Gasser-Leutwyler parameters of the chiral Lagrangian, and (modulo rather large perturbative errors) find the light and strange quark masses.

The masses and decay constants of light pseudoscalars can be computed with high precision at fixed lattice spacing and quark mass. Assuming that the chiral and continuum extrapolations can be performed with controlled systematic errors, such a calculation therefore can provide an accurate determination of the physical properties of the π-K system, as well as a sensitive check on our algorithms and methods.

Our simulations have three light dynamical flavors with the Asqtad action[1], with $\mathcal{O}(\alpha_s a^2)$ errors in general, but $\mathcal{O}(\alpha_s^2 a^2)$ taste-violations, which require 2-gluon exchange. One group of lattices ("coarse") has $a \approx 0.12$ fm, the dynamical strange quark mass held fixed at a nominal value $am_s = 0.05$, and five different values of dynamical light quark mass $m_u = m_d \equiv m_q$. The second group ("fine") has $a \approx 0.09$ fm, a nominal value $am_s = 0.031$, and two values of m_q. The nominal strange quark mass turned out to be about 18% (10%) larger than the physical mass, $m_s^{\rm phys}$, on the coarse (fine) lattices. Table 1 shows the lattice parameters used.

The relative scale among coarse (or, separately, fine) lattices is kept fixed using the length r_1 [2,3] from the static quark potential. We reduce statistical fluctuations in r_1/a by using a "smoothed" r_1/a coming from a 4-parameter fit to a smooth function of $10/g^2$ and $m_{\rm tot} \equiv 2m_q + m_s$. The absolute scale is taken from Υ $2S$-$1S$ or $1P$-$1S$ splittings, determined by the HPQCD group [5] on the lattices with $m_q = 0.2 m_s$. We extrapolate to the continuum linearly in $\alpha_s a^2$ and get $r_1 = 0.317(7)$ fm. Since the physical r_1 has some (small) dependence on sea quark masses, fixing a using r_1/a can introduce some spurious dependence on mass [4]. This has a negligible effect on quantities like f_π, but could be a significant

am_q / am_s	$10/g^2$	size	number
0.03 / 0.05	6.81	$20^3 \times 64$	262
0.02 / 0.05	6.79	$20^3 \times 64$	485
0.01 / 0.05	6.76	$20^3 \times 64$	608
0.007 / 0.05	6.76	$20^3 \times 64$	447
0.005 / 0.05	6.76	$24^3 \times 64$	137
0.0124 / 0.031	7.11	$28^3 \times 96$	531
0.0062 / 0.031	7.09	$28^3 \times 96$	583

Table 1
Lattice parameters. The lattice sets above the double line are "coarse;" those below are "fine."

*presented by C. Bernard at *Lattice 2003*, Tsukuba, Japan, July 15-19, 2003 systematic for the Gasser-Leutwyler parameters (L_i). Estimates of this effect are in progress.

On the coarse (fine) lattices, quark propagators are computed every 6 time units for 9 (8) values of valence mass between $0.1 m_s$ ($0.14 m_s$) and m_s. We determine meson masses and decay constants from simultaneous fits to wall-point and point-point correlators. The complete covariance matrix of decay constants and masses on each lattice set is computed using single elimination jackknife; we then adjust for autocorrelations by scaling the matrix by a factor estimated by blocking lattices in groups of four.

We first fit squared masses of various-taste pions to the tree-level form [6]: linear in quark mass plus a constant splitting (at fixed a) for each taste multiplet. Although the fits are poor, they do give the masses within $\sim 5\%$. The slopes and splittings determined may then be consistently used as inputs to the one-loop terms in the chiral log fits below. The ratio of splittings on the fine lattices to those on the coarse lattices is ≈ 0.35. This is consistent with expectation that taste violations go like $\mathcal{O}(\alpha_s^2 a^2)$: using $\alpha_s = \alpha_V(q^*)$ at one-loop [7] and $q^* = 3.33/a$, this ratio is 0.375.

We then compare the (partially quenched) decay constants and squared meson masses to one-loop chiral log forms (including finite volume corrections) computed in staggered chiral perturbation theory (SχPT) [8]. Up through NLO, we have 10 free parameters in the chiral expressions. Of these, two appeared previously at tree level: the decay constant f and the ratio, μ, of squared Goldstone-meson mass to the sum of valence quark masses, $m_x + m_y$. There are also analytic contributions proportional to L_4, L_5, $2L_8 - L_5$, and $2L_6 - L_4$. Finally there are four $\mathcal{O}(a^2)$ parameters that appear because of taste violation [8]: $a^2\delta'_A$ and $a^2\delta'_V$, which are the taste-violating "hairpin" parameters that enter into NLO chiral logs, and a^2C and a^2F, which give analytic, $\mathcal{O}(ma^2)$ contributions.

Since the data is very precise ($\sim$0.1–0.4% statistical errors) and we include quark masses as large as $m_s^{\rm phys}$ (to extract K physics), we must go beyond NLO to get good fits. We include all $\mathcal{O}(m^3)$ NNLO analytic terms. NNLO chiral logs and taste-violating analytic terms are unknown and not included. But for larger masses, where NNLO terms are non-negligible, the logarithms should be changing slowly and thus well represented by analytic terms. Similarly, for larger masses, $\mathcal{O}(m^3)$ terms should be more important than taste-violating $\mathcal{O}(m^2a^2)$ or $\mathcal{O}(ma^4)$ terms. When expressed in "natural" chiral units, the 10 NNLO $\mathcal{O}(m^3)$ parameters (5 for the decay constant and 5 for the mass) should have coefficients of $\mathcal{O}(1)$ if chiral perturbation theory (χPT) is well behaved. We constrain them to have standard deviation $\sigma = 1$ around 0 using Baysean priors [9].

The strength of the chiral log terms is governed by the "chiral coupling," $1/(16\pi^2\tilde{f}^2)$. Typically, one takes $\tilde{f} = f$, where f is the bare (tree-level) decay constant. For better convergence of χPT, it seems reasonable to put a *physical* parameter here, *e.g.*, , $\tilde{f} = f_\pi$ or $\tilde{f} = f_K$ (the difference is NNLO). In practice, we try 3 approaches: (1) $\tilde{f} = f$, (2) $\tilde{f} = f_\pi$, and (3) $\tilde{f} = f_\pi/\sqrt{\omega}$, where ω is a new fit parameter allowed to vary around 1 with $\sigma = 0.1$. So far, the best fits have been obtained with choice (3); we currently use such fits for central values. Choices (1) and (2) are now also giving acceptable fits. Only choice (2) has been included in systematic error estimates below, but choice (1) will be added in the future.

To test convergence of χPT, a NNNLO term in valence quark mass is included for both decay constants and masses. Such terms are found to have small coefficients; we plan to eliminate them for the fits that determine central values.

We fit both coarse and fine lattice data simultaneously. The 4 taste-violating parameters are forced to change by the factor of 0.375 (ratio of $\alpha_s^2 a^2$) in going from coarse to fine. The remaining physical parameters are expected to differ between coarse and fine lattices by $\alpha_s a^2 \Lambda_{QCD}^2 \sim 2\%$. We therefore also include an additional "scaling" parameter for each physical parameter. The scaling parameter is the fractional difference between the physical parameter on the coarse and fine lattices. The scaling parameters are constrained to be 0 with σ of 0.02 or 0.025 in central value fits; this is changed to 0.01 or 0.04 in fits used to estimate systematics.

Finally, the 4 parameters used in smoothing r_1/a are allowed to vary by 1 σ. The total is 46

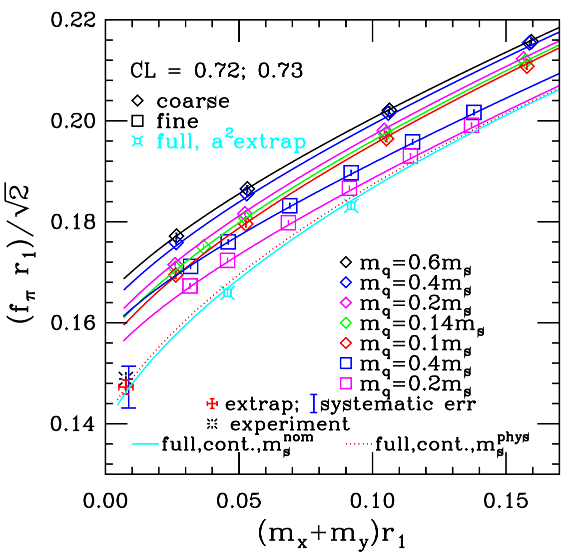

Figure 1. Decay constants with $m_x = m_y$. The two cyan "fancy squares" are full QCD ($m_x = m_y = m_q$) points, continuum extrapolated at fixed quark mass. The lowest two lines (dotted red and cyan) use continuum-extrapolated fit parameters, $m_x = m_y = m_q$, and either the physical or the (fine lattice) nominal strange mass. The first confidence level (CL) is computed in the standard way; the second treats the Baysean priors as if they were additional data points.

parameters (or 44 with choices (1) or (2) above), although many are tightly constrained.

Figure 1 shows some decay constants from our central-value fit to the partially quenched data with $m_x, m_y \lesssim 0.8 m_s^{\rm phys}$ (194 points). We determine physical quark masses from the meson-mass part of this fit (see Ref. [10]) with m_π and m_K as input; the decay constants are then extrapolated to the physical masses and to the continuum.

Our preliminary results for decay constants are:

$$\begin{aligned} f_\pi &= 129.3 \pm 1.1 \pm 3.5 \text{ MeV} \\ f_K &= 155.0 \pm 1.8 \pm 3.7 \text{ MeV} \\ f_K/f_\pi &= 1.201(8)(15)\ , \end{aligned} \qquad (1)$$

where the first error is statistical; the second, systematic. The largest error on f_π and f_K is the 2.2% scale uncertainty. A chiral and continuum extrapolation error of $\approx 1.5\%$ has been estimated by considering alternative fits. The results agree with experiment within errors.

Preliminary results (in units of 10^{-3}) for the L_i at chiral scale m_η are:

$$\begin{aligned} 2L_8 - L_5 &= -0.1(1)(^{+1}_{-3})\ ; \qquad L_5 = 1.9(3)(^{+6}_{-2}) \\ 2L_6 - L_4 &= 0.5(2)(^{+1}_{-3})\ ; \qquad L_4 = 0.3(3)(^{+7}_{-2})\ . \end{aligned} \qquad (2)$$

The errors here are dominated by differences over The result for $2L_8 - L_5$ is well outside the range that allows for $m_u = 0$ [11].

Finally, our preliminary results for quark masses at scale 2 GeV are $m_s^{\overline{MS}} = 70(15)$ MeV and $\hat{m}^{\overline{MS}} = 2.7(6)$ MeV, where $\hat{m}$ is the average of the u and d masses. These results use the 1-loop perturbative result for mass renormalization [12,13]. The rather large error is dominated by the $\mathcal{O}(\alpha_s^2)$ correction to the renormalization constant, estimated in Ref. [12] to be $\sim$20%.

Additional discussion of the calculation and results appears in Ref. [10].

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