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Status of the MILC light pseudoscalar meson project

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We discuss the current status of our calculation of the physics of π and K mesons using three dynamical flavors of improved staggered quarks. This year, we have a new ensemble with a lattice spacing of 0.06 fm and a light sea mass of $0.2m_s$, as well as significant increases in statistics at several coarser lattice spacings and/or heavier sea masses. Results for decay constants, quark masses, low energy constants, condensates, and V_{us} are presented.

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a (fm)	$a\hat{m}'$ / am'_s	L (fm)	m_π / m_ρ	$m_\pi L$	$10/g^2$	Lat Dim	# Lats
≈ 0.15	0.0290 / 0.0484	2.4	0.522	6.7	6.600	$16^3 \times 48$	600
≈ 0.15	0.0194 / 0.0484	2.4	0.454	5.5	6.586	$16^3 \times 48$	600
≈ 0.15	0.0097 / 0.0484	2.4	0.348	3.9	6.572	$16^3 \times 48$	600
≈ 0.15	0.00484 / 0.0484	3.0	0.256	3.4	6.566	$20^3 \times 48$	600
≈ 0.12	0.03 / 0.05	2.4	0.582	7.6	6.81	$20^3 \times 64$	362
≈ 0.12	0.02 / 0.05	2.4	0.509	6.2	6.79	$20^3 \times 64$	485
≈ 0.12	0.01 / 0.05	2.4	0.394	4.5	6.76	$20^3 \times 64$	894
≈ 0.12	0.01 / 0.05	3.4	0.395	6.3	6.76	$28^3 \times 64$	275
≈ 0.12	0.007 / 0.05	2.4	0.342	3.8	6.76	$20^3 \times 64$	836
≈ 0.12	0.005 / 0.05	2.9	0.299	3.8	6.76	$24^3 \times 64$	527
≈ 0.12	0.03 / 0.03	2.4	0.590	7.6	6.81	$20^3 \times 64$	360
≈ 0.12	0.01 / 0.03	2.4	0.398	4.5	6.76	$20^3 \times 64$	349
≈ 0.09	0.0124 / 0.031	2.4	0.495	5.8	7.11	$28^3 \times 96$	531
≈ 0.09	0.0062 / 0.031	2.4	0.380	4.1	7.09	$28^3 \times 96$	583
≈ 0.09	0.0031 / 0.031	3.4	0.297	4.2	7.08	$40^3 \times 96$	503
≈ 0.06	0.0072 / 0.018	2.9	0.474	6.3	7.48	$48^3 \times 144$	556
≈ 0.06	0.0036 / 0.018	2.9	0.370	4.5	7.47	$48^3 \times 144$	334

Table 1: Lattice parameters. The lattice spacings are the "nominal" scales (see text). The π and ρ referred to are those formed out of the sea quarks for each lattice; valence quark masses however go down to the lightest sea-quark values in the table.

We are using improved staggered quarks [1] with $N_f = 3$ dynamical flavors (both unquenched ("full") QCD and partially quenched) to study the physics of light pseudoscalars (π, K). Since our original published work [2], we have continued to add data sets with lighter sea quark masses and/or finer lattice spacings and to improve the analysis. This is the latest in a series of periodic updates [3, 4]. We concentrate here on those aspects that have changed since last year.

Table 1 gives the parameters of our lattices. The quantities m'_s and $\hat{m}' = m'_u = m'_d$ denote the values of sea quark masses chosen in each run. (The corresponding masses without the primes, *e.g.*, m_s and $\hat{m} \equiv (m_u + m_d)/2$, are the physical values.)

The $a \approx 0.06$ fm lattice with masses 0.0036 / 0.018 is a new ensemble this year, as is (for this analysis) the large-volume $a \approx 0.12$ fm lattice with masses 0.01 / 0.05 and spatial size 28^3. The numbers of configurations for the $a \approx 0.06$ fm lattice with masses 0.0072 / 0.018 and for several of the $a \approx 0.12$ fm lattices have almost doubled since last year. Running on an $a \approx 0.06$ fm lattice with $\hat{m}' = 0.1 m'_s$ (masses 0.0018 / 0.018) has recently begun but is not included here.

On each ensemble, we determine r_1/a, where $r_1(\hat{m}', m'_s, g^2)$ [5] is a length scale from the static quark potential, similar to r_0 [6]. The quantity $r_1^{\rm phys}$, defined as the continuum r_1 at physical quark masses ($\hat{m}$, m_s) may be determined from the r_1/a values and the Υ 2S-1S splitting [7]. We obtain $r_1^{\rm phys} = 0.318(7)$ fm [4, 8].

For generic chiral and continuum extrapolations, it is convenient to define the lattice scale by $a \equiv r_1^{\rm phys}/(r_1(\hat{m}', m'_s, g^2)/a)$. We call this the "nominal" scale-setting procedure. In choosing the input lattice coupling g^2, we kept r_1/a fixed as $\hat{m}'$ and m'_s changed over a given set of ensembles

(*e.g.*, the $a \approx 0.12$ fm ensembles). Thus, up to tuning errors, each ensemble grouped within a box in Table 1 has the same nominal scale. However, fixing the scale this way is not completely correct for applying chiral perturbation theory (χPT) to quantities such as f_π since r_1 has some (small, but physical) dependence on the dynamical quark masses that is not included in χPT.

A mass-independent procedure to set the scale is preferable [9]. A convenient procedure is to replace $r_1(\hat{m}', m_s', g^2)/a$ by $r_1(\hat{m}, m_s, g^2)/a$, where the value of $r_1(\hat{m}, m_s, g^2)/a$ at physical masses $\hat{m}, m_s$ is obtained by a smooth interpolation/extrapolation from $r_1(\hat{m}', m_s', g^2)/a$. We tried this mass-independent scheme in Ref. [2], but the differences with the nominal approach were smaller than other systematic errors for all quantities. With better data, we now find significant differences in a few low energy constants (LECs). In addition, the mass-independent scheme tends to have better confidence levels in our χPT fits. Therefore we use this scheme exclusively here.

As in Refs. [2, 3, 4] we fit the partially quenched (PQ) lattice data to rooted staggered chiral perturbation theory (rSχPT) forms [10, 11, 12]. We always fit multiple lattice spacings, and both masses and decay constants, simultaneously. To determine the LO and NLO LECs and chiral-limit quantities, we fit to the low quark-mass region, and omit the $a \approx 0.15$ fm lattices, where taste violations are large. Denoting the valence quark masses in the mesons by m_x and m_y, the low-mass cuts are: $am_x + am_y \lesssim 0.39\ am_s$ (at $a \approx 0.12$ fm); $am_x + am_y \lesssim 0.51\ am_s$ (at $a \approx 0.09$ fm); and $am_x + am_y \lesssim 0.56\ am_s$ (at $a \approx 0.06$ fm). We can tolerate a higher cutoff at smaller lattice spacing because the taste violations, and hence the masses of non-Goldstone pions, are smaller. In these fits, we also cut on sea-quark mass and remove the $a \approx 0.12$ fm sets with masses $0.03/0.05$, $0.02/0.05$, and $0.03/0.03$. Because the statistical errors are so small, we still need to add in the NNLO analytic terms to the complete NLO forms in order to get good fits [2].

For interpolation around m_s, we must include higher quark masses. Once LO and NLO parameters are determined, we fix them (up to statistical errors) and fit to all sea mass sets, all lattice spacings, and valence masses $m_x + m_y \lesssim 1.2\ m_s$. We now also need to add in NNNLO analytic terms to get good fits. These NNNLO fits are used for central values of f_π, f_K and quark masses.

Figure 1 shows results for the squared pseudoscalar masses as a function of quark mass. "Pions" have valence masses $m_y = m_x$; while "kaons" have m_y held fixed at various (arbitrary) values while m_x varies. The fit is to the full quark-mass range and uses NNNLO terms.

For the pions, the relative values of the results on various lattices is determined largely by the relation between the simulation strange mass m_s' in the sea and the physical mass m_s. For example, m_s'/m_s is largest for the $a \approx 0.12$ fm lattices, which makes the slope of the pion data greatest for these lattices. For kaons, the biggest effect is simply the choice of the values of the fixed valence mass m_y, typically chosen to be various fixed fractions of m_s'.

Extrapolating to the continuum and setting valence and sea quark masses equal, we get the dashed red lines; m_s' has been adjusted so that both the kaon and the pion hit their physical values at the same value of m_x. This gives the physical quark masses $\hat{m}$ and m_s (after renormalization).

Note that the fit lines in Fig. 1 are remarkably straight on this scale. To see curvature coming from the NLO chiral logs as well as the analytic higher order terms, we plot $m_\pi^2/(m_x + m_y)$ in Fig. 2 (left). As the lattice spacing decreases, the PQ log at small mass becomes more evident. At larger lattice spacing, the PQ log is largely washed out by staggered taste violations. The continuum dashed red line has sea and valence masses equal, so no PQ log is expected.

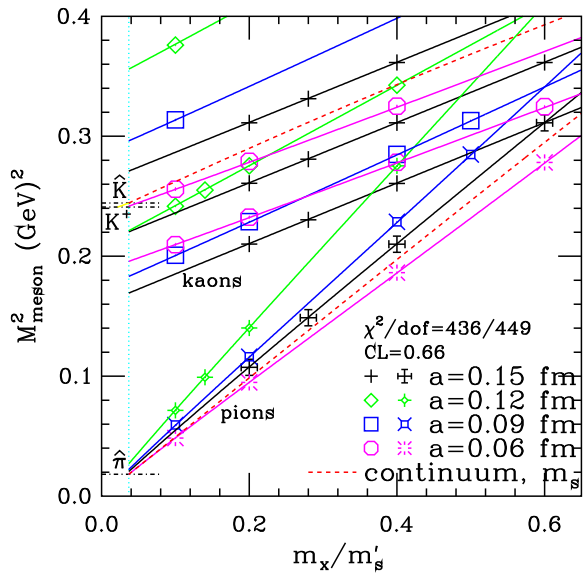

Figure 1: Comparison of NNNLO fit to partially-quenched squared meson masses. For clarity only the lightest sea-quark ensemble for each lattice spacing is shown.

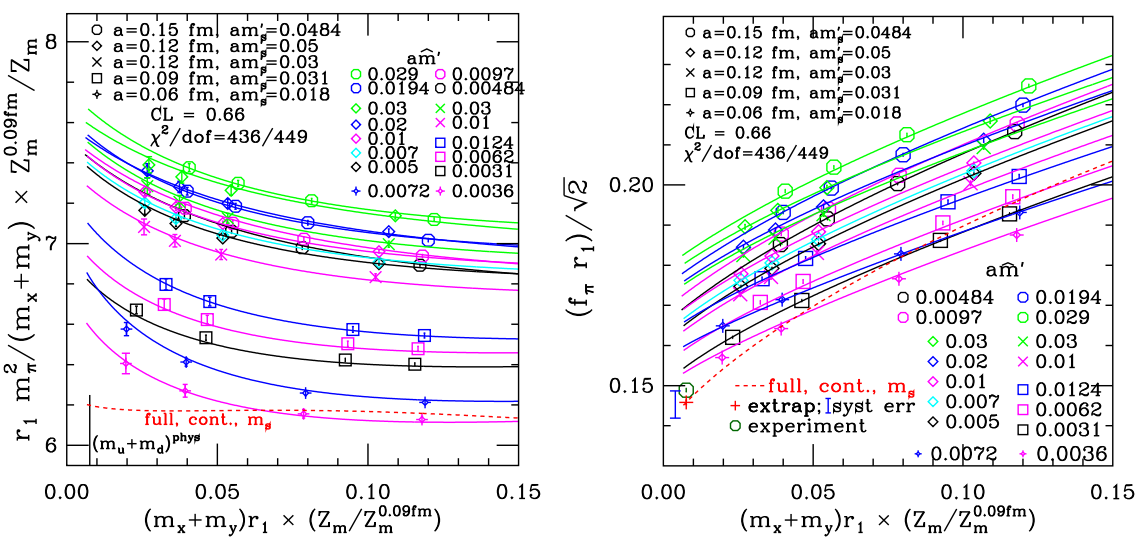

Figure 2: Data for $m_\pi^2/(m_x+m_y)$ is plotted at left; while that for f_π is plotted at right. This is the same fit as in Fig. 1. All sea-quark ensembles are represented, but only "pion" points ($m_x = m_y$) are shown.

In Fig. 2 (right), we show the behavior of the decay constant. Extrapolating to the continuum, setting $m'_s = m_s$, and setting light valence and sea masses equal gives the dashed red line. The final result for f_π after extrapolating $m_x, m_y \to \hat{m}$ is marked by a $+$. The experimental result is indicated by the $\circ$; it comes from the $\pi^+ \to \mu^+ + \nu_\mu$ decay width and $V_{ud} = 0.97377(27)$ [13].

All points and fit lines above have been corrected for finite volume effects using the rSχPT forms at one loop. However, it is known [14] that finite volume effects coming from higher orders in χPT can be a large ($\sim 50\%$) correction to the one-loop effects in the current ranges of quark mass and volume. We therefore study this issue directly by comparing results on the spatial size 20^3 and 28^3 $a \approx 0.12$ fm ensembles with $a\hat{m}' = 0.01$, $am'_s = 0.05$. These lattices have spatial length

quantity	% difference	boosted % diff.	1-loop % diff.
af_π	1.4(2)%	1.6(2)%	1.1%
af_K	0.4(3) %	0.4(3) %	0.3%
$(am_\pi)^2$	-1.0(4)%	-1.2(4)%	-0.9%
$(am_K)^2$	-0.4(2) %	-0.4(2) %	-0.2%

Table 2: Finite volume effects. The second column is the % difference between the values on the 28^3 and the 20^3 lattice. In the third column, we "boost" the difference, to take into account the (small) further difference between 28^3 and infinite volume. (We use one-loop results to make the adjustment.) The last column shows the % difference between 20^3 and infinite volume as predicted by one-loop rSχPT. "Pion" quantities are from lattice valence masses 0.005, 0.005; "kaon" quantities are from lattice valence masses 0.005, 0.04.

2.4 fm and 3.4 fm, respectively. The comparison is shown in Table 2.

As expected from Ref. [14], the true finite volume effects are larger than those predicted at one-loop, although only for f_π are the relative errors small enough to make the comparison unambiguous. We define the "residual finite volume effect" on the 20^3 lattice as that effect not taken into account at one-loop, *i.e.*, the difference between columns three and four in Table 2. In practice, the 20^3, $a\hat{m}' = 0.01$, $am'_s = 0.05$, $a \approx 0.12$ fm lattice is close to the worst case in our data set, since the volumes both at the lightest sea quark masses and at the finest lattice spacings are larger.

Judging by the one-loop results, we expect the overall (and hence residual) finite volume effects closest to the chiral and continuum limits in our data set to be about half those seen in the table. We therefore correct our data by $1/2$ the residual finite volume effects from Table 2, and take the full size of the correction as a systematic error. We note that the size of the error determined here is very similar to that estimated by us previously [2] using Ref. [14].

The fact that we can get good fits to the forms predicted by rSχPT (and not to those of continuum χPT [2]) is an overall test of staggered chiral perturbation theory, including the "replica trick" to represent rooting. As a more focused test of the replica trick in rSχPT, we allow n_r, the number of replicas per staggered flavor, to be a free fit parameter. If rSχPT is correct, we should find $n_r = 1/4$. On the low-mass data set described above, we obtain $n_r = 0.28(2)(3)$, where the errors are statistical and systematic (describing the variation over details of the chiral fits). While the ability of rSχPT to describe rooted staggered data cannot prove the correctness of the rooting trick itself, it does indicate that no problems occur in the chiral sector of the rooted theory [12]. This is because rSχPT reproduces continuum χPT in the limit $a \to 0$.

Using $r_1 = 0.318(7)$ fm from Υ splittings, we obtain (still preliminary)

$$\begin{aligned} f_\pi &= 128.3 \pm 0.5\,{}^{+2.4}_{-3.5}\ \text{MeV} \\ f_K &= 154.3 \pm 0.4\,{}^{+2.1}_{-3.4}\ \text{MeV} \\ f_K/f_\pi &= 1.202(3)({}^{+\ 8}_{-14})\ , \end{aligned}$$

where the errors are from statistics and lattice systematics. These results are consistent with our previous results [2], with 20–30% smaller errrors. Our value for f_π is consistent with the experimental result, $f_\pi^{\rm expt} = 130.7 \pm 0.1 \pm 0.36$ MeV [13].

Instead of setting the scale from Υ splittings, we can set the scale from f_π itself, which gives smaller errors for π–K quantities. Note that even dimensionless quantities can change with the new scale, due to changes in physical quark masses. We then obtain (preliminary):

$$
\begin{aligned}
f_K &= 156.5 \pm 0.4\,{}^{+1.0}_{-2.7}\ \text{MeV} & f_K/f_\pi &= 1.197(3)({}^{+\ 6}_{-13}) \\
f_\pi/f_2 &= 1.052(2)({}^{+6}_{-3}) & \langle\bar{u}u\rangle_2 &= -(\,278(1)({}^{+2}_{-3})(5)\ \text{MeV}\,)^3 \\
f_\pi/f_3 &= 1.21(5)({}^{+13}_{-\ 3}) & \langle\bar{u}u\rangle_3 &= -(\,242(9)({}^{+\ 5}_{-17})(4)\ \text{MeV}\,)^3 \\
f_2/f_3 &= 1.15(5)({}^{+13}_{-\ 3}) & \langle\bar{u}u\rangle_2/\langle\bar{u}u\rangle_3 &= 1.52(17)({}^{+38}_{-15}) \\
2L_6 - L_4 &= 0.4(1)({}^{+2}_{-3}) & 2L_8 - L_5 &= -0.1(1)(1) \\
L_4 &= 0.4(3)({}^{+3}_{-1}) & L_5 &= 2.2(2)({}^{+2}_{-1}) \\
L_6 &= 0.4(2)({}^{+2}_{-1}) & L_8 &= 1.0(1)(1) \\
m_s &= 88(0)(3)(4)(0)\ \text{MeV} & \hat{m} &= 3.2(0)(1)(2)(0)\ \text{MeV} \\
m_u &= 1.9(0)(1)(1)(1)\ \text{MeV} & m_d &= 4.6(0)(2)(2)(1)\ \text{MeV} \\
m_s/\hat{m} &= 27.2(1)(3)(0)(0) & m_u/m_d &= 0.42(0)(1)(0)(4)\ .
\end{aligned}
$$

The errors are statistical, lattice-systematic, perturbative (for masses and condensates; from two-loop perturbation theory [15]) and electromagnetic (for masses; from continuum estimates). f_2 (f_3) represents the three-flavor decay constant in the two (three) flavor chiral limit, and $\langle\bar{u}u\rangle_2$ ($\langle\bar{u}u\rangle_3$) is the corresponding condensate. The low energy constants L_i are in units of 10^{-3} and are evaluated at chiral scale m_η; the condensates and masses are in the $\overline{\text{MS}}$ scheme at scale 2 GeV.

We also obtain

$$r_1 = 0.3108(15)({}^{+26}_{-79})\ \text{fm}\ ,$$

which is 1-σ lower (and with somewhat smaller errors) than the value from the Υ system. There is a 2-σ conflict between our r_1 result from f_π and the HPQCD Collaboration [7] value from Υ splittings, $r_1 = 0.321(5)$ fm. If instead we compare to our own evaluation of r_1 from the Υ spectrum, $r_1 = 0.318(7)$ [4], the difference is only 1-σ. We emphasize, however, that the evaluations of r_1 from the Υ splittings both by us and by the HPQCD Collaboration use the same lattice data: HPQCD Υ splittings [7] and MILC values of r_1/a [8]. The difference is only in how we extrapolate to the physical point and estimate the systematic error. Our result is consistent, though, with the ($N_f = 2$) result from the ETM Collaboration [16], $r_0 = 0.454(7)$ fm. Converting from r_0 to r_1 using the ratio $r_0/r_1 = 1.46(1)(2)$ (from Ref. [8], adjusted for the slight difference between $N_f = 3$ and $N_f = 2$), this gives $r_1 = 0.311(7)$ fm.

Together with the experimental result for the kaon leptonic branching fraction [17], our result for f_K/f_π implies $|V_{us}| = 0.2246({}^{+25}_{-13})$, which is consistent with (and competitive with) the world-average value $|V_{us}| = 0.2257(21)$ [13] coming from semileptonic K-decay coupled with non-lattice theory.

The change in the perturbative mass renormalization constant Z_m from one to two loops accounts for almost all of the difference between the mass values quoted here and those in Ref. [18, 2]. A non-perturbative evaluation of Z_m is in progress.

We stress that our extraction of the L_i uses fits that include (analytic) NNLO terms. Therefore, a comparison to other evaluations, either phenomenological or on the lattice, that stop at NLO terms is problematic. Indeed, NNLO terms of "natural size" in χPT can produce changes in the L_i (relative to a pure NLO evaluation) that are as large as, or even somewhat larger than, our current systematic errors. This is confirmed by NLO fits to our data. Such fits have very poor confidence levels, however, which is why we do not include them in the analysis.

The $SU(2)_L \times SU(2)_R$ LECs $\bar{l}_3$, $\bar{l}_4$ that are extracted [19] from our $SU(3)_L \times SU(3)_R$ results using one-loop (NLO) formulae are therefore quite sensitive to the NNLO terms, particularly for $\bar{l}_3$. An NLO fit, on the other hand, gives $\bar{l}_3 = 2.85(7)$ (statistical errors only), which is comparable to the results from groups [16, 20] performing two-flavor simulations with NLO $SU(2)_L \times SU(2)_R$ fits. Indeed, this must be true, because the m_π^2 data are so linear (see Fig. 1), which requires $\bar{l}_3$ to have roughly this value [19]. Alternative fits using $SU(2)_L \times SU(2)_R$ rSχPT are in progress. Since the strange sea-quark is omitted from the chiral theory, the approach should make possible good NLO fits on light-mass data, and thereby bypass this issue. Inclusion of two-loop (continuum) chiral logs [21] in $SU(3)_L \times SU(3)_R$ fits is also in progress.

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