Lattice results for the decay constant of heavy-light vector mesons

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(September 17, 2001)

Abstract

We compute the leptonic decay constants of heavy-light vector mesons in the quenched approximation. The reliability of lattice computations for heavy quarks is checked by comparing the ratio of the vector to the pseudoscalar decay constant with the prediction of Heavy Quark Effective Theory in the limit of infinitely heavy quark mass. We then calculate the decay constant ratio for B mesons: $f_{B^*}/f_B = 1.01(0.01)(^{+0.04}_{-0.01})$. We also quote quenched $f_{B^*} = 177(6)(17)$ MeV.

The symmetries of Heavy Quark Effective Theory (HQET) [1] show how Quantum Chromodynamics (QCD) simplifies in the limit of infinite quark mass. For a mesonic system such as the neutral B, consisting of a heavy, but finite mass, anti-b quark and a light d quark, HQET can be applied with the inverse b mass as a small perturbation parameter. In particular, the ratio of the decay constants of the B^* and B can be calculated. Heavy quark spin symmetry implies that in the limit of an infinite quark mass, the spins of the quarks decouple and the vector and pseudoscalar mesons are degenerate, so the ratio of their decay constants is 1. Perturbative corrections to this ratio are also calculable within the HQET framework.

In this paper, we study the heavy-light vector and pseudoscalar decay constants in quenched lattice QCD. Because computational restrictions limit the range of heavy quark masses that are used in our simulations, the data must be extrapolated to the B mass (or interpolated between the heavy-light data and a static-light point). The lattice calculations also inherently require extrapolations to the continuum limit of zero lattice spacing. The comparison of the lattice calculation and the HQET calculation of the ratio of the vector and pseudoscalar decay constants tests the consistency of the treatment of heavy quarks in lattice QCD. Calculations of heavy-light vector decay constants on the lattice have been carried out previously by several other groups [2].

In earlier work [3], we computed pseudoscalar decay constants only. Here we extend that analysis to include vector mesons. Since our aim is to test the consistency of these lattice simulations with the results of HQET, we confine the analysis to the quenched data sets, the details of which are summarized in Table I. The parameters of the generation of these lattices, gauge fixing, and quark propagator determination are found in [4,3]. We use unimproved Wilson valence quarks. "Smeared-local" (SL) and "smeared-smeared" (SS) vector meson propagators are calculated for the heavy-lights. These propagators are also calculated for the heavy-light and light-light pseudoscalar mesons. The heavy-light pseudoscalar decay constant f_{Qq} is defined by

$$\left\langle 0 \left| A_0^{cont}(0) \right| P, \vec{p}=0 \right\rangle = -i f_{Qq} M_{Qq}$$

where

$$A_0^{cont}(0) = \overline{Q}\gamma_0\gamma_5 q$$

is the 0^{th} component of the of the axial current at point $\mathbf{x}=0$, and $|P,\vec{p}=0\rangle$ is a zero 3-momentum pseudoscalar bound state of heavy quark $\overline{Q}$ and light quark q with mass M_{Qq}. We define the vector decay constant in exact analogy to the pseudoscalar decay constant (this is standard in HQET) to simplify the interpretation of the ratio:

$$\left\langle 0 \left| V_i^{cont}(0) \right| V, \vec{p}=0, \epsilon \right\rangle = \epsilon_i f^*_{Qq} M^*_{Qq}$$

where V_i^{cont} is a spatial component of the continuum vector current,

$$V_\mu^{cont} = \overline{Q}\gamma_\mu q$$

and $|V,\vec{p}=0,\epsilon\rangle$ is the vector meson state with zero 3-momentum, mass M^*_{Qq}, and polarization ϵ.

The vector propagators have the same relation to f_{B*} as the pseudoscalar propagators have to f_B. The light-light pseudoscalars are used to set the scale (through f_π) and the physical value of the hopping parameter of the degenerate up/down quarks (through m_π).

Each pair of SL and SS propagators for a particular mass combination is fit simultaneously and covariantly to single exponential forms sharing the same mass; *i.e.*, we make three-parameter fits. The time ranges used in these fits were varied to produce different fits (typically 8–10 of them) that provided reasonable confidence levels for both vector and pseudoscalar decay constants. The alternate fit ranges were then used to fit the ratio of the decay constants as discussed below. A preferred fit range was selected from the acceptable alternatives by choosing a range that provided a good blend of high confidence level and small statistical error for the ratio fit. For each ratio derived from these fits, the standard deviation of the alternate fit ranges was added in quadrature to the raw statistical error of the preferred fit to produce a measure of the statistical uncertainty in the ratio that reflects the different possible plateau regions.

To relate the matrix elements measured on the lattice to their continuum counterparts we use the perturbative renormalization factors for heavy-light currents calculated by Kuramashi [5]. These renormalization factors include a dependence on the quark mass, which for large quark masses produces an approximately 100% difference in the one loop coefficients compared to those in the massless quark limit. We adjust the values calculated in [5] to correspond to our definition of the mean link, u_0, in terms of the critical hopping parameter, $u_0 = 1/8\kappa_c$. The mass dependence of the renormalization factors is more important for the individual decay constants, but still provides a meaningful improvement to the calculation of the decay constant ratios, where only the ratio of the vector and axial current renormalizations is relevant. As in [3], we adjust the measured meson pole mass upwards by the difference of the heavy quark kinematic mass and the heavy quark pole mass. This allows us to estimate the kinetic mass of the meson while only looking at its zero-momentum state.

A new element that is added to the previous analysis of this data is the choice of q^*, the momentum scale that satisfies [6]:

$$ln(q^{*2}) = \frac{\int d^4q f(q) ln(q^2)}{\int d^4q f(q)}$$

where $f(q)$ is the integrand of a 1-loop current renormalization. Evaluating the coupling (we use g_V^2 defined in terms of the plaquette [6,7]) at q^* and using that coupling to evaluate the renormalization should reduce higher order effects. The values of q^* for the heavy-light currents have not been calculated. In Ref. [3], Hernandez and Hill's result [8] for the tadpole-improved static-light axial current scale ($q^* = 2.18/a$, a value close to the tadpole-improved light-light axial current scale, $q^* = 2.32/a$ [9]) was used to argue that q^* was only mildly mass dependent. The light-light q^* was then used for the heavy-lights. Hernandez and Hill's calculation has recently been repeated [10] (see also [11]), with a rather different result, $q^* \sim 1.4/a$. We believe this static-light q^* is likely to be more appropriate for the heavy-lights than the light-light q^*, and we use it here. The new calculation of the static-light q^* includes the continuum part of static-light current, which gives rise to an am_Q dependence. For the axial current this dependence is weak enough that a constant value of q^* from [10] ($q^* \approx 1.43/a$) can be used reliably, and this has been done in [12]. However, the am_Q dependence of q^* for the static-light vector current is more pronounced, so here the scale was calculated for each heavy kappa for both the vector and pseudoscalar case. We compare the value of f_B obtained from the mass dependent q^* scheme with the results [12] for the $q^* \cong 1.43/a$ scheme[1] as a consistency check.

To help estimate systematic uncertainties we use three different chiral fits, in which we extrapolate the results at the light quark kappas used in the simulation to the kappa appropriate to physical light quarks, as determined by the pion mass. The first of these, from which the central value for the ratio is taken and which will be referred to as the standard analysis, uses quadratic fits vs. am_2 (light quark kinematic mass) for m_π^2, and linear fits vs. am_2 for f_π, M_{Qq}, and f_{Qq}. The rationale for these choices is discussed in [3]. The first of the alternate analyses has quadratic fits for m_π^2 and f_π with all other fits linear, and the second has quadratic fits for m_π^2, f_π, and f_{Qq}. The difference of these chiral fits is used to assess the systematic error in the choice for the standard analysis.

For each set of lattices, the ratio of $f\sqrt{M}$ for the vector and pseudoscalar mesons at each heavy kappa is calculated. For each heavy κ, we then divide out $1 - g^2/(6\pi^2)$, the leading order HQET correction to the ratio [13], using g_V^2 evaluated on the lattice at the q^* appropriate to m_B. The resulting data is fit to the three parameter function

$$\frac{b + c/M}{1 + d/M} \tag{1}$$

Since the decay constants each have a $1/M$ expansion in HQET, this fitting function can be viewed as the ratio of the first two terms from the individual expansions. If the data produce the correct static limit, the constant term in the numerator, b, should be 1. Table II shows the value of b for the standard analysis of each set of lattices. Note that all the results are consistent with 1. The errors are quite large on the coarsest lattices at $\beta = 5.7$, but are much smaller on the finer lattices.

From now on we assume consistency with HQET and use the two parameter fitting function Eq. (1) with $b = 1$ to extract the ratio f_{B^*}/f_B. The difference of these fitting methods can be seen in Fig. 1. The final two-parameter fit is then interpolated to the B mass and the leading order perturbative correction is reinserted. This result still includes the ratio of the square roots of the B^* and B masses. Removing this gives us a value of the ratio for each set of lattices, which must then be combined and extrapolated to zero lattice spacing.

The fits shown in Fig. 2 are different possible lattice spacing extrapolations for the continuum value of the ratio. These data are the result of the standard analysis on each set of lattices, but the general features of the plot are generic for all the analyses, as can be seen in Fig. 3. We use constant fits over different intervals in the lattice spacing: 0.2 to 0.4, 0.2 to 0.5, and 0.2 to 0.75 $(\mathrm{GeV})^{-1}$. We do not include a linear fit, as analysis of the new data sets described in [12] suggests that the constant fits provide a good measure of the lattice spacing extrapolation uncertainty. For each set of analyses, the fit to the interval containing

[1]To quote f_B we had to maintain the distinction between lattice sets C & CP, since the static points of these lattices are calculated differently. This is irrelevant for the ratio of the decay constants computed here, but is necessary for this consistency check.

only the values from the two sets with finest lattice spacing ($0.2 < a < 0.4(\mathrm{GeV})^{-1}$) is taken to be the central value for that ratio.

The systematic errors are obtained from various alternative analyses (see Table III). The discretization error is estimated by computing the difference between the average of the two finest lattices and the average over all lattice spacings. The three constant fits for the standard analysis are shown in Fig. 2, and the results of the three fits for each of the alternate analyses can be seen in Fig. 3. We estimate the lattice spacing extrapolation error as the largest difference of the three constant fits, which is $\cong^{+0.02}_{-0}$ for the standard analysis.

Higher order perturbative effects are a second source of systematic error. This error is estimated by taking the difference of the standard analysis with the analysis performed at different values of q^*. In particular, we compare the standard analysis results to the two alternate analyses 4 and 7 of Table III, where q^* is adjusted down and up, respectively. The comparison can be seen in Fig. 3. We estimate the perturbative error to be $\cong^{+0.03}_{-0.01}$.

The final significant contribution to the systematic error comes from the chiral extrapolations. Our estimate of the systematic error involved in this extrapolation procedure is found by taking the larger difference of the central value and the two alternate chiral fits described above. This systematic error can be seen in fig. 3 by comparing fit number 1 to 3, 4 to 6, or 7 to 9. We estimate the error from chiral extrapolation in our central value of the ratio as $\cong^{+0.02}_{-0}$. We do not include an analysis of the other sources of error mentioned in [3] (difference of magnetic mass and kinetic mass, higher order lattice extrapolation fits, and finite volume effects) because they are negligible for the ratio f_{B^*}/f_B.

We combine the three sources of systematic error as if they were completely independent, because we see in Fig. 3 that the results of the different changes made in the analysis are not significantly correlated. This gives us our final quenched value of f_{B^*}/f_B:

$$1.01(0.01)(^{+0.04}_{-0.01})\,.$$

In HQET the leading order (in $1/M$) value of the ratio is $1-\frac{g^2(m_B)}{6\pi^2} \approx 0.96$, using $\Lambda^{(5)} = 0.208$ GeV [14]. Note that this is less than 1 because the perturbative correction is negative. However the results of our simulations suggest that the ratio for the B is more likely to be greater than or equal to 1. Neubert has calculated that the value of the ratio f_{B^*}/f_B using the subleading order terms in the $1/M$ expansion to be 1.07 ± 0.03 [15], which is consistent with our result.

Using the same analysis, we find $f_B \cong 175(7)$MeV, where statistical error only is shown. This should be compared with the current MILC value $f_B = 173(6)(16)$MeV [12]. The latter includes improved action data and a complete systematic error analysis and, therefore, should be taken as the most up-to-date MILC value[2] for quenched f_B. However, the consistency of the current analysis with the previous calculation is comforting and indicates, among other things, that the use of a scheme in which q^* varies with heavy quark mass has no drastic effects.

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[2]We note that the central value of f_B in [12] is considerably higher than the value $157(11)(^{+22}_{-9})(^{+21}_{-0})$MeV quoted in [3]. That difference is due to new data (including improved action) and new analysis (including update of q^*) as explained in [11] and [12].

We also report a value for f_{B^*}. This quantity was not calculated in [3] or [12], so we perform a more detailed analysis including estimates of systematic errors analogous to that performed in [12]. This gives $f_{B^*} = 177(6)(17)$MeV in the quenched approximation.

These results for f_{B^*} and f_{B^*}/f_B are in qualitative agreement with what is expected. However, the main point of this paper is not the computation of the quenched decay constants at the B mass, but the extrapolation of our results to infinite heavy quark mass. The agreement of f^*_{Qq}/f_{Qq} in this limit with the HQET prediction is an indication that the present treatment of the heavy quark on the lattice is consistent.

Calculations for this project were performed at ORNL CCS, SDSC, Indiana University, NCSA, PSC, MHPCC, CTC, CHPC (Utah), and Sandia Natl. Lab. This work was supported by the U.S. Department of Energy under grants DE-FG02-91ER-40628, DE-FG02-91ER-40661, DE-FG02-97ER-41022 and DE-FG03-95ER-40906, and by the National Science Foundation under grants PHY99-70701 and PHY97–22022.

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TABLES

TABLE I. Summary of quenched Wilson action lattices

set	β	size	# confs.
A	5.7	$8^3 \times 48$	200
B	5.7	$16^3 \times 48$	100
E	5.85	$12^3 \times 48$	100
C	6.0	$16^3 \times 48$	100
CP	6.0	$16^3 \times 48$	305
D	6.3	$24^3 \times 80$	100
H	6.52	$32^3 \times 100$	60

TABLE II. Agreement of the static limit of the 3 parameter ratio fits with 1.

Set of Lattices (β)	Static limit of fitting function (including plateau uncertainties)
H(6.52)	0.99 ± 0.05
D(6.3)	0.99 ± 0.04
CP(6.0)	1.05 ± 0.04
C(6.0)	1.08 ± 0.08
E(5.85)	0.89 ± 0.08
B(5.7/16)	0.70 ± 0.42
A(5.7/8)	1.04 ± 0.25

TABLE III. Explanation of the Analysis types from Fig 3

Analysis Number	Details of Analysis
1	standard analysis
2	Change f_π fits from linear to quadratic
3	Change f_π and f_{Qq} fits from linear to quadratic
4–6	Same as 1–3 with q^* chosen so that its mean value for the heavy kappas is $1/a$
7–9	Same as 1–3 with q^* chosen twice as large as the standard analysis

FIGURES

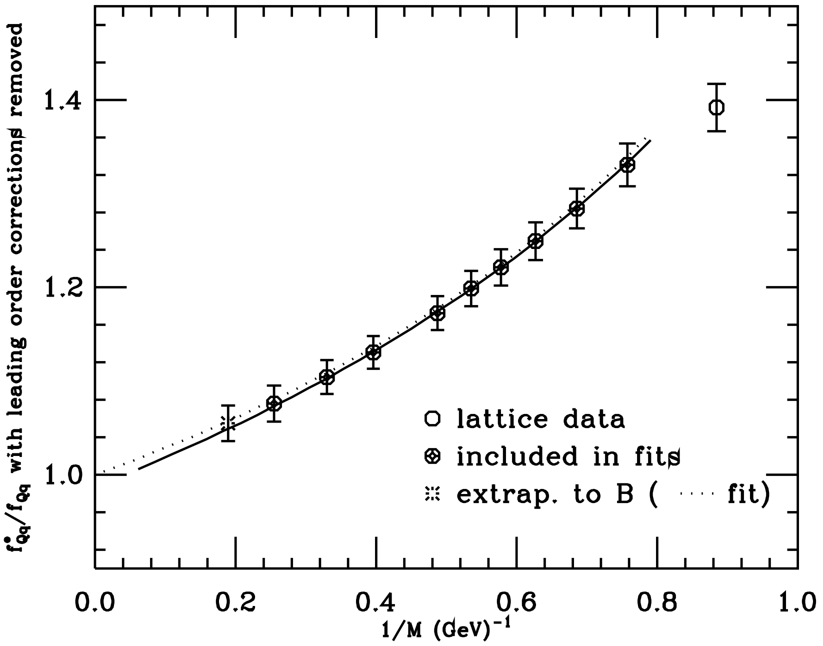

FIG. 1. Comparison of three-parameter fits and two-parameter fits for the $\beta = 6.52$ lattices. Perturbative HQET corrections have been removed. Dotted line enforces the HQET result that $f^*_{Qq}/f_{Qq} = 1$ at $M = \infty$. Solid line allows the $M = \infty$ value to be free.

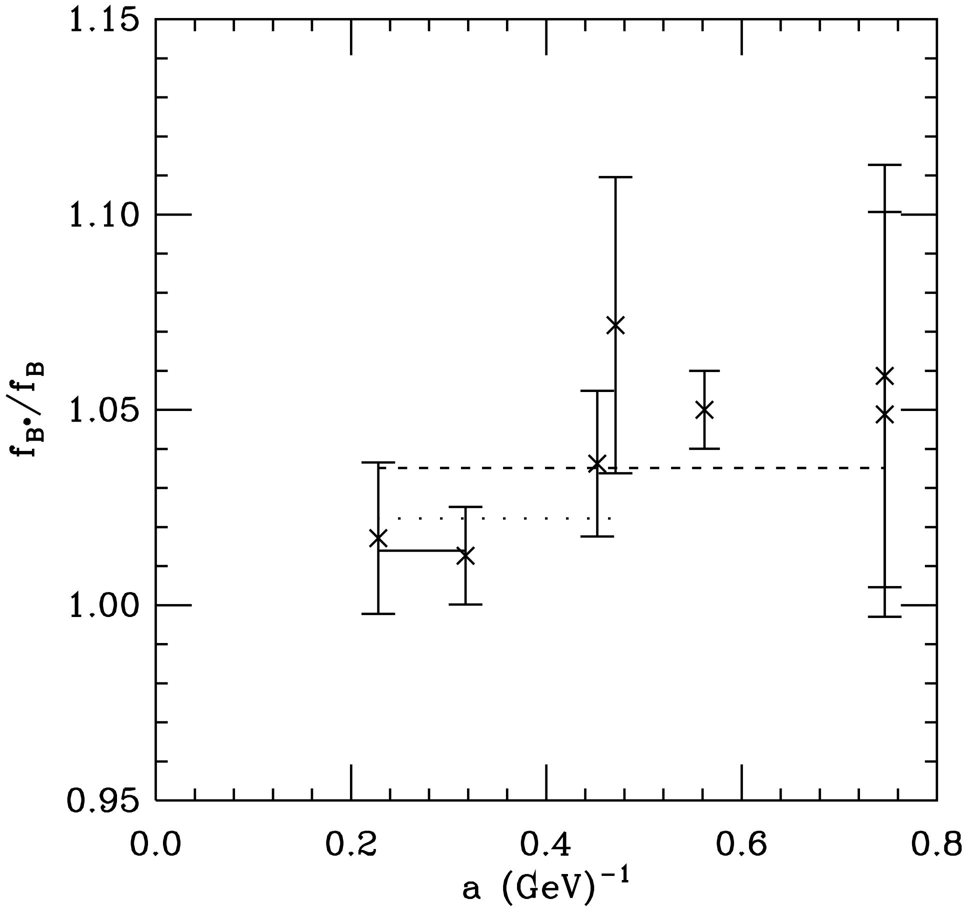

FIG. 2. Three constant fits to estimate the lattice spacing extrapolation error in the ratio. The fit to $a < 0.4$ (solid line) is taken as the central value. The data are for the standard analysis. The fit ranges are 0.2 to 0.4 $(\mathrm{GeV})^{-1}$ (solid line) , 0.2 to 0.5 $(\mathrm{GeV})^{-1}$ (dotted line), and 0.2 to 0.75 $(\mathrm{GeV})^{-1}$ (dashed line)

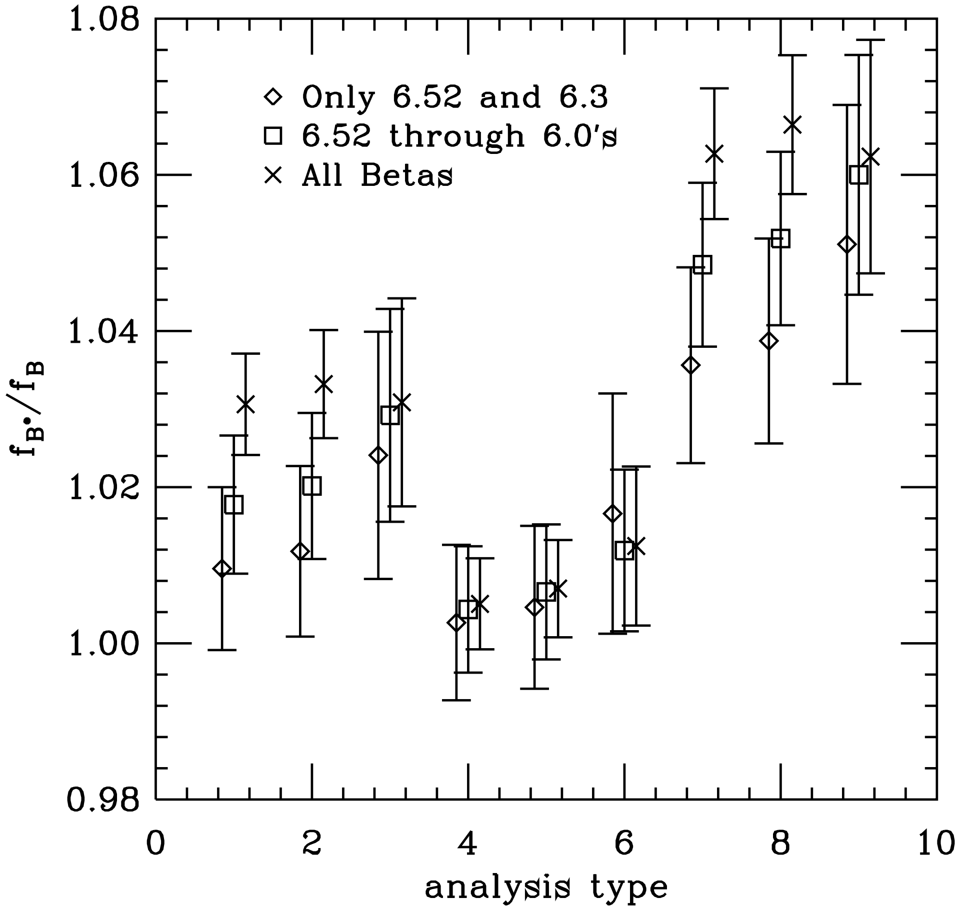

FIG. 3. Plot of the final data from the different types of analyses. See Table III for descriptions of each analysis.