Lattice QCD:
adding virtual quark loops
and
other vexing issues

Claude Bernard

— also —

The MILC Collaboration:

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Feb. 4, 1999: complete talk now follows

Focus on heavy-light physics, largely leptonic decay constant. Concentrate on MILC results.

MILC = "MIMD Lattice Computations"

For general reviews see:

S. Sharpe, ICHEP '98, hep-lat/9811006 [all of lattice QCD]

T. Draper, Lattice 98, hep-lat/9810065 [heavy-light physics on the lattice]

A. Kronfeld, Heavy Quarks at Fixed Target '98, hep-ph/9812288 [heavy-light physics on the lattice]

Outline:

- motivation to calculate f_B, *etc.*

- history of f_B (quenched)

Note: "quenched" $\equiv$ omitted virtual quark loops

- published MILC results (quenched; dynamical quarks only to estimate quenching errors)

Note: "dynamical quarks" $\equiv$ virtual quark loops

- what do we expect for quenching effects?

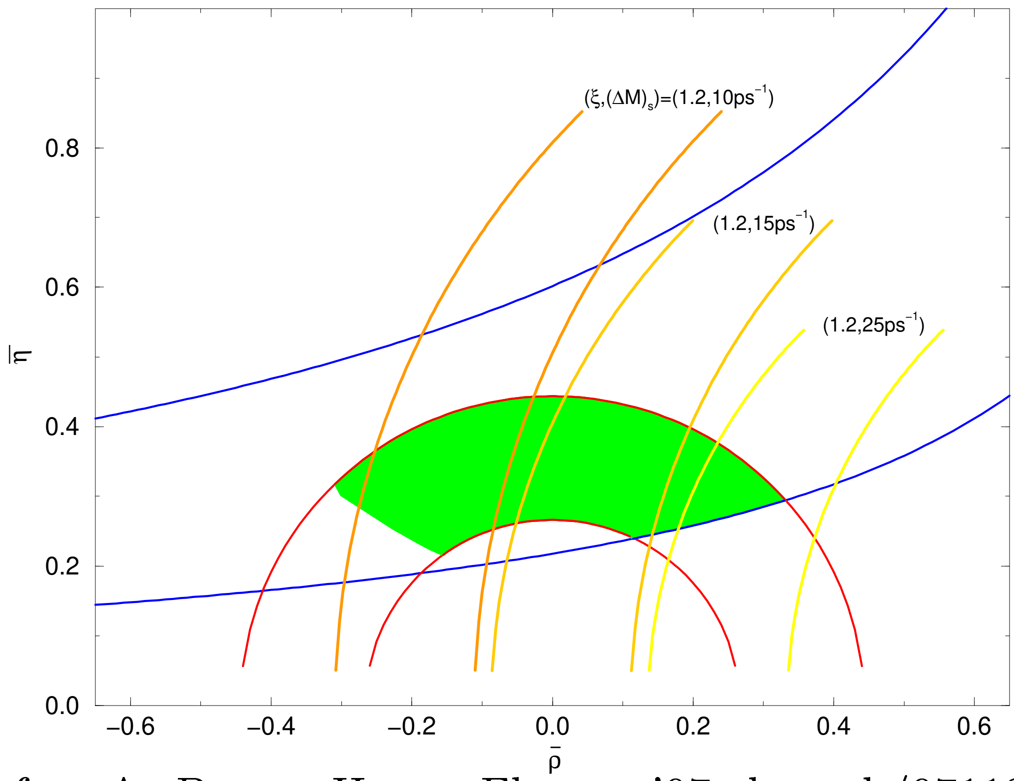
- new MILC results; preliminary analysis of f_B with 2 flavors of dynamical quarks ($f_B^{N_F=2}$)

- results on "unquenching" f_B from other groups

- comments on $\hat{B}_B$, $\hat{B}_{B_s}$

- interesting new results (quenched) on semileptonic form factors: $B \to D$ (FNAL), $B \to \pi$ (UKQCD, FNAL)

- where do we go from here?



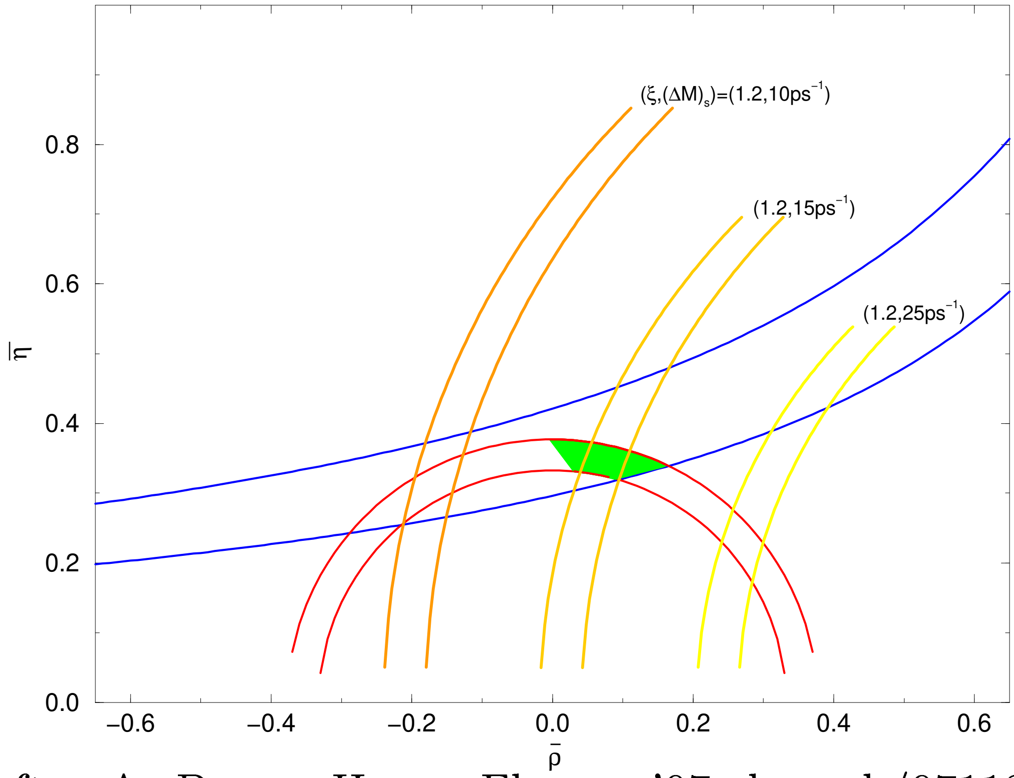
after A. Buras, Heavy Flavors '97, hep-ph/9711217: "Unitarity Triangle 1997"

Assumes: 20% errors on $f_B\sqrt{\hat{B}_B}$,

$$10\% \text{ errors on } \xi \equiv \frac{f_{B_s}\sqrt{B_{B_s}}}{f_B\sqrt{B_B}}$$

orange/yellow pairs show constraints on ρ, η (with current errors on ξ) if B_s-$\bar{B}_s$ mixing were observed at the given levels.

Expect constraint from B-$\bar{B}$ mixing to be pair of circles (like the orange pairs). Actually B-$\bar{B}$ mixing just cuts off the lower left hand corner of the allowed region — due to correlations with the constraints from K-$\bar{K}$ mixing and V_{ub}/V_{cb}. B-$\bar{B}$ has minimal effect at present due to large theoretical errors in $f_B\sqrt{\hat{B}_B}$

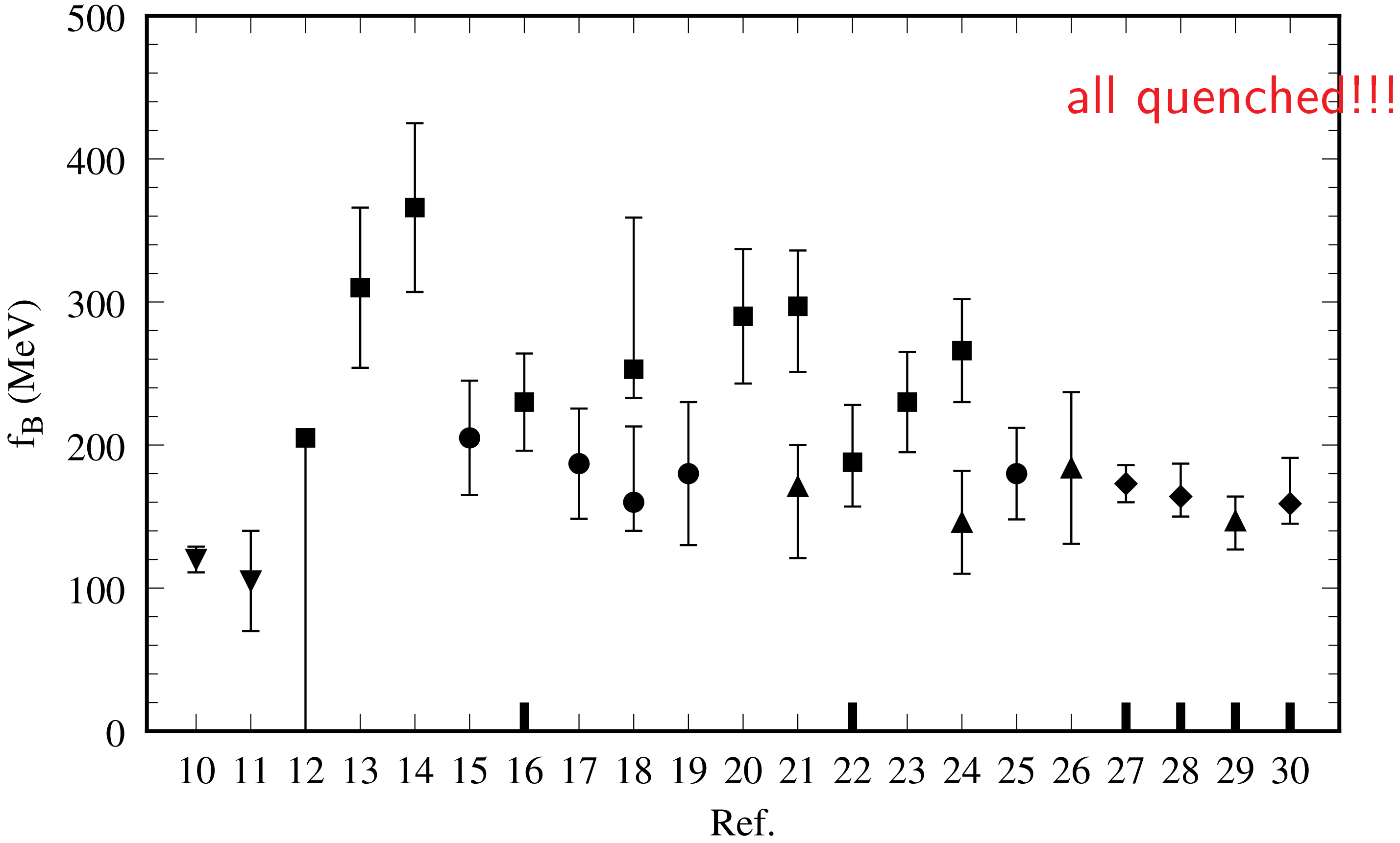


after A. Buras, Heavy Flavors '97, hep-ph/9711217: "Unitarity Triangle 2007"

possible scenario for 2007.

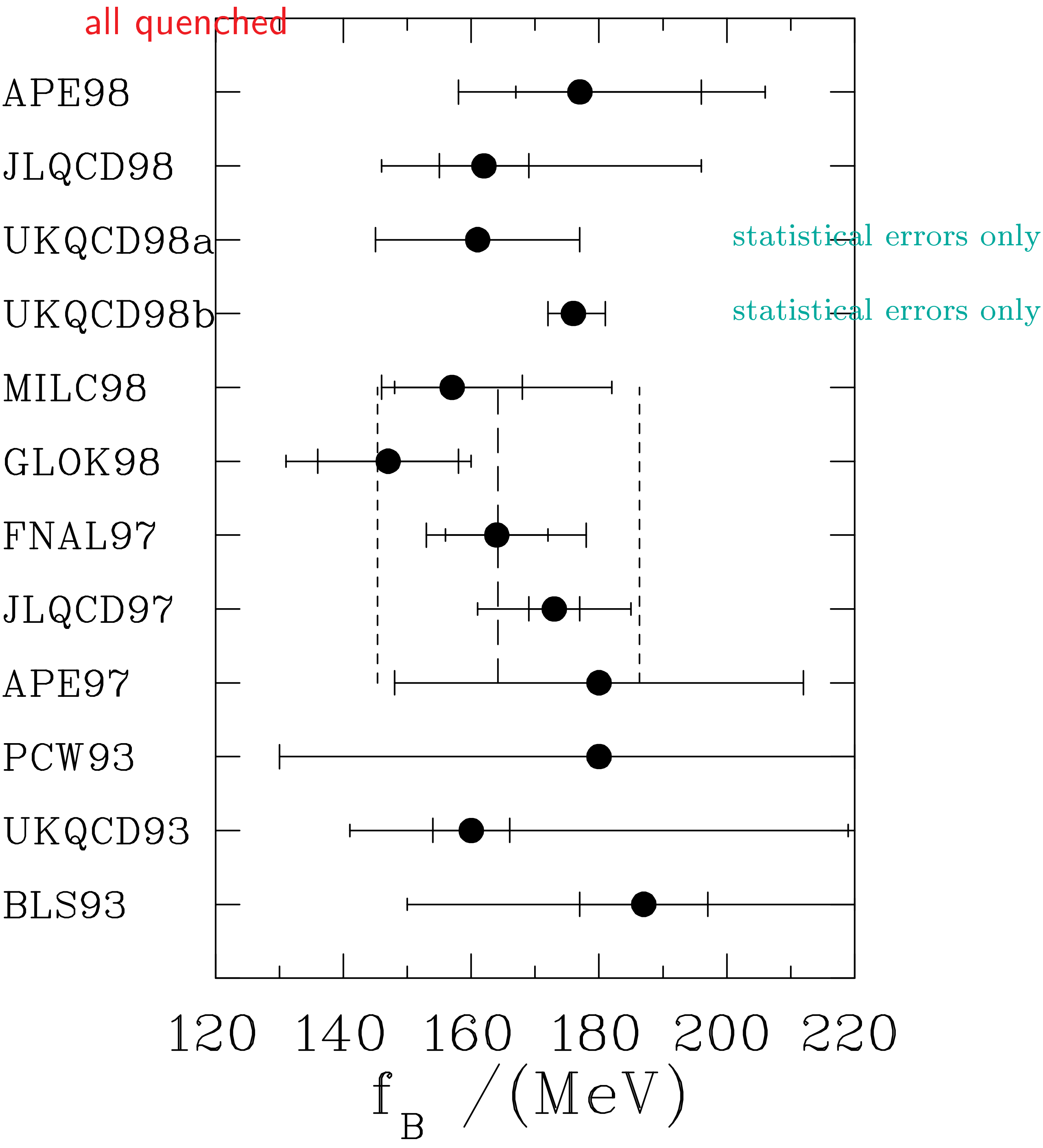
Assumes: 5% errors on $f_B\sqrt{\hat{B}_B}$,

3% errors on ξ



Time-line of calculations of f_B with lattice QCD. Methods shown are extrapolation from $m_Q \leq m_c$ to m_b (inverted triangles), the static limit $m_Q \to \infty$ (squares), interpolation between m_c and ∞ (circles), NRQCD (triangles), and that of Ref. [6]. Entries with a thick tick-mark on the horizontal axis control lattice spacing effects, as explained in the text.

from: A. Kronfeld, Heavy Quarks at Fixed Target 98, hep-ph/9812288 from: T. Draper, Lattice 98, hep-lat/9810065



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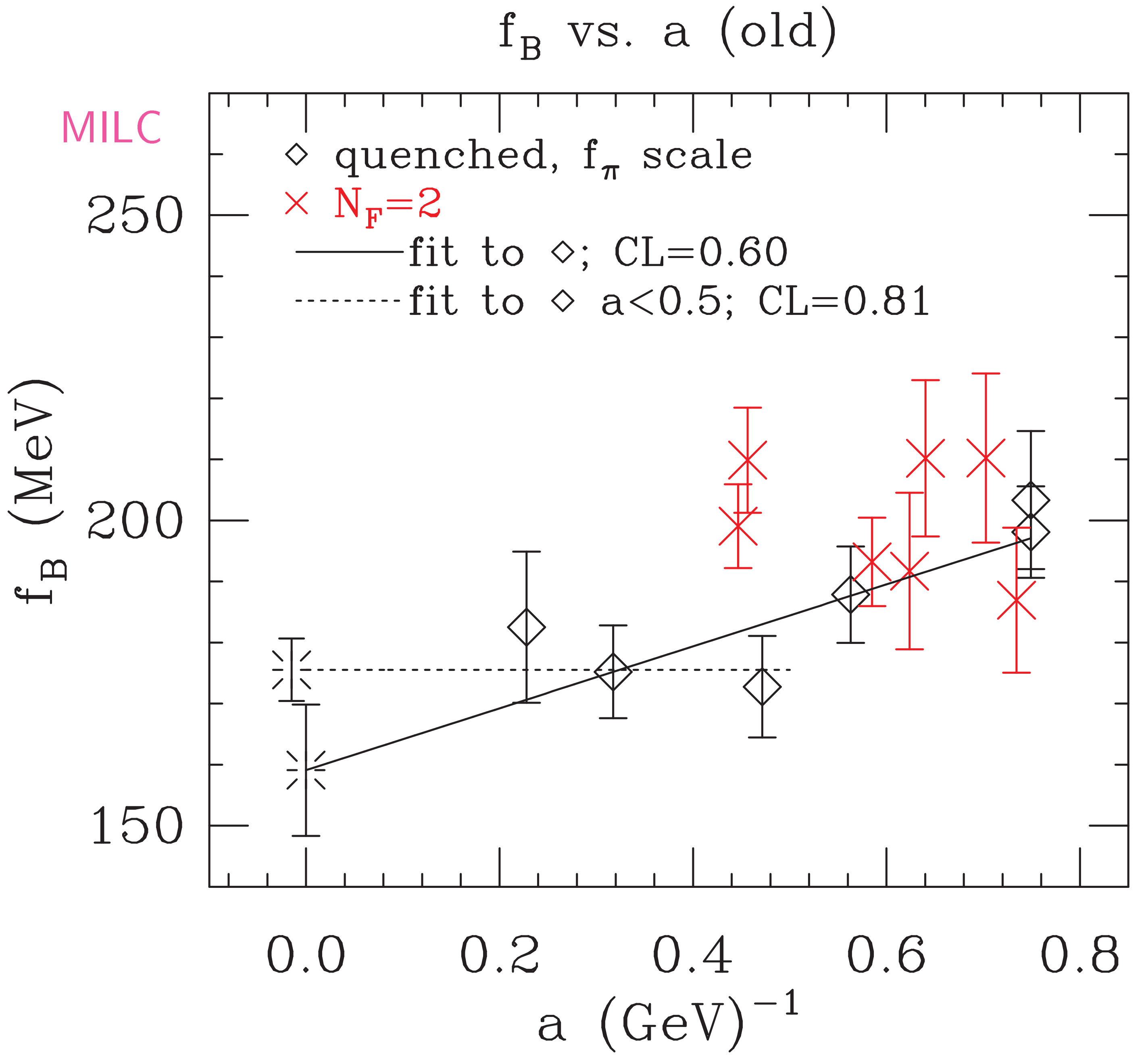
LATTICES:

Name	β	**size**	**# configs.**
A	$5.7(quench)$	$8^3 \times 48$	200
B	$5.7(quench)$	$16^3 \times 48$	100
E	$5.85(quench)$	$12^3 \times 48$	100
C	$6.0(quench)$	$16^3 \times 48$	100
CP†	$6.0(quench)$	$16^3 \times 48$	305
D	$6.3(quench)$	$24^3 \times 80$	100
H	$6.52(quench)$	$32^3 \times 100$	60
L	$5.445,\ m = .025$	$16^3 \times 48$	100
N	$5.5,\ m = .1$	$24^3 \times 64$	100
O	$5.5,\ m = .05$	$24^3 \times 64$	100
M†	$5.5,\ m = .025$	$20^3 \times 64$	199
P†	$5.5,\ m = .0125$	$20^3 \times 64$	199
G*	$5.6,\ m = .01$	$16^3 \times 32$	200
R	$5.6,\ m = .01$	$24^3 \times 64$	200
S†	$5.6,\ m = .02$	$24^3 \times 64$	201
T†	$5.6,\ m = .04$	$24^3 \times 64$	202
U†	$5.6,\ m = .08$	$24^3 \times 64$	201
F**	$5.7,\ m = .01$	$16^3 \times 32$	49

* from HEMCGC
** from Columbia group
† new

Additional set "5.7–large" at $\beta = 5.7$ is used to study light quark physics only. Set includes $12^3 \times 48$ (403 configurations), $16^3 \times 48$ (390 confs.), $20^3 \times 48$ (200 confs.), $24^3 \times 48$ (184 confs.)



The $N_F = 2$ results have fixed dynamical masses (in the range $m_s/2 \lesssim m_{dyn} \lesssim 3m_s$) but valence masses extrapolated to the chiral limit. This is often called "partial quenching."

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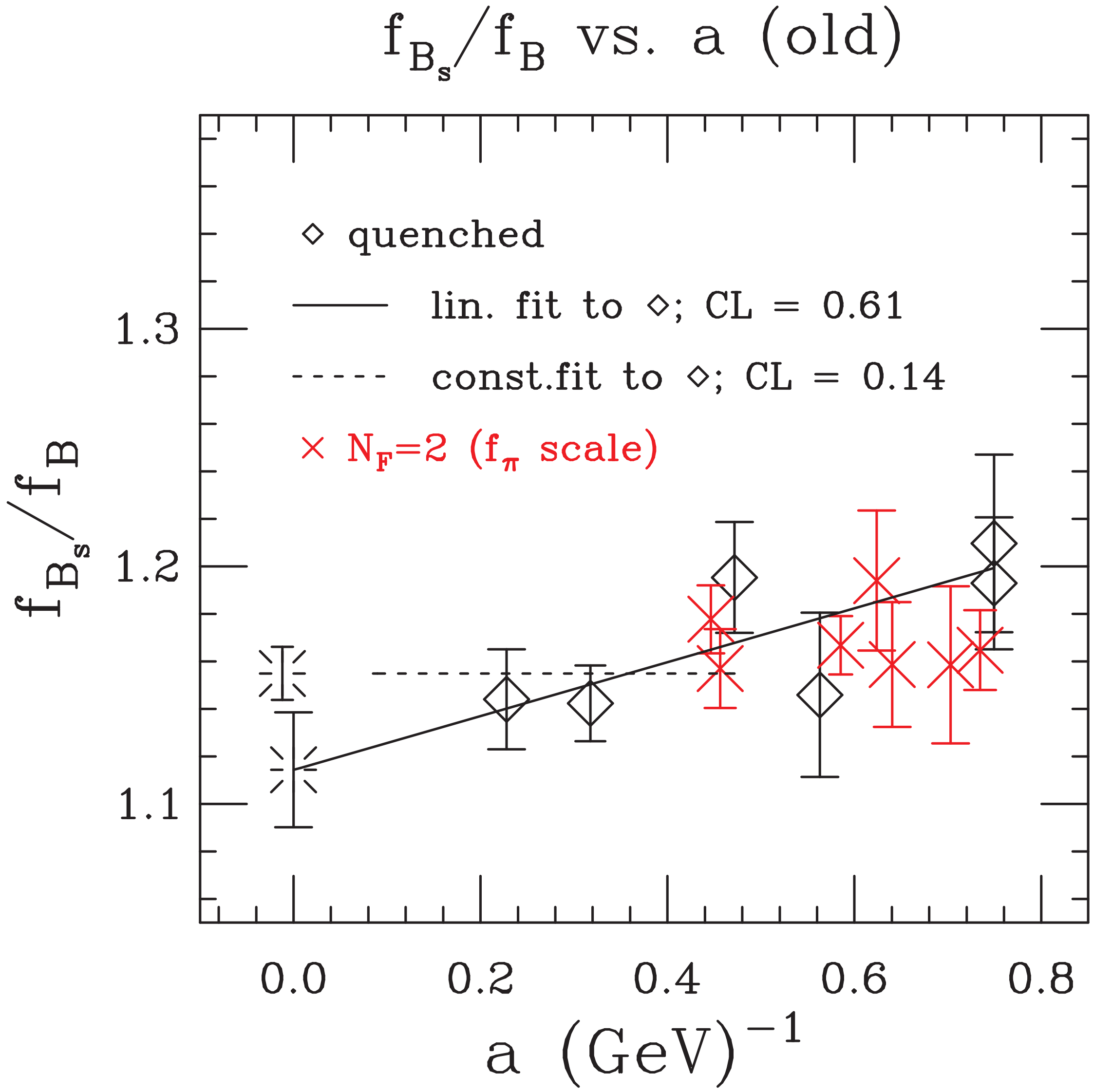
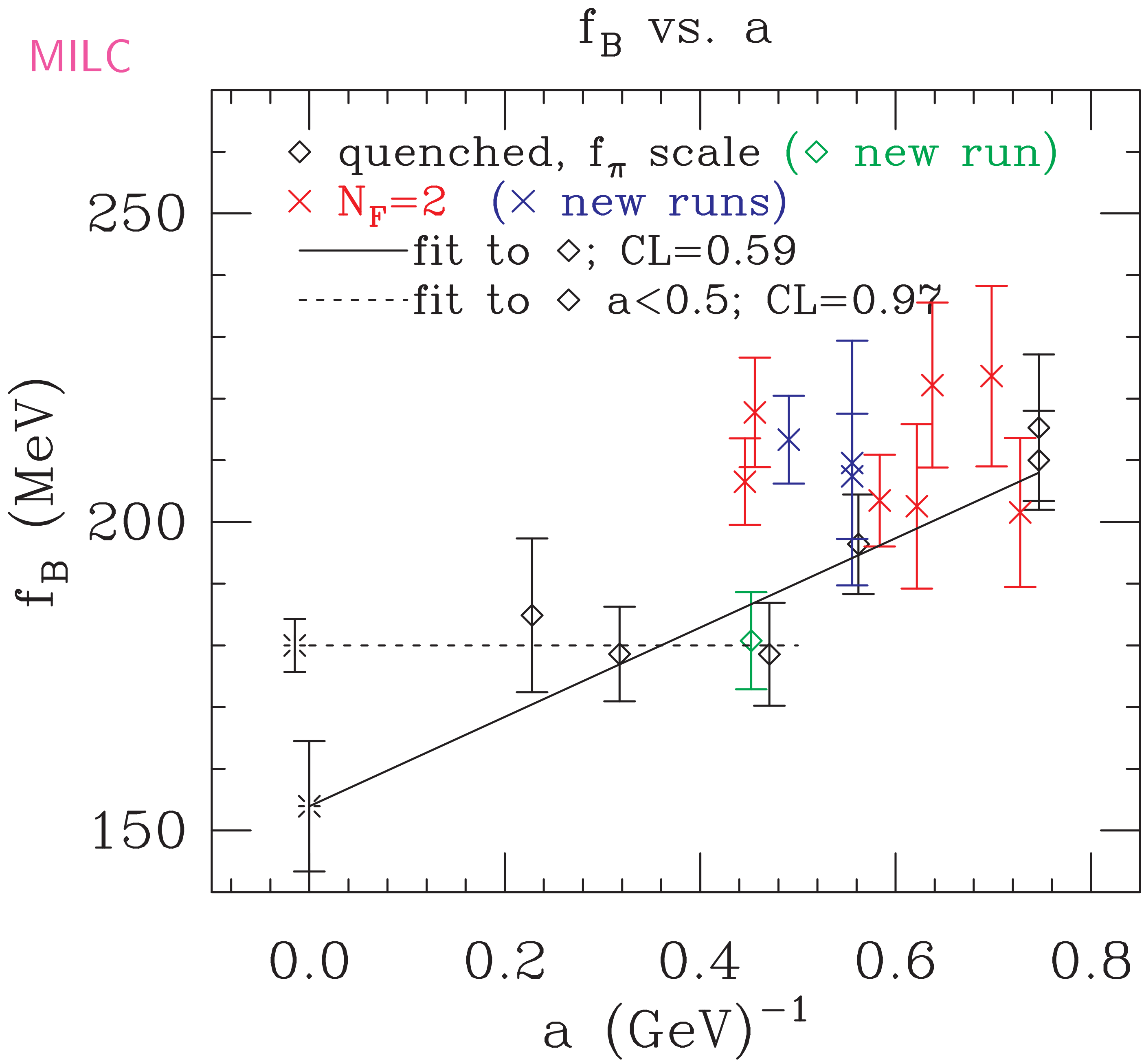


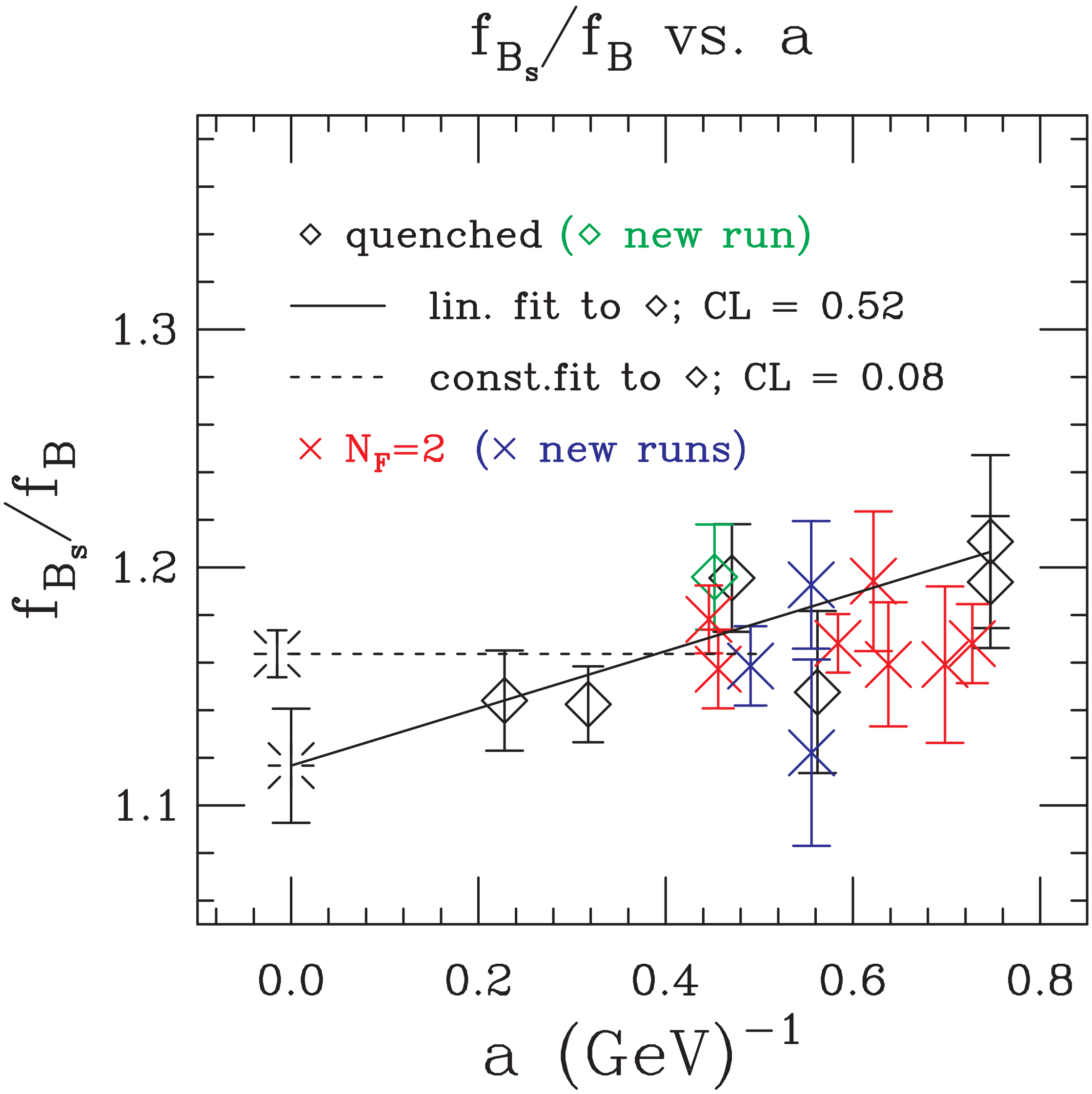
f_{B_s}/f_B vs. a (old)
quenched
lin. fit to ◇; CL = 0.61
const.fit to ◇; CL = 0.14
N_F=2 (f_π scale)
1.3
1.2
1.1
f_{B_s}/f_B
0.0
0.2
0.4
0.6
0.8
a $(\mathrm{GeV})^{-1}$

MILC



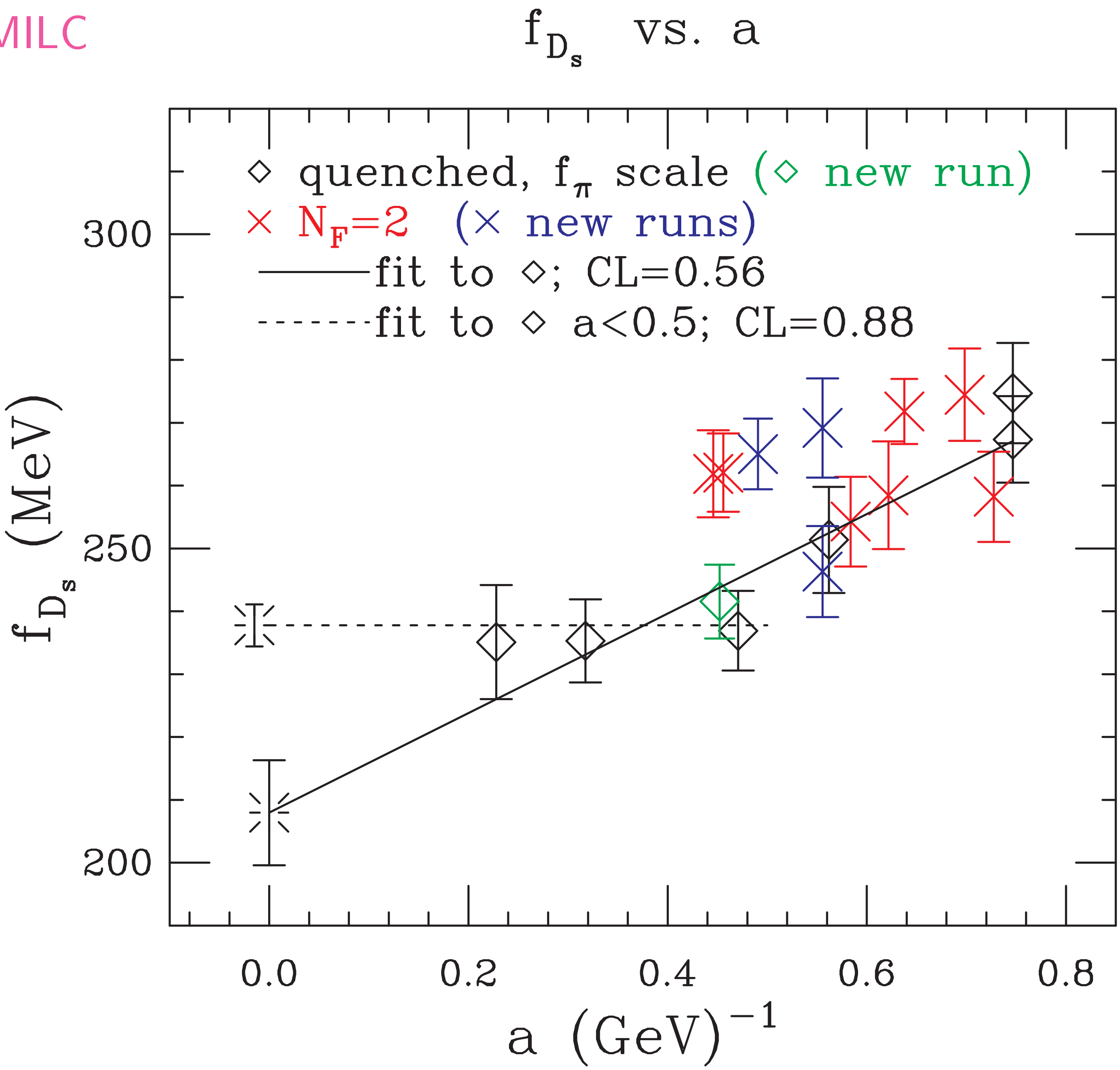
Constant fit is to quenched data better than linear fit here, but see f_{B_s}/f_B (next slide).

MILC



Constant fit to quenched data is poor here. We therefore use linear fit here. For consistency, have to use linear fit for decay constants also (f_B, $f_{B_s}, \ldots$)

MILC



f_{D_s} vs. a
quenched, f_π scale (◇ new run)
N_F=2 (× new runs)
fit to ◇; CL=0.56
fit to ◇ a<0.5; CL=0.88
300
250
200
f_{D_s} (MeV)
0.0
0.2
0.4
0.6
0.8
a $(\mathrm{GeV})^{-1}$

MILC

Results from quenched approx, with dynamical runs to estimate errors:

$$
\begin{aligned}
f_B &= 154 \ \pm 11 \ {}^{+27}_{-\ 7} \ {}^{+23}_{-\ 0} \quad \mathrm{MeV} \\
f_{B_s} &= 169 \ \pm \ \ 9 \ {}^{+35}_{-8} \ {}^{+27}_{-\ 0} \quad \mathrm{MeV} \\
f_D &= 191 \ \pm 10 \ {}^{+19}_{-10} \ {}^{+15}_{-\ 0} \quad \mathrm{MeV} \\
f_{D_s} &= 208 \ \pm \ \ 8 \ {}^{+26}_{-\ 8} \ {}^{+17}_{-\ 0} \quad \mathrm{MeV}
\end{aligned}
$$

$$
\begin{aligned}
f_{B_s}/f_B &= 1.12 \ \pm 0.02 \ {}^{+0.04}_{-0.03} \ {}^{+0.03}_{-0.03} \\
f_{D_s}/f_D &= 1.10 \ \pm 0.02 \ {}^{+0.04}_{-0.02} \ {}^{+0.02}_{-0.03} \\
f_B/f_{D_s} &= 0.75 \ \pm 0.03 \ {}^{+0.04}_{-0.02} \ {}^{+0.08}_{-0.00} \\
f_{B_s}/f_{D_s} &= 0.85 \ \pm 0.03 \ {}^{+0.05}_{-0.03} \ {}^{+0.05}_{-0.00}
\end{aligned}
$$

$$
c_1 = 0.8 \ \pm 0.2 \ {}^{+0.3}_{-0.1} \ {}^{+0.4}_{-0.2} \quad \mathrm{GeV}
$$

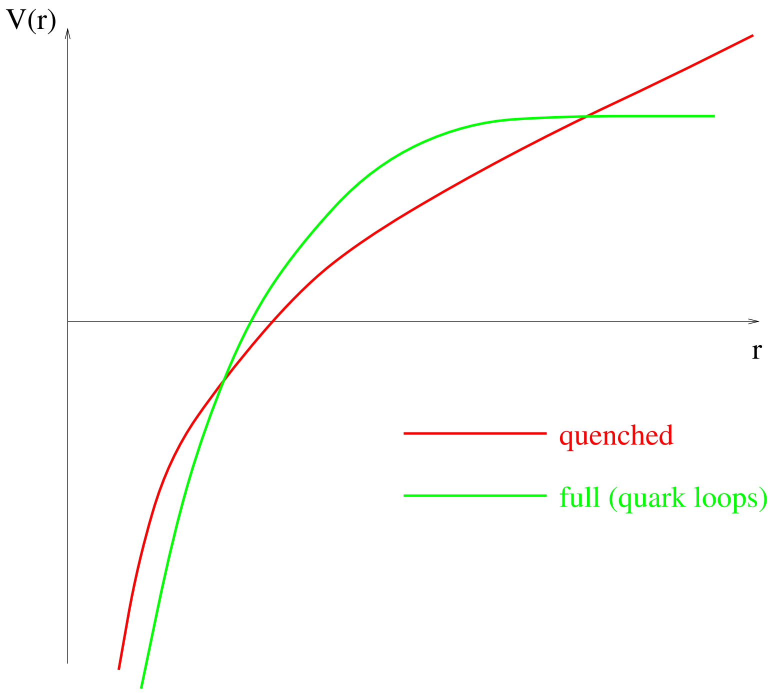
where the errors are statistical, systematic (within the quenched approximation), and systematic (of quenching), respectively.

The above results are slightly different from our published results [Phys. Rev. Lett. 81, 4812 (1998)] because some new quenched running is included.

What do we expect for quenching effects on heavy-light (HL) decay constants?

Quark Model

states are small. so Coulomb-like part of potential $V_{\mathrm{coul}} = \alpha_S(r)/r$ is probably most important



Asymptotic freedom softens potential at small r

Quenched α_S runs faster $\Rightarrow$ softened more

$\Rightarrow f_B^{\mathrm{quench}} < f_B^{\mathrm{full}}$

(decay constant is basically a wave function at the origin)

Note, however, that we set the scale by f_π (for our central values) so we are really calculating f_B/f_π.

Does this change the conclusion?

Ans: No. For hydrogen-like wave functions, radius $r_0 \sim 1/\mu$, with μ the reduced mass.

$\Rightarrow B$ is smaller state than π, so B is more affected by wave function at the origin.

$$\Rightarrow \left(\frac{f_B}{f_\pi}\right)^{\text{quench}} < \left(\frac{f_B}{f_\pi}\right)^{\text{full}}$$

Similarly expect

$$\left(\frac{f_{B_s}}{f_B}\right)^{\text{quench}} < \left(\frac{f_{B_s}}{f_B}\right)^{\text{full}}$$

Chiral Perturbation Theory

quenched chiral perturbation theory (Sharpe; C.B. & Golterman) has been applied to HL system by Booth; Sharpe & Zhang

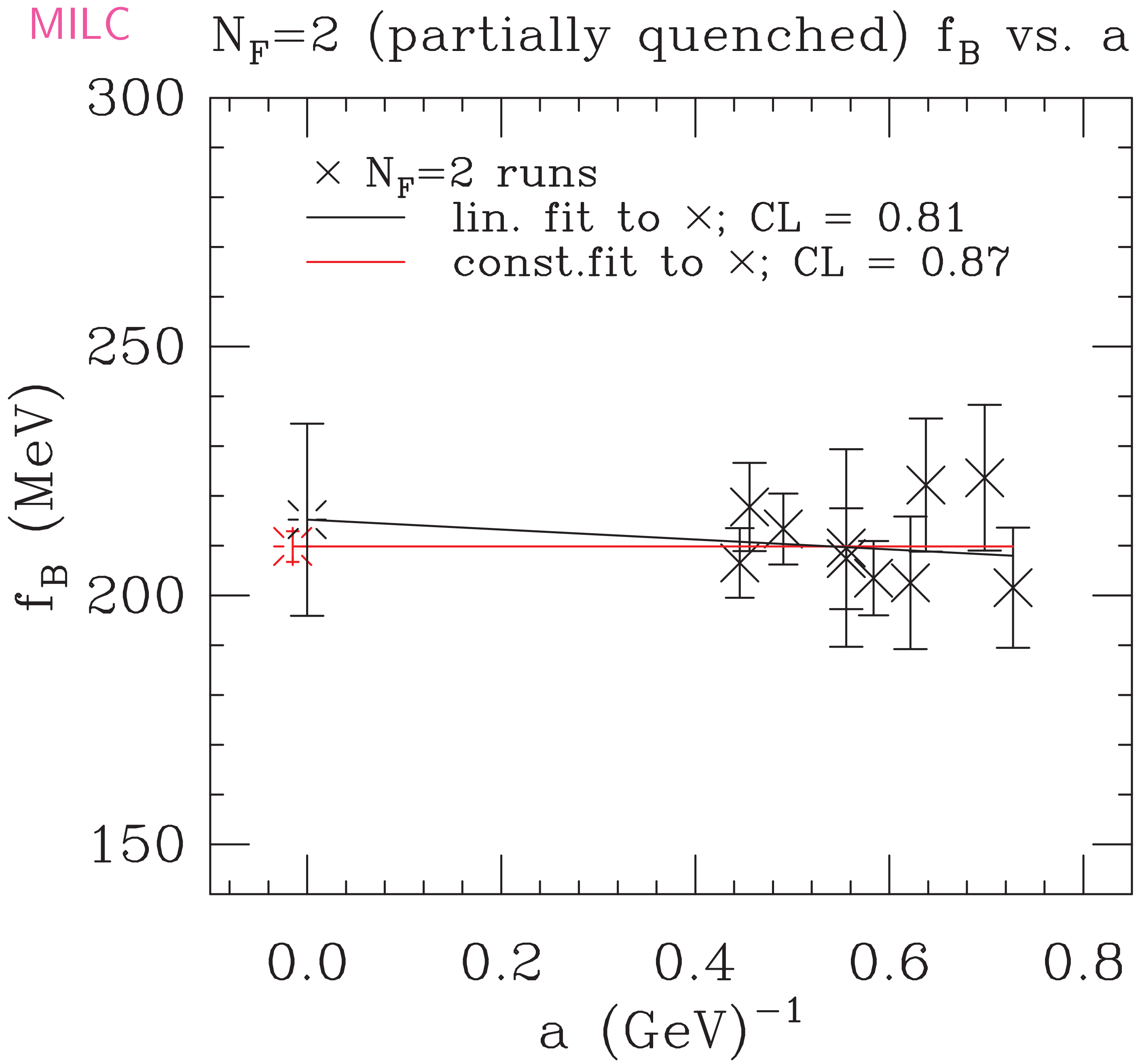
quenched chiral logs for decay constants and bag parameters are different from chiral logs of full theory

unfortunately, many results are qualitative only — sensitive to unknown couplings and analytic terms

Qualitatively, expect:

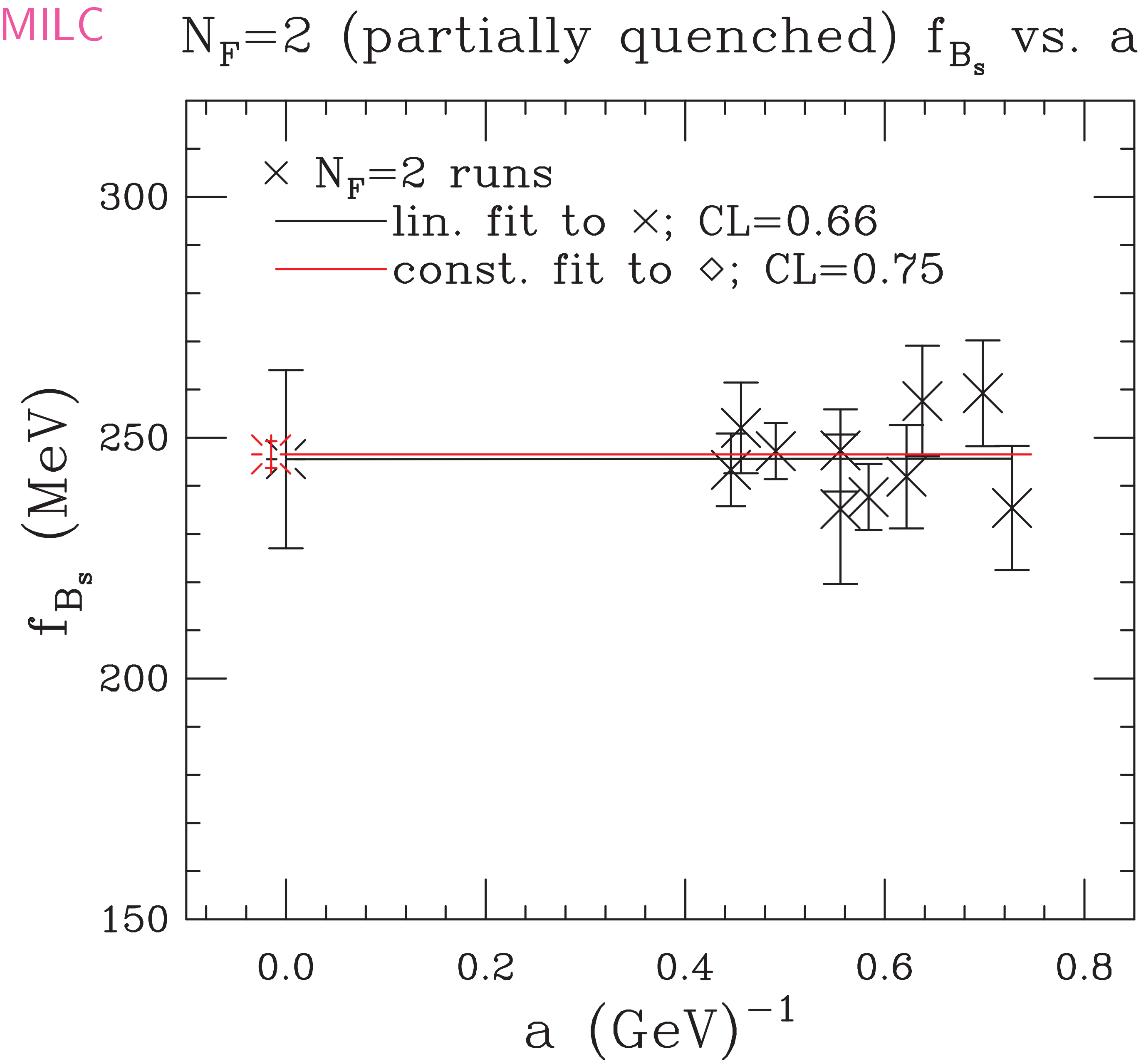
- quenching effects on f_B, f_{B_s} can be large (~ 15–40%)
- $f_B^{\rm quench} < f_B^{\rm full}$; $\qquad f_{B_s}^{\rm quench} < f_{B_s}^{\rm full}$
- $\left(\frac{f_{B_s}}{f_B}\right)^{\rm quench} < \left(\frac{f_{B_s}}{f_B}\right)^{\rm full}$
- bag paramters (B_B, B_{B_s}) have smaller quenching errors
- B_{B_s/B_B} has still smaller quenching errors
- errors in "partially quenched" theory are considerably less, but still may be significant

recall "partially quenched " $\Rightarrow m_{\rm dynamical}$ fixed, but $m_{\rm valence} \to$ chiral limit.



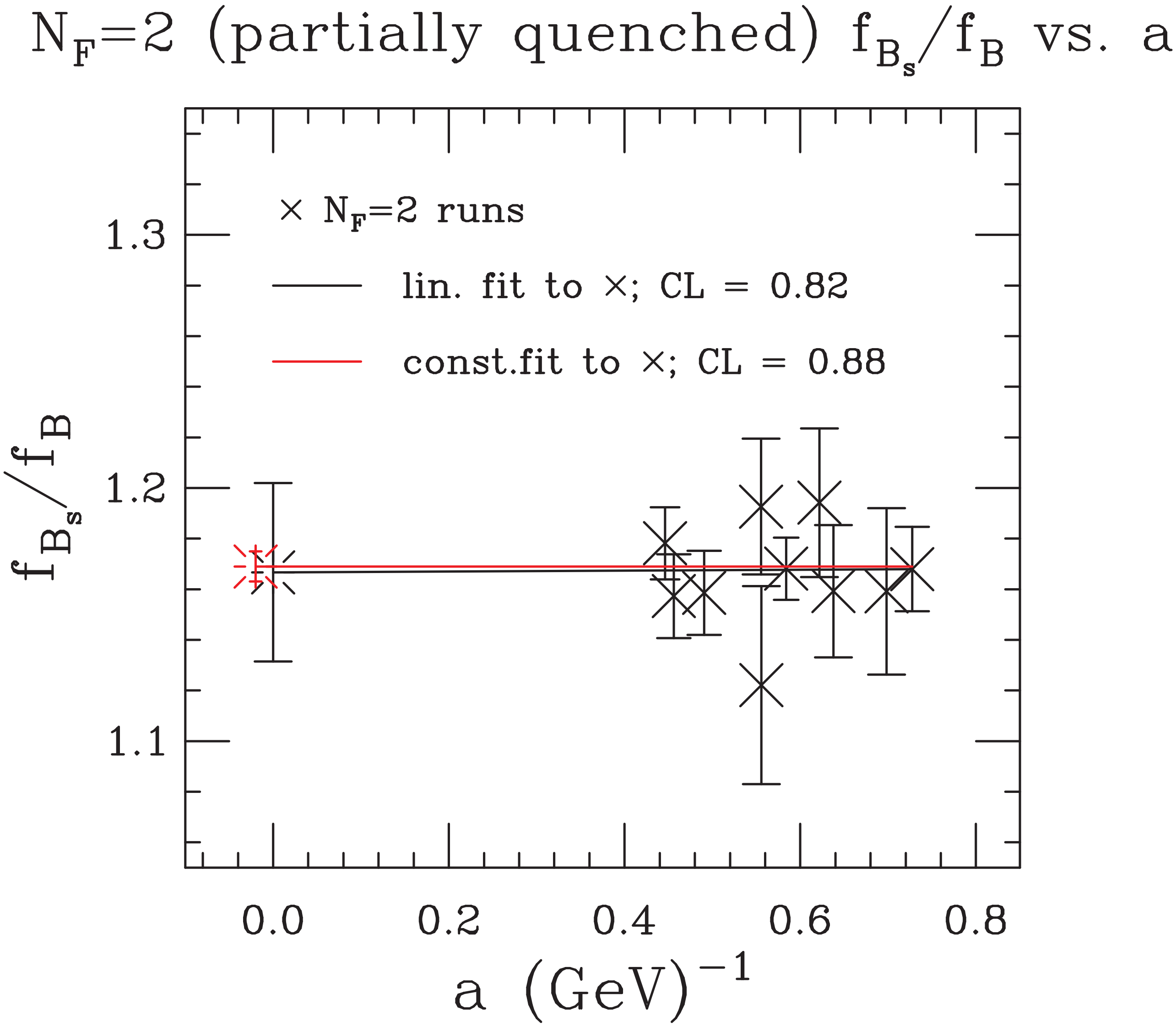
MILC
N_F=2 (partially quenched) f_B vs. a
× N_F=2 runs
lin. fit to ×; CL = 0.81
const.fit to ×; CL = 0.87
300
250
200
150
f_B (MeV)
0.0
0.2
0.4
0.6
0.8
a $(\mathrm{GeV})^{-1}$

MILC

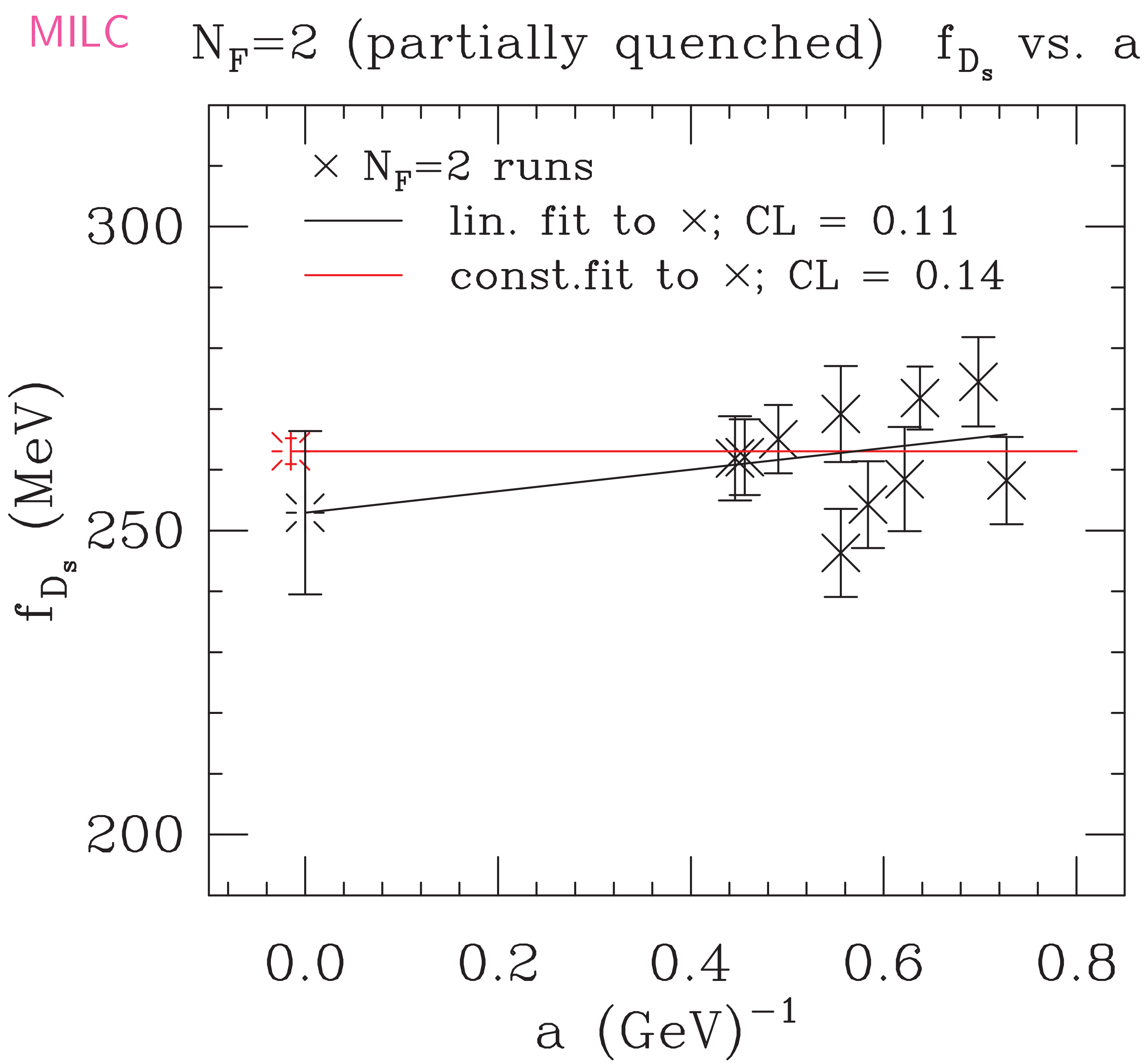


N_F=2 (partially quenched) f_{B_s} vs. a
× N_F=2 runs
lin. fit to ×; CL=0.66
const. fit to ◇; CL=0.75
f_{B_s} (MeV)
300
250
200
150
0.0
0.2
0.4
0.6
0.8
a (GeV)^{-1}

MILC

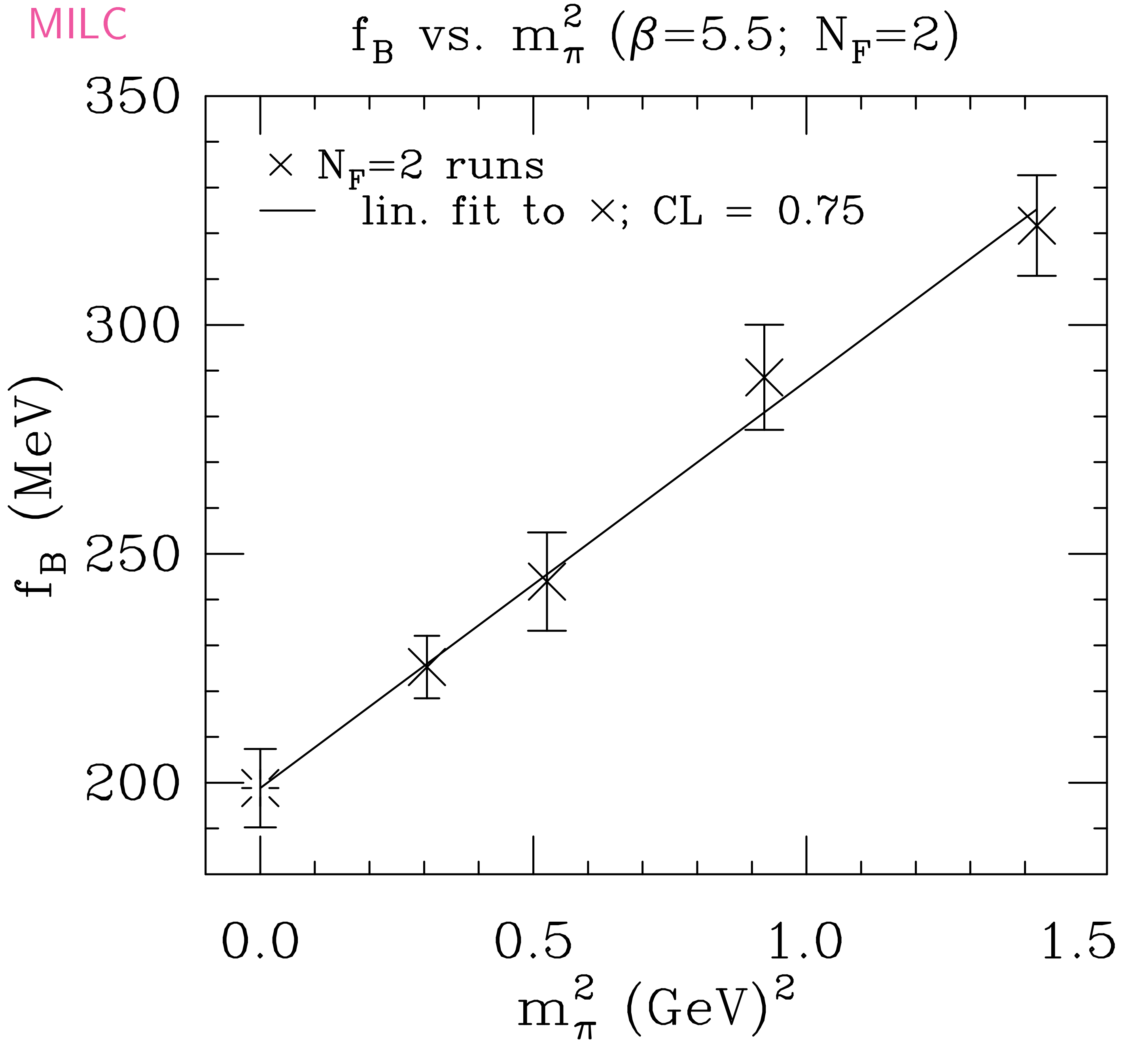


N_F=2 (partially quenched) f_{B_s}/f_B vs. a
× N_F=2 runs
lin. fit to ×; CL = 0.82
const.fit to ×; CL = 0.88
1.3
1.2
1.1
f_{B_s}/f_B
0.0
0.2
0.4
0.6
0.8
a $(\mathrm{GeV})^{-1}$



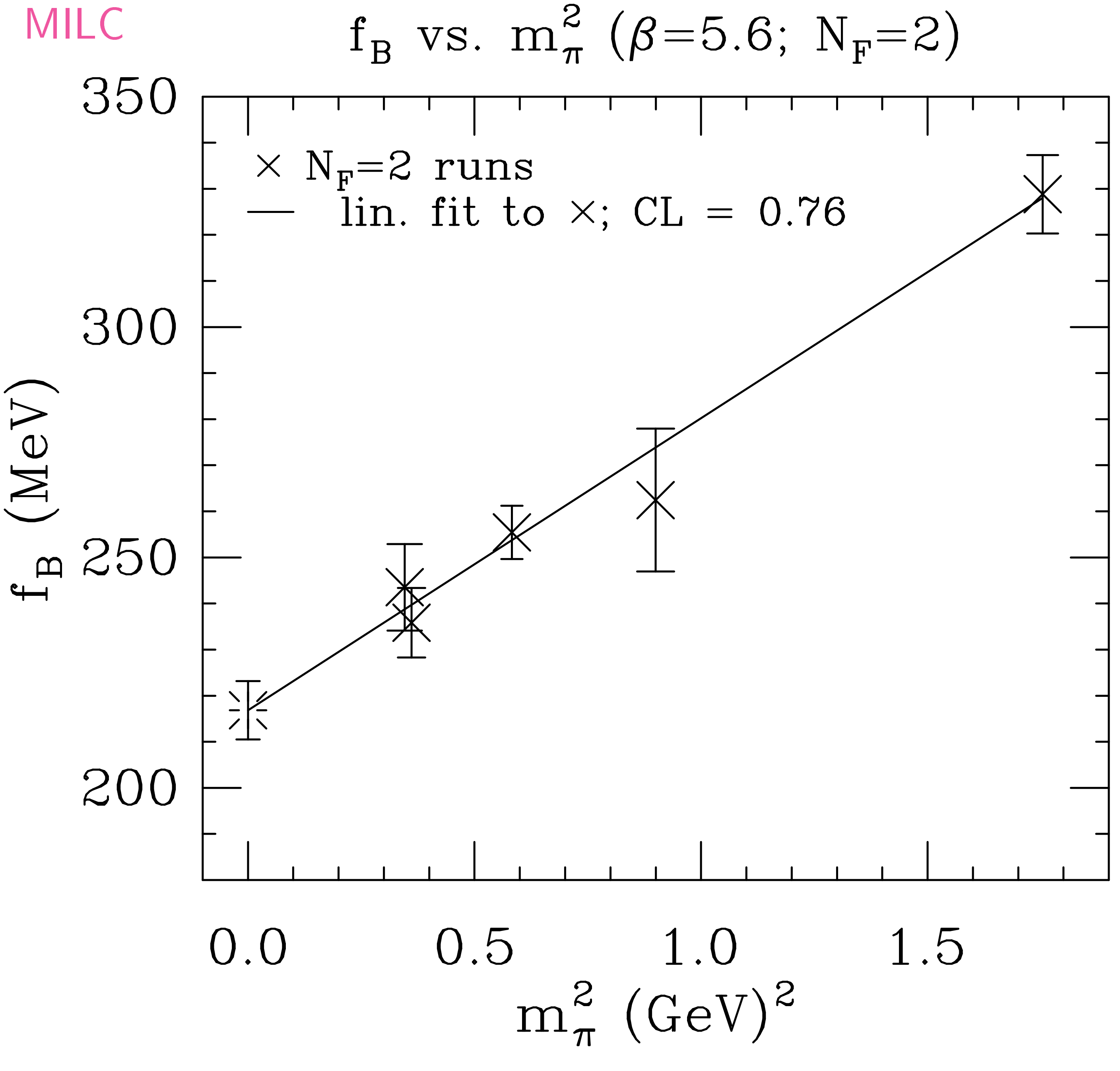
MILC
N_F=2 (partially quenched) f_{D_s} vs. a
× N_F=2 runs
lin. fit to ×; CL = 0.11
const.fit to ×; CL = 0.14
300
250
200
f_{D_s} (MeV)
0.0
0.2
0.4
0.6
0.8
a $(\mathrm{GeV})^{-1}$

MILC



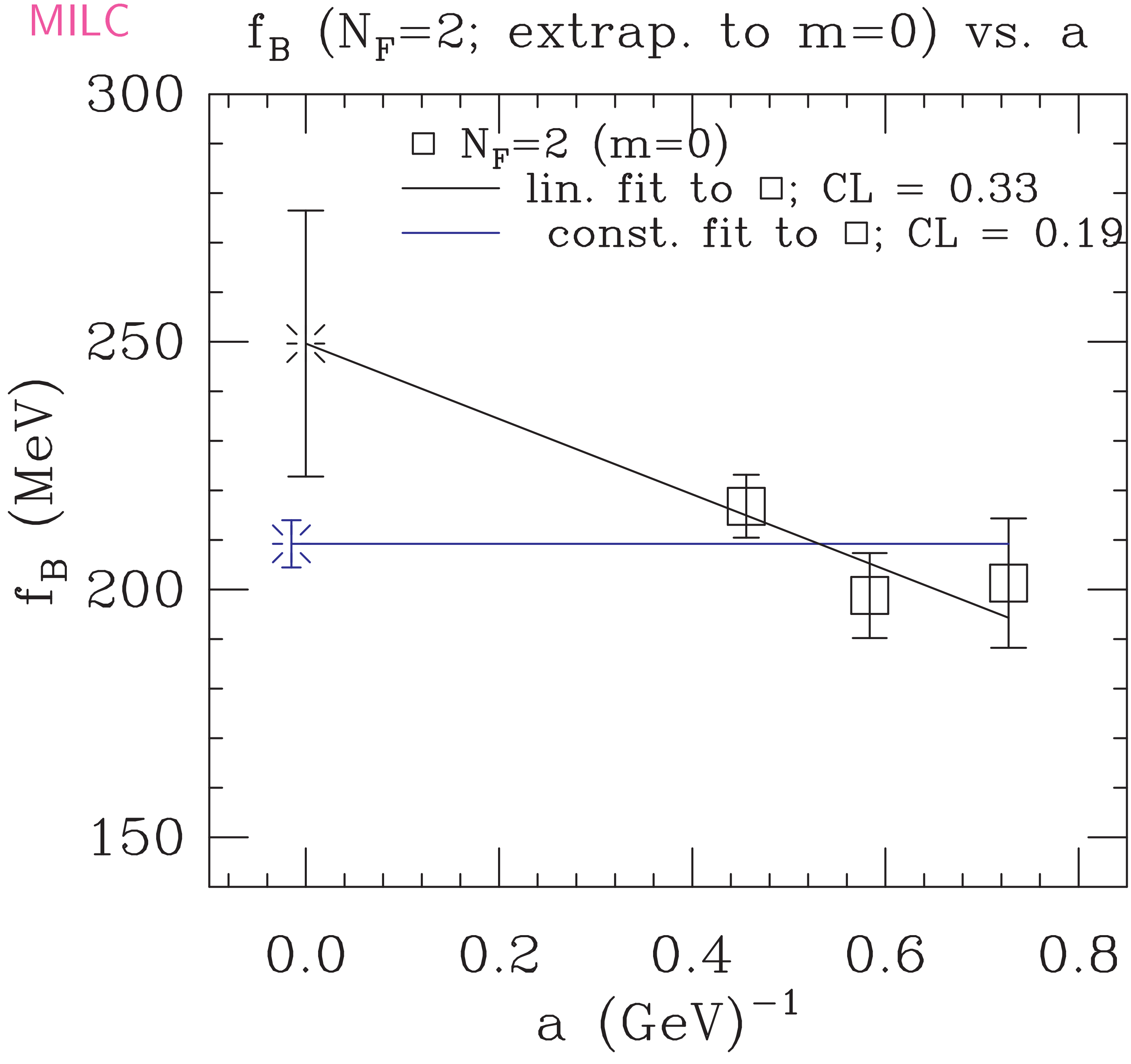
true unquenching: $m_{\text{valence}} = m_{\text{dynamical}}$; they are extrapolated to chiral limit together

MILC



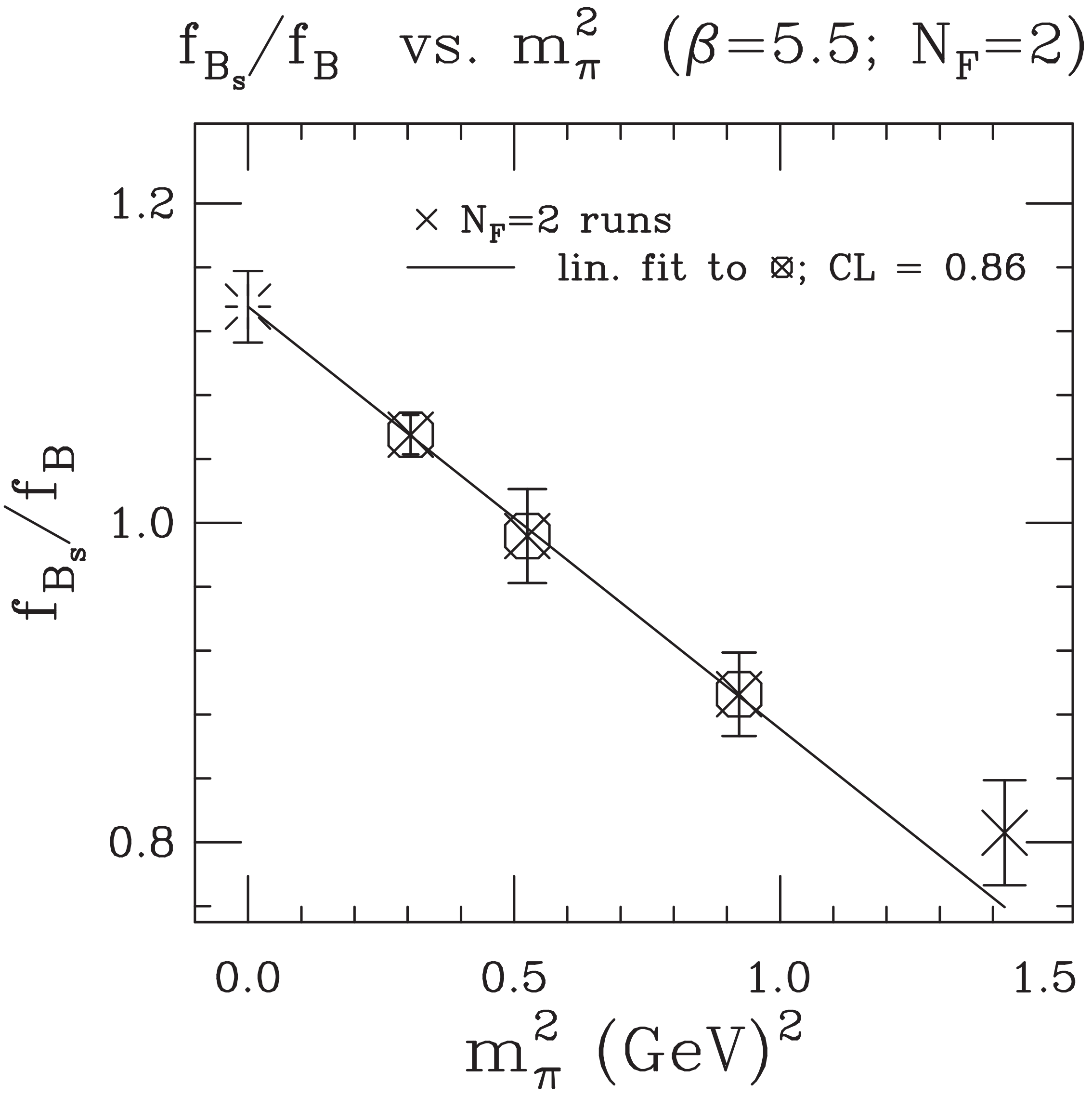
true unquenching: $m_{\rm valence} = m_{\rm dynamical}$; they are extrapolated to chiral limit together

MILC



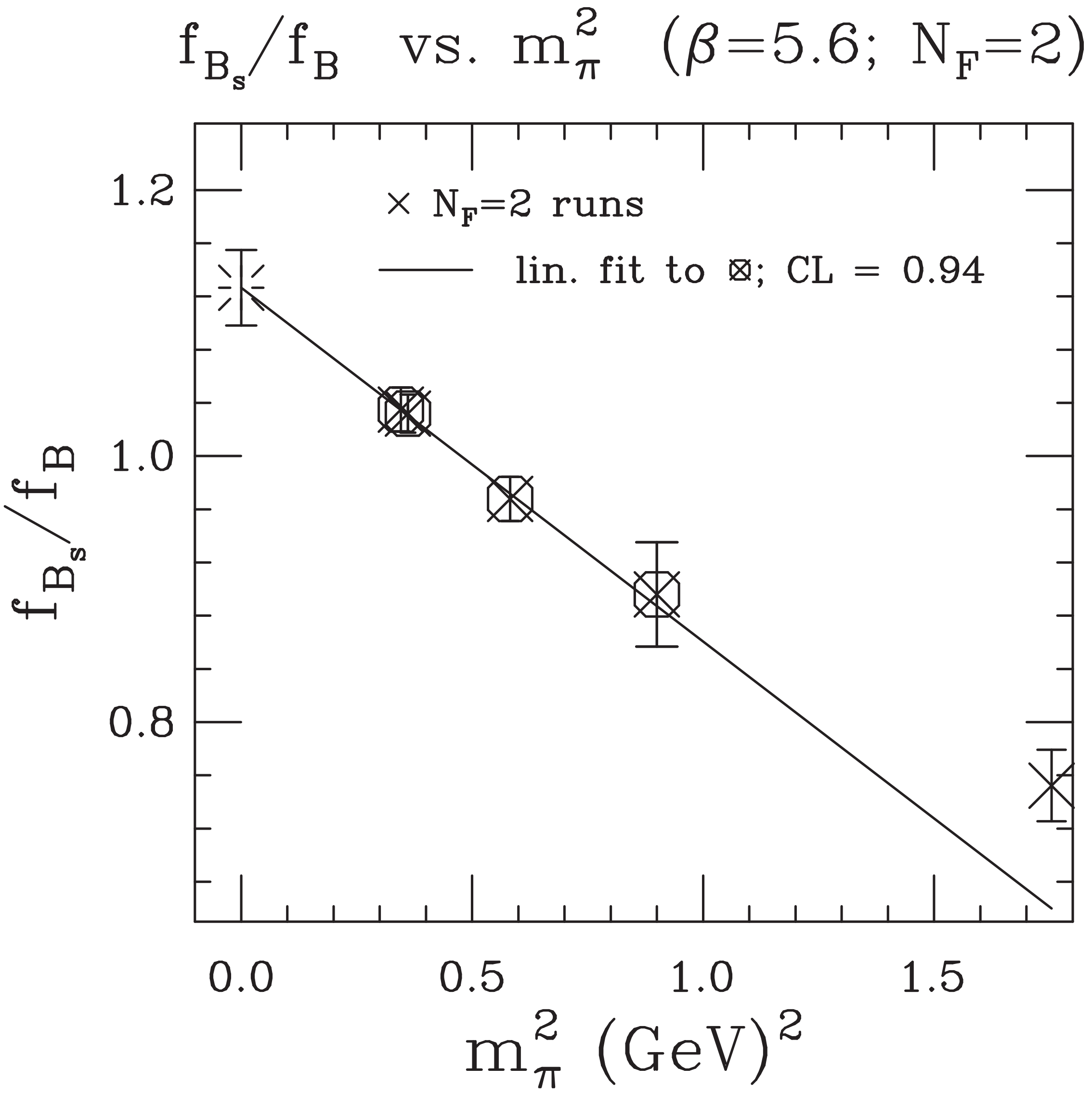
true unquenching: $m_{\text{valence}} = m_{\text{dynamical}}$

MILC



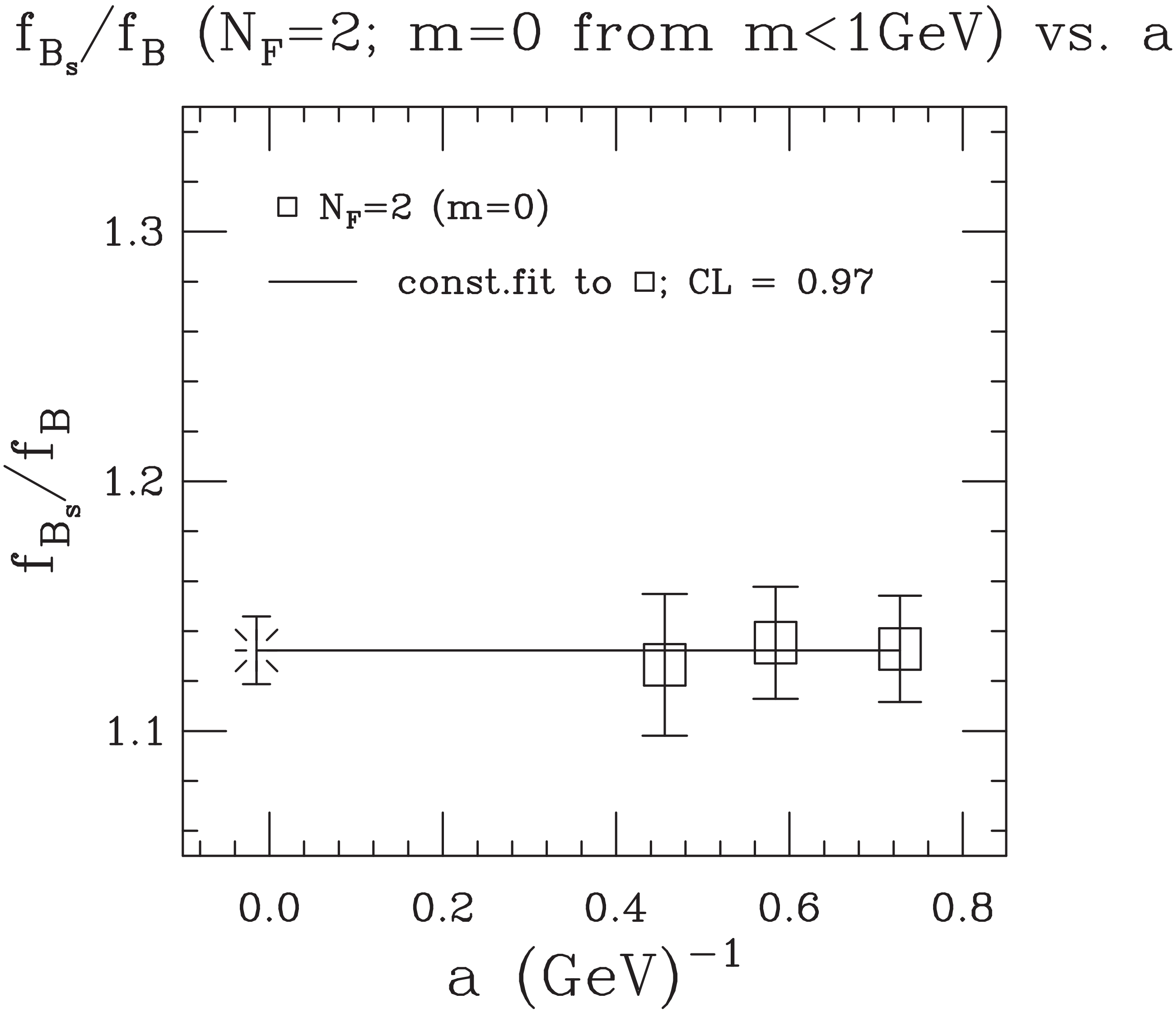
true unquenching: $m_{\text{valence}} = m_{\text{dynamical}}$; they are extrapolated to chiral limit together

MILC



true unquenching: $m_{\rm valence} = m_{\rm dynamical}$; they are extrapolated to chiral limit together

MILC



true unquenching: $m_{\rm valence} = m_{\rm dynamical}$

MILC

Preliminary results with virtual quark loops (2 light flavors):

$$
\begin{aligned}
f_B &= 210 \ \pm 3 \ \pm 26 \ \pm 28 \quad \text{MeV} \\
f_{B_s} &= 246 \ \pm 3 \ \pm 25 \pm 38 \quad \text{MeV} \\
f_{B_s}/f_B &= 1.17 \ \pm 0.02 \ {}^{+0.03}_{-0.04} \pm 0.03 \\
f_{D_s} &= 263 \ \pm 2 \ \pm 17 \pm 28 \quad \text{MeV}
\end{aligned}
$$

where the errors are statistical, systematic (within $N_F = 2$), and systematic (of missing virtual strange quark), respectively.

The last error is an (educated) guess at this point

Experiment:

J. Richman, ICHEP 1996:

$$f_{D_s} = 241 \ \pm 21 \ \pm 30 \quad \text{MeV}$$

Parodi, Roudeau, Stocchi, hep-ph/9802289:

$$f_{D_s} = 254 \ \pm 31 \quad \text{MeV}$$

The effect of changing from the quenched approx. to virtual quark loops ($N_F = 2$) is considerably bigger than we estimated previously.

Difference: the $N_F = 2$ do not have the same lattice spacing dependence as the quenched results (luckily, the lattice spacing dependence in the $N_F = 2$ case appears milder). Therefore the previous comparison between the two, which was done at fixed lattice spacing, was misleading.

NRQCD collab: (S. Collins et al.), hep-lat/9901001:

TABLES

TABLE XVIII. A comparison of the decay constants of the B and B_s mesons from this work and those on quenched configurations at $\beta^{n_f=0} = 6.0$ [3]. The results have been converted into physical units (MeV) using a^{-1} from m_ρ. The errors are purely statistical, with the exception of f_{B_s}/f_B where the second error comes from the uncertainty in κ_s.

		$\beta^{n_f=0} = 6.0$	$\beta^{n_f=2} = 5.6$
f_B	$q^* = 1.0$	145(10)	180(5)
	$q^* = \pi$	154(10)	192(5)
	tree-level	176(11)	217(6)
f_{B_s}	$q^* = 1.0$	175(6)(+7)	208(2)(-4)
	$q^* = \pi$	186(6)(+7)	222(2)(-4)
	tree-level	205(6)(+7)	247(2)(-4)
f_{B_s}/f_B		1.20(4)(+4)	1.14(2)(-2)

$$\text{quenched}: \quad f_B = 147(11)\left({}^{+8}_{-12}\right)(9)(6)(+40?)$$

$$f_{B_s}/f_B = 1.20(4)\left({}^{+1}_{-0}\right)$$

$$N_F = 2 \text{ (part. quench.)}: \quad f_B = 186(5)(19)(9)(13)(+50)$$

$$f_{B_s}/f_B = 1.14(2)\left({}^{+0}_{-2}\right)$$

Question: How significant statistically is the big difference seen? Some of the systematic errors are different in the two cases, so comparing the results with just statistical errors (as in the table) is somewhat misleading.

CP-PACS collab. at Lattice '98 (S. Aoki, *et al.*, hep-lat/9809118):

- range of dynamical mass is comparable (but slightly smaller) than that of MILC.

(recall: MILC's range is $m_s/2 \lesssim m_{\rm dynamical} \lesssim 3m_s$, where m_s is the strange quark mass.)

- find little dependence (~ 2–3%) in HL decay constants as they vary dynamical mass

- this has been taken (Draper, Lattice '98) as evidence of small effect of quenching

- but it is completely consistent with what MILC sees: little dependence of decay constants on $m_{\rm dynamical}$ for fixed bare coupling.

As $m_{\rm dynamical}$ increases the dynamical results do approach the quenched results because the lattice spacing is increasing, and the quenched results rise with increasing lattice spacing to meet the dynamical results.

B (bag) parameters for heavy-light mesons

- very mild lattice spacing dependence
- expect relatively small quenching effects

From Draper, Lattice '98, and Sharpe, ICHEP '98, (and a little diddling of my own) we have:

$$\hat{B}_B = 1.4(1)(1)$$
$$\frac{\hat{B}_{B_s}}{\hat{B}_B} = 1.00(1)(2)(2) \quad ,$$

where the error in red is an estimate of the effects of quenching.

Semileptonic decays

- $B \to D\ell\nu$ form factors

FNAL (S. Hashimoto, *et al.*, hep-lat/9810056) have a pretty method involving calculation of

$$\frac{\langle D|V_0|B\rangle\langle B|V_0|D\rangle}{\langle B|V_0|B\rangle\langle D|V_0|D\rangle}$$

at zero recoil. The denominator is fixed by current conservation. The ratio has smaller statistical and systematic errors, and better behaved perturbative corrections (lattice to continuum matching) than $\langle B|V_0|D\rangle$ alone.

They compute $\mathcal{F}_{B\to D}(1)$, where differential decay rate is proportional to $\mathcal{F}_{B\to D}(\omega)$, with $\omega = v \cdot v'$.

They find

$$\mathcal{F}_{B\to D}(1) = 1.069(8)(2)(25)$$

This includes the corrections to HQET.

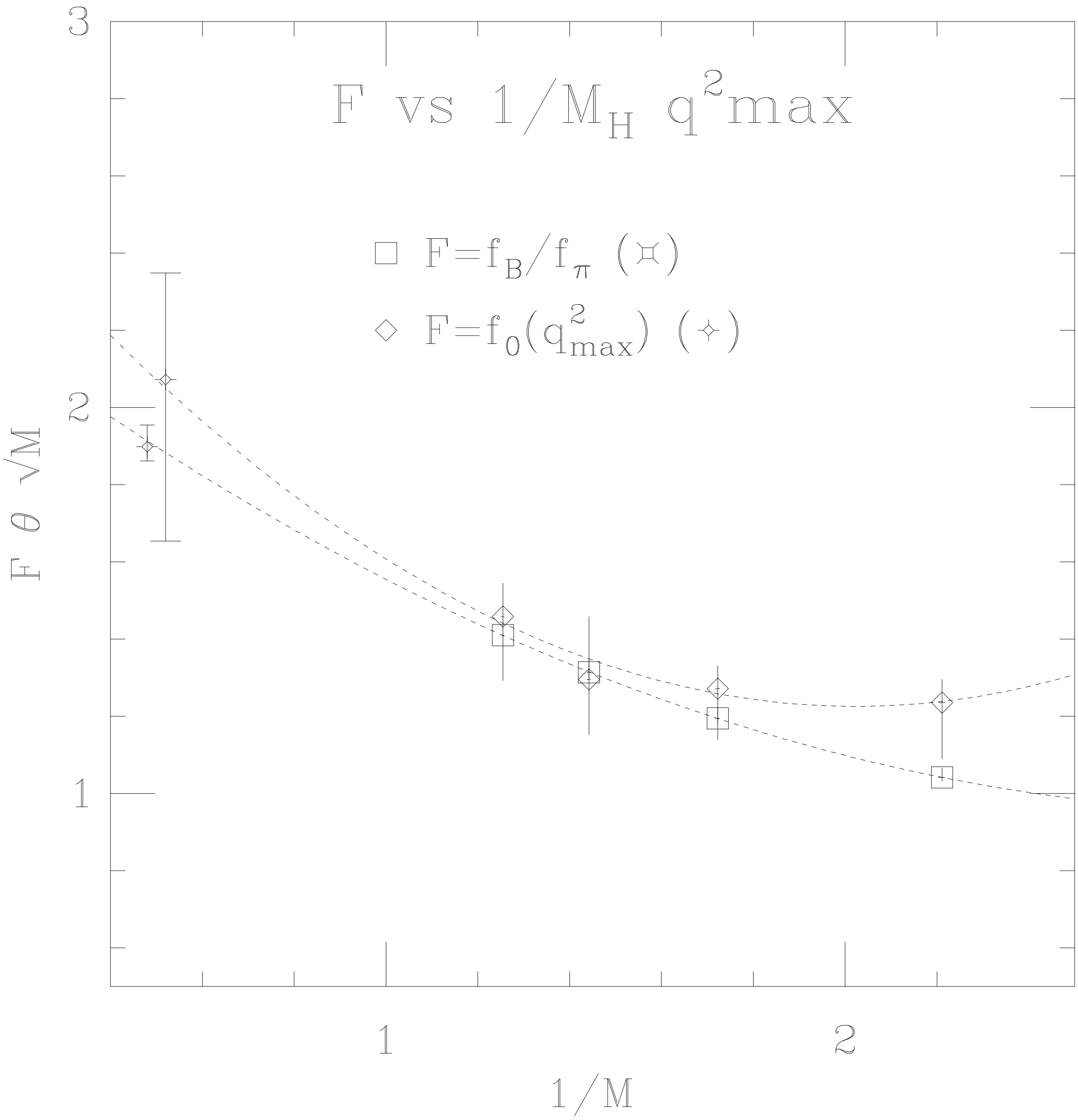
"Uncertainties from lattice artifacts and from quenching have not yet been taken into account."

- $B \to \pi\ell\nu$ form factors

Both UKQCD and FNAL have some nice new results (see following graphs)

C. Maynard (UKQCD) hep-lat/9809064

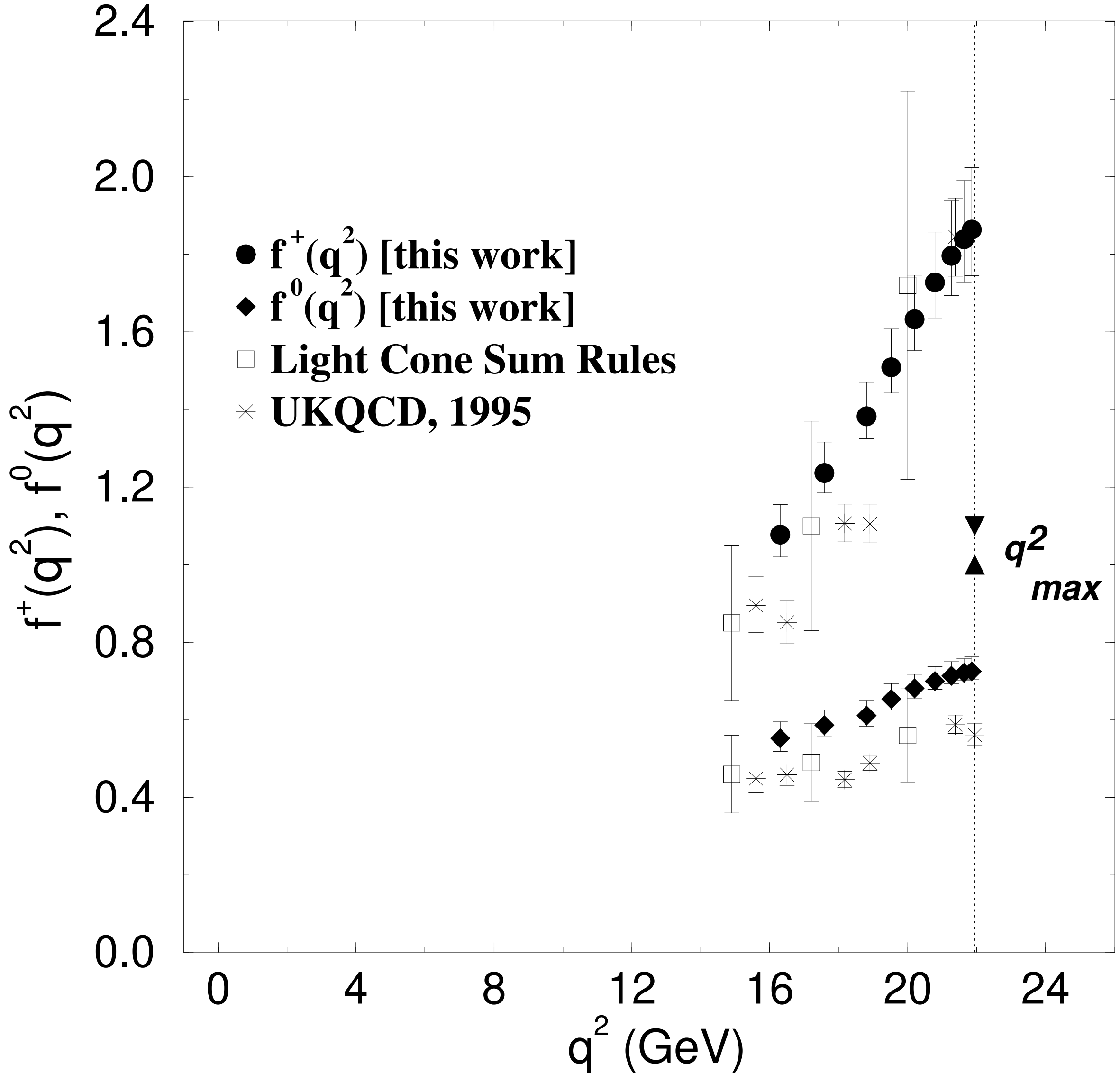
difference between f_B/f_π and $f_0(q^2_{max})$ seen in Tominaga *et al.*, NPB 63A (1998), 380 and Matsufura, *et al.*, *ibid*, 368 is absent in this newer calculation with smaller lattice spacing, renormalization constants included, and better(?) action.



$B \to \pi \ell \nu$

FNAL (S. Ryan et al.) hep-lat/9810041

"... our results agree with previous calculations using different approaches..." (?)



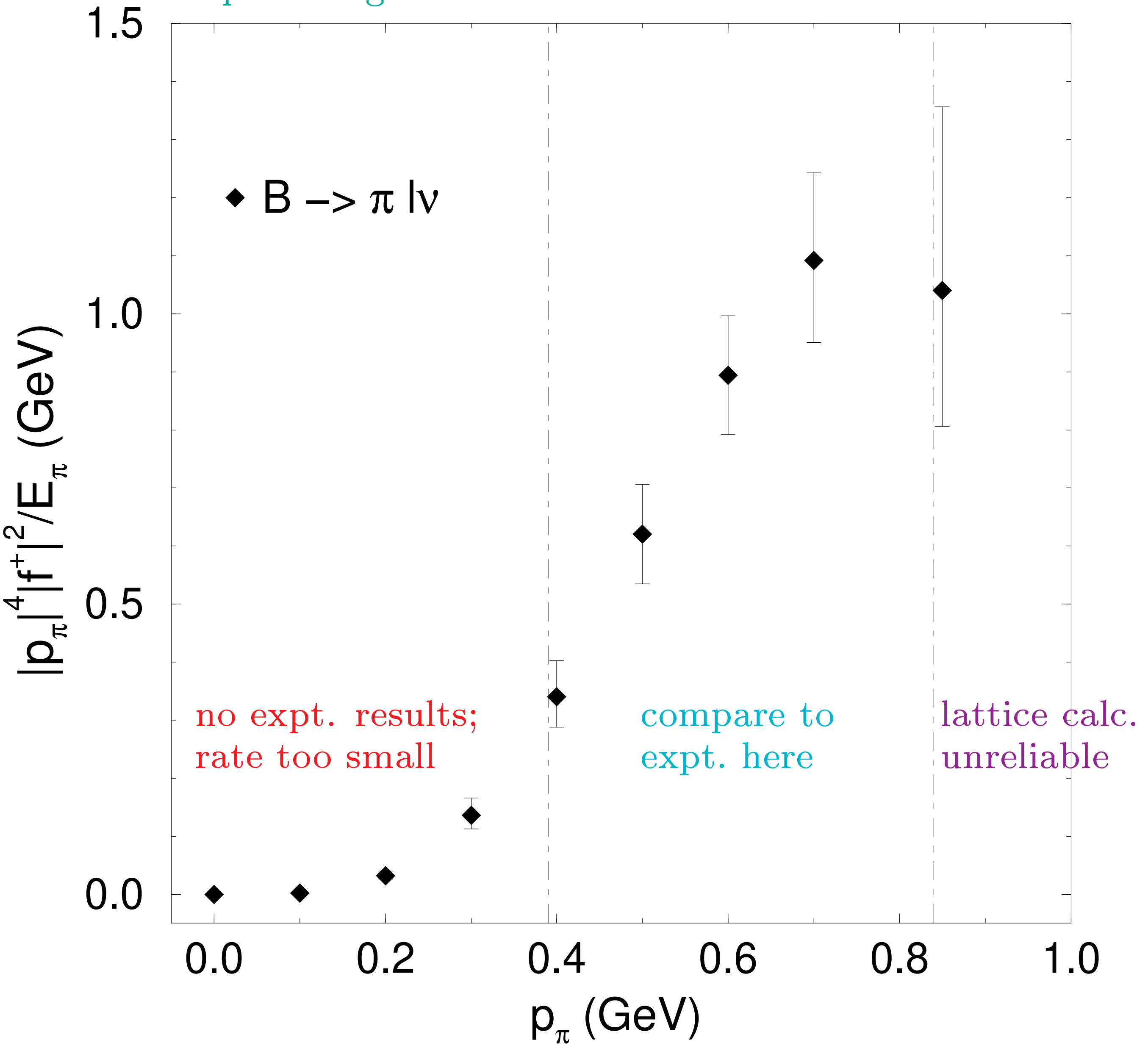
since extrapolation to $q^2 = 0$ is dangerous, they advocate comparing differential decay rate for $q^2 > 16$ (GeV)^2

FNAL (S. Ryan et al.) hep-lat/9810041

expected theoretical errors in $|V_{ub}|$:

statistics 8%
pert. th. 5%
chiral extrap. 7%
quenching ???

lattice spacing 5%
excited states 2%
m_Q tuning 1%



Conclusions:

- f_B, f_{B_s}, f_{D_s}, . . . are under good control in the quenched approximation
- First calculation of decay constants including dynamical quark effects ($N_F = 2$) is nearing completion.
- B parameters also doing well, but inclusion of quark loops still some way off. (Loop effects are expected to be smaller here.)
- Semileptonic form factors: encouraging progress, but still not controlled like decay constants, even in quenched approx.

MILC goals for the future:

- Reduce discretization errors: $N_F = 2$ & quenched ($\sim$ 2 years)
- Include dynamical strange quark (*i.e.*, strange sea) ($\sim$ 2 years)
- Do semileptonic case like decay constants ($\sim$ 2 years)

We hope we can accomplish these in parallel!

Other groups have similar goals. Expect new dynamical quark results on heavy-lights from CP-PACS (Japan) and UKQCD very soon.