Lattice Determination of Heavy-Light Decay Constants

C. Bernard[1], T. DeGrand[2], C. DeTar[3], Steven Gottlieb[4], Urs M. Heller[5], J. E. Hetrick[6], N. Ishizuka[7], C. McNeile[3], R. Sugar[8], D. Toussaint[9], M. Wingate[10]

[1] *Washington University, St. Louis, Missouri 63130, USA*

[2] *University of Colorado, Boulder, Colorado 80309, USA*

[3] *University of Utah, Salt Lake City, Utah 84112, USA*

[4] *Indiana University, Bloomington, Indiana 47405, USA*

[5] *SCRI, The Florida State University, Tallahassee, Florida 32306-4130, USA*

[6] *University of the Pacific, Stockton, CA 95211-0197, USA*

[7] *University of Tsukuba, Tsukuba Ibaraki 305, Japan*

[8] *University of California, Santa Barbara, California 93106, USA*

[9] *University of Arizona, Tucson, Arizona 85721, USA*

[10] *RIKEN BNL Research Center, Upton, New York 11973, USA*

(October 19, 1998)

We report on the MILC collaboration's calculation of f_B, f_{B_s}, f_D, f_{D_s} and their ratios. Our central values come from the quenched approximation, but the quenching error is estimated from $N_F = 2$ dynamical staggered lattices. We use Wilson light valence quarks and Wilson and static heavy quarks. We find, for example, $f_B = 157 \pm 11\ {}^{+25}_{-9}\ {}^{+23}_{-0}$ MeV, $f_{B_s}/f_B = 1.11 \pm 0.02\ {}^{+0.04}_{-0.03} \pm 0.03$, $f_{D_s} = 210 \pm 9\ {}^{+25}_{-9}\ {}^{+17}_{-1}$ MeV and $f_B/f_{D_s} = 0.75 \pm 0.03\ {}^{+0.04}_{-0.02}\ {}^{+0.08}_{-0.00}$, where the errors are statistical, systematic (within the quenched approximation), and systematic (of quenching), respectively.

PACS numbers: 12.15.Hh, 12.38.Gc, 13.20.-v

The neutral B meson (B_d), a bound state of a d quark and an anti-b quark, is known to mix with its antiparticle, $\overline{B}_d$. In the Standard Model, x_d, the ratio of the mixing oscillation to the decay rate, is proportional to the absolute square of the fundamental quantity V_{td}. However, despite the fact that the x_d is well measured [1], V_{td} remains poorly determined because the proportionality constant between x_d and $|V_{td}|^2$ depends on nonperturbative strong interaction effects. These effects are parameterized by f_B, the pseudoscalar decay constant of the B_d meson, and B_B, the corresponding "bag parameter." Accurate computations of f_B and B_B therefore put tight constraints on the Standard Model. Similarly, a measurement of x_s for B_s mesons would determine a second fundamental quantity, V_{ts}, if f_{B_s} and B_{B_s} were known, or $|V_{td}/V_{ts}|$, if the ratios f_{B_s}/f_B and B_{B_s}/B_B were known.

Lattice QCD offers a way to compute quantities like f_B and B_B from first principles. Here, we present a computation by the MILC collaboration of the decay constants f_B, f_{B_s}, f_D, f_{D_s}, and their ratios. Ref. [2] gives additional details; preliminary results were described in Refs. [3,4].

Table I shows the lattice parameters used. See [3] for details of the lattice generation, gauge fixing, and determination of the quark propagators. We compute "smeared-local" and "smeared-smeared" pseudoscalar meson propagators in each of three cases: heavy-light, static-light, and light-light (with degenerate masses only). Light Wilson quark propagators are computed by a minimal residual algorithm for three values of the hopping parameter, giving light quark masses (m_q) in the range $0.7m_s \lesssim m_q \lesssim 2.0m_s$, where m_s is the strange quark mass. The light-light pseudoscalars are used to set the scale (through f_π) and to find the physical values of κ_{ud} and κ_s, the hopping parameters of the up/down and strange quarks. We determine κ_s by adjusting the degenerate pseudoscalar mass to $\sqrt{2m_K^2 - m_\pi^2}$, the lowest order chiral perturbation theory value. We also compute smeared-local light-light vector meson propagators, which we use for alternative determinations of the scale (through m_ρ) and κ_s (through m_ϕ).

Heavy quark propagators are computed by the hopping parameter expansion [5]. Because of practical limitations to this approach [3], we sum the sink point of the smeared-smeared correlators only over a subset of points in a spatial volume. This means that intermediate states of nonzero 3-momenta can contribute. For the heavy-light mesons studied here, these higher momentum states are suppressed sufficiently at asymptotic Euclidean time t by their higher energy, although in the largest volumes (sets N and O) this can require $t_{\min}/a$ as large as 25 (a is the lattice spacing).

The static-light mesons have no such suppression. However on our smallest volumes (sets A, C, D, F, G, H) the contamination by higher momentum states is small ($\approx 0.7\%$, which we estimate using static-light wavefunctions from Ref. [6]). On all other sets the contamination is expected to be large. We therefore have performed a dedicated static-light computation on those lattices, with relative smearing functions taken from [7] and zero momentum intermediate states enforced by a complete FFT sum over spatial slices. In addition, the dedicated static light computation has been run on set A (because the plateaus from the hopping method proved to be poor) and set G (as a check of the hopping method). On the latter set, the two methods give consistent results.

For all pseudoscalars, we fit the smeared-local and smeared-smeared correlators simultaneously and covariantly to single exponential forms, with the same mass in both channels. We vary the fit range (in t) in each channel over several choices that have reasonable confidence level (CL). Combining such choices for the light-light, heavy-light and static-light cases, we have approximately 25 different versions of the analysis on each data set. Our central values are taken from the version which has the best blend of high CL and small statistical errors. We then find the standard deviation of the result over the other versions and add it in quadrature with the raw jackknife error of the central value. The resulting error will be called, henceforth, "the statistical error."

We employ the EKM norm [8] throughout. In the heavy-light case we also adjust the measured meson pole mass upward by the difference between the heavy quark kinematic mass (m_2) and the heavy quark pole mass (m_1) as calculated in the tadpole-improved tree approximation [8], fixing the mean link from κ_c. We use the one-loop tadpole-improved, mass-dependent perturbative renormalization of the axial current [9], with coupling $\alpha_V(3.4018/a)$ defined in terms of the plaquette [10]. We adjust the result of [9] for our matching point (m_2 rather than m_1) and for our choice of the mean link. Our central values use "scale choice *i*": $q^*_{HL} = 2.32/a$ for the heavy-light corrections [11] and $q^*_{SL} = 2.18/a$ for the static-light corrections [12]. The heavy-light scale was calculated in the massless limit; however, since it differs little from the static-light scale, it seems reasonable to use it for all mass values. The effects of two other choices of scale (*ii*: $q^*_{HL} = q^*_{SL} = 1/a$; *iii*: $q^*_{HL} = 4.63/a$, $q^*_{SL} = 4.36/a$) give an estimate of the perturbative errors.

In our chiral fits we use am_2 as the independent variable rather than the more standard $1/\kappa$. Although the two are formally equivalent at this order in a, m_2 has the advantage that CL of linear fits to M_{Qq} and f_{Qq} are quite good. (Here, Q is a generic — possibly static — heavy quark, and q is a generic light quark.) Further, linear fits to f_π also have reasonable CL for quenched $\beta \geq 6.0$. For m_π^2, however, the CL of the linear fits is uniformly very poor whether $1/\kappa$ or am_2 is the independent variable. To study this problem in more detail, we have examined the pseudoscalar mesons for 6 light quark masses at $\beta = 5.7$ on additional lattices (set "5.7-large") of size $12^3 \times 48$ (403 configurations), $16^3 \times 48$ (390 confs.), $20^3 \times 48$ (200 confs.), and $24^3 \times 48$ (184 confs.). The lightest two meson masses in this set (≈ 385 and ≈ 515 MeV), are below those used in the full computation. On set 5.7-large, linear fits of m_π^2 *vs.* either $1/\kappa$ or am_2 are still poor, but quadratic fits are good. Indeed, a quadratic fit of m_π^2 *vs.* am_2 using the 5 heaviest masses goes right through the lightest m_π^2 in all but the 12^3 volume. For our central values we thus employ quadratic fits *vs.* am_2 for m_π^2, and linear fits *vs.* am_2 for f_π, M_{Qq}, and f_{Qq}. We call this "chiral fit *I*." Three other fit choices (*II*: all linear; *III*: m_π^2 and f_π quadratic, all others linear; *IV*: m_π^2, f_π and f_{Qq} quadratic, all others linear) are used to assess the systematic error. One set, F, undergoes very large ($\sim 50\%$) variation when the chiral fit choice is changed, possibly because of finite size effects. Set F is therefore dropped from further analysis.

To find f_B on a given data set, we divide out the perturbative logarithms [9] from $f_{Qq}\sqrt{M_{Qq}}$, fit to a polynomial in $1/M_{Qq}$, interpolate to m_B, and then replace the logarithms. We do three versions of the polynomial fit: (1) a quadratic fit to the mesons in the approximate mass range 2 to 4 GeV ("heavier heavies") (2) a quadratic fit to the mesons in the approximate mass range 1.25 to 2 GeV ("lighter heavies") (3) a cubic fit to the mesons in the approximate mass range 1.25 to 4 GeV. We include the static-light point in all three fits. We use range (1) in central values for f_B and f_{B_s}; range (2), for f_D and f_{D_s}. The alternative ranges go into the systematic error estimates.

The final extrapolation is in lattice spacing. Since the Wilson action's leading errors are $\mathcal{O}(a)$, we attempt a linear extrapolation in a for all our quenched results. Figure 1 shows the extrapolation for f_B, with the central choices of the perturbative scale (choice *i*) and of the chiral fits (fit *I*). An alternative possibility, with which the data are also consistent, is that the $\mathcal{O}(a)$ effects are small enough for $6/g^2 \geq 6.0$ ($a \lesssim 0.5$ GeV^{-1}) that one may extrapolate with a constant fit in this region. For the decay constants, both fits have acceptable CL, but the constant fit is better. However, for f_{B_s}/f_B and f_{D_s}/f_D, the linear fits (CL ≈ 0.6) are much better than the constant fits (CL ≈ 0.1). (See Ref. [2] for plots of the ratios and additional details.) Since it would be inconsistent to treat the decay constants as independent of a, yet fit the ratios linearly, and since we in any case expect significant $\mathcal{O}(a)$ errors for Wilson fermions, we take the linear fits to the quenched results for our central values. The differences with the constant fits are included in the systematic errors. At this point, the dynamical $N_F = 2$ data is not good enough to extrapolate to the continuum, even for unphysically large dynamical quark mass. We use the dynamical data only to assess the error due to quenching.

The systematic errors are computed as follows:

(1) The three largest sources of error within the quenched approximation are the continuum extrapolation, the chiral extrapolation, and the $\mathcal{O}(\alpha_s^2)$ perturbative corrections (as estimated from a change in scale in the $\mathcal{O}(\alpha_s)$ terms). With our data, these errors cannot be computed independently. For example, when the chiral extrapolations are changed to fit *IV* (see Fig. 2), the difference between the linear and constant (not shown) continuum extrapolations gets smaller (15 MeV instead of 23 MeV). Further, while the systematic error in the final results would be very small if the only source of uncertainty were the next higher order perturbative cor-

rection ($\mathcal{O}(\alpha_s^2)$), this is not the case once the interplay between perturbative uncertainties and other continuum extrapolation errors is included. Indeed, assume there exists "perfect" data which are linear in a with slope and intercept as in Fig. 1, and then add on an $\mathcal{O}(\alpha_s^2)$ correction with a coefficient chosen to give the same change at $\beta = 5.7$ as would be produced by reducing q^*_{HL} and q^*_{SL} to $1/a$ (choice *ii*). Although this gives a 17% change at $\beta = 5.7$, a linear extrapolation of the changes to $a = 0$ results in a residual error of less than 1%. With the real data, however, changing q^* to choice *ii* raises the linearly extrapolated value by 10% (see Fig. 2); while it reduces the constant fit by 3%. Choice *iii* reduces the linear value and raises the constant value by $\sim$2%. We therefore estimate the errors from the continuum extrapolation, chiral extrapolation and perturbative corrections together. We compute each quantity 24 times (2 continuum extrapolations $\times$ 4 chiral fits $\times$ 3 scale choices), giving a central value and 23 alternatives. The alternatives are divided into two groups depending on whether the result is greater or less than the central value, and the standard deviation of each group about the central value is then taken as the positive or negative combined error.

(2) The "magnetic mass" m_3, which divides the chromomagnetic interaction in the effective non-relativistic Hamiltonian for Wilson fermions, is not equal to the kinetic mass m_2 [8]. This introduces an error at fixed a of $\mathcal{O}((c_{mag} - 1)\Lambda_{QCD}/M_{Qq})$, where $c_{mag} \equiv m_2/m_3$. The error is not completely removed by the linear extrapolation to $a = 0$. Following [13], we estimate the residual error by using the tree level expression for c_{mag} (with our values of am_2) and extrapolating c_{mag} linearly in a. With our preferred choices for the mass range in the $f_{Qq}\sqrt{M_{Qq}}$ fit, this gives an error of $\sim 2\%$ for f_B and $\sim 3\%$ for f_D. The error on f_B can be reduced to less than 1% by switching to the "lighter heavies" (+ static) mass range: the static-light point, for which $m_3 \neq m_2$ is not an issue, becomes particularly important in this case. In practice, we assess the errors due to $m_3 \neq m_2$ as the larger of: (a) the 2 or 3% model estimate with our preferred mass ranges and (b) the actual difference in the final result caused by switching from "heavier heavies" to "lighter heavies" or *vice versa.*

(3) Our preferred fits of $f_{Qq}\sqrt{M_{Qq}}$ *vs.* $1/M_{Qq}$ are truncated at quadratic order. A scale of $\sim$0.75 GeV for $1/M_{Qq}$ is expected in the omitted cubic term, since this is roughly the scale size found in the linear and quadratic terms. We calculate that the existence of such a cubic term in the data would lead to an error, in the analysis that uses only quadratic fits, of $\sim$1% in the decay constants. In practice we estimate this error by changing to cubic fits (using the entire mass range 1.25 to 4 GeV); the errors found are indeed $\lesssim$1%.

(4) The finite volume effects are estimated by comparing results on sets A (spatial size ~ 1.2 fm) and B (~ 2.5 fm) and applying the fractional difference to the final results. Set A is smaller than all other quenched lattices; B, much larger. Therefore the difference should give a conservative bound on the finite volume error. In practice, we take the larger of: (a) the difference when all quantities are computed individually on sets A and B and (b) the difference when all light-light quantities are taken from set 5.7-large. Since there is some cancellation of error between f_{Qq} and f_π, (b) is generally larger. We find an error of $\sim$ 2–3% on decay constants, $\sim$ 4% on f_B/f_{D_s}, and $\sim$1–2% on other ratios.

Errors (2)–(4) do not have definite signs and appear to be largely independent of each other and of error (1). We thus take the error within the quenched approximation to be the sum, in quadrature, of errors (1) through (4). For decay constants, error (1) always dominates; while for the ratios, error (2) (and for f_B/f_{D_s}, (4)) is (are) comparable to (1).

(5) The quenching error is estimated in three ways: (a) We set the scale by using m_ρ instead of f_π. (b) For quantities involving the strange quark, we fix κ_s from m_ϕ instead of the pseudoscalars. (c) We compare the results from the weakest coupling $N_F = 2$ lattices (sets G and R) with the quenched results interpolated (*via* the linear fit) to the same value of the lattice spacing ($a \approx 0.45$ GeV^{-1} — see Fig. 1). We average this difference over 12 analysis choices (4 chiral fits $\times$ 3 scale choices), plus (for strange quark quantities) the preferred choices but with κ_s fixed from m_ϕ. For the decay constants, this difference has a definite sign over all 12 or 13 choices. We then take the signed error (c) to be just the average difference. However, for some of the ratios, the standard deviation of the difference is larger than the average difference. In that case, the positive (negative) error (c) is taken to be average difference plus (minus) the standard deviation. Finally, the quenching error in the positive or negative direction is defined to be the largest of errors (a), (b), and (c) in that direction. In almost all cases, (c) is largest.

Note that our quenching error estimate is still rather crude. Our $N_F = 2$ simulations are not "full QCD" because they are not extrapolated to the continuum or to the physical quark mass, and they do not have a dynamical strange quark. For these reasons we prefer to quote the central values as the quenched results and to treat the difference (c) as a signed error, not a correction. (See [2] for further discussion.) Additional dynamical simulations, which we hope will allow us to improve this situation, are in progress. Note, however, that the sign we find of the difference is what is expected from intuitive arguments about the wave function at the origin [14].

We then have:

$$
\begin{aligned}
f_B &= 157 \ \pm 11 \ {}^{+25}_{-9} \ {}^{+23}_{-0} \ \ \text{MeV} \\
f_{B_s} &= 171 \ \pm 10 \ {}^{+34}_{-9} \ {}^{+27}_{-2} \ \ \text{MeV} \\
f_D &= 192 \ \pm 11 \ {}^{+16}_{-8} \ {}^{+15}_{-0} \ \ \text{MeV} \\
f_{D_s} &= 210 \ \pm 9 \ {}^{+25}_{-9} \ {}^{+17}_{-1} \ \ \text{MeV} \\
f_{B_s}/f_B &= 1.11 \ \pm 0.02 \ {}^{+0.04}_{-0.03} \pm 0.03
\end{aligned}
$$

$$
\begin{aligned}
f_{D_s}/f_D &= 1.10\ \pm 0.02\ {}^{+0.04}_{-0.02}\ {}^{+0.02}_{-0.03} \\
f_B/f_{D_s} &= 0.75\ \pm 0.03\ {}^{+0.04}_{-0.02}\ {}^{+0.08}_{-0.00} \\
f_{B_s}/f_{D_s} &= 0.85\ \pm 0.03\ {}^{+0.05}_{-0.03}\ {}^{+0.05}_{-0.00}\ ,
\end{aligned} \tag{1}
$$

where the errors are statistical, systematic (within the quenched approximation), and systematic (of quenching), respectively. The result for f_{D_s} is consistent with the experimental value [15] of $241 \pm 21 \pm 30$ MeV. Our quenched approximation values are consistent with recent quenched results using improved actions [13,16].

This work was supported by the U.S. DOE and NSF. Calculations for this project were performed at ORNL CCS, SDSC, Indiana University, NCSA, PSC, MHPCC, CTC, CHPC (Utah), and Sandia Natl. Lab. We thank Y. Kuramashi for discussions, and the Columbia group and the HEMCGC collaboration for lattice sets F and G, respectively.

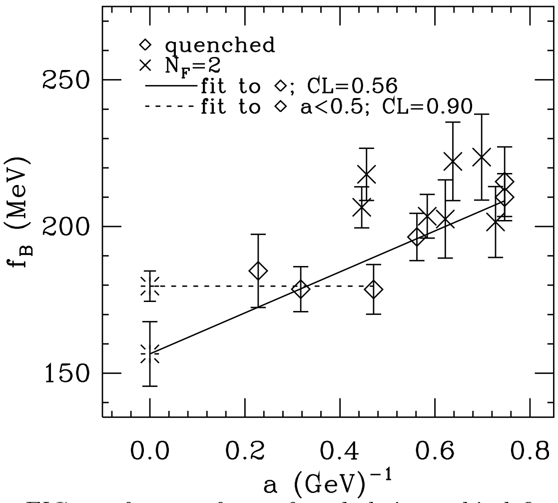

FIG. 1. f_B *vs.* a for preferred choices: chiral fit *I*, perturbative scale *i*, and "heavier heavies" mass range. The scale is set by f_π. The linear fit to all quenched points (solid line) gives the central value.

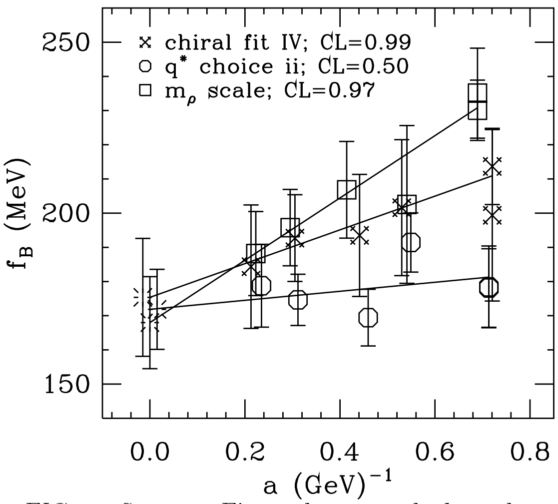

FIG. 2. Same as Fig. 1, but quenched results only, with alternative analysis choices. Some points have been displaced slightly horizontally for clarity.

[1] See Phys. Rev. **D54** (Review of Particle Properties), 506 (1996).
[2] C. Bernard *et al.*, hep-lat/9809109, to be published.
[3] C. Bernard *et al.*, Nucl. Phys. **B** (Proc. Suppl.) **42**, 388 (1995).
[4] C. Bernard *et al.*, Nucl. Phys. **B** (Proc. Suppl.) **47**, 459 (1996); **58**, 358 (1997); **60A**, 106 (1998); **63**, 362 (1998); AIP Conference Proceedings **424**, 227 (1998).
[5] D. Henty and R. Kenway, Phys. Lett. **289B**, 106 (1992).
[6] A. Duncan, E. Eichten, and H. Thacker, Nucl. Phys. **B** (Proc. Suppl.) **30**, 441 (1993).
[7] T. Draper and C. McNeile, Nucl. Phys. **B** (Proc. Suppl.) **34**, 453 (1994).
[8] A. El-Khadra, A. Kronfeld, P. Mackenzie, Phys. Rev. **D55**, 3933 (1997).
[9] Y. Kuramashi, Phys. Rev. **D58**, 034507 (1998).
[10] G. P. Lepage and P. B. Mackenzie, Phys. Rev. D **48**, 2250 (1993); K. Bitar et al., Phys. Rev. **D48**, 370 (1993); T. Klassen, Phys. Rev. **D51**, 5130 (1995).
[11] C. Bernard, M. Golterman, C. McNeile, hep-lat/9808032.
[12] O. Hernandez and B. Hill, Phys. Rev. **D50**, 495 (1994).
[13] S. Aoki *et al.*, Phys. Rev. Lett. **80**, 5711 (1998).
[14] See, *e.g.*, F. Butler *et al.*, Nucl. Phys. **B421**, 217 (1994). Note that although we are really computing f_B/f_π, one would expect f_B to be more affected by the potential at the origin than f_π, because the B is a smaller state.
[15] J. Richman, *Proceedings of the 28th International Conference on High Energy Physics*, Warsaw, 1996, Z. Ajdik and A.K. Wroblewski, Eds., World Scientific, 1997.
[16] A. El-Khadra *et al.*, Phys. Rev. **D58**, 014506 (1998)

TABLE I. Lattice parameters. Sets F, G, and L–R use variable-mass Wilson valence quarks and two flavors of fixed-mass staggered dynamical fermions; all other runs use quenched Wilson quarks.

set	β (am_q)	size	# confs.
A	5.7	$8^3 \times 48$	200
B	5.7	$16^3 \times 48$	100
E	5.85	$12^3 \times 48$	100
C	6.0	$16^3 \times 48$	100
D	6.3	$24^3 \times 80$	100
H	6.52	$32^3 \times 100$	60
L	5.445 (0.025)	$16^3 \times 48$	100
N	5.5 (0.1)	$24^3 \times 64$	101
O	5.5 (0.05)	$24^3 \times 64$	100
M	5.5 (0.025)	$20^3 \times 64$	199
P	5.5 (0.0125)	$20^3 \times 64$	199
G	5.6 (0.01)	$16^3 \times 32$	200
R	5.6 (0.01)	$24^3 \times 64$	200
F	5.7 (0.01)	$16^3 \times 32$	49