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Update on pi and K Physics

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We present an update of the MILC studies of the physics of light pseudoscalars using improved staggered fermions. New runs at lighter quark mass, as well as increased statistics for older sets, are enabling us to improve the results for decay constants in full QCD. In addition, we have analyzed quenched runs at two different lattice spacings. This makes possible a test of the applicability of staggered chiral perturbation theory in a different context.

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Since our previously published work [1] on the decay constants and masses of the π-K system, we have continued to generate and analyze lattices with $N_f = 3$ flavors of dynamical quarks, increasing the statistics and moving to lighter u,d, and (separately) lighter s masses. We use improved staggered fermions [2]. With the same action, we have also investigated these quantities in the quenched approximation. Assuming the chiral fits are good and the continuum extrapolations are accurate, such calculations make possible:

- A sensitive check of algorithms and methods — including the $\sqrt[4]{\mathrm{Det}}$ trick for dynamical staggered quarks — by comparing full QCD f_π to the well-determined experimental value.
- A precise extraction of the CKM matrix element V_{us} from f_K or f_K/f_π, competitive with the world-average from alternative methods.
- A determination of the light quark masses and their ratios with high lattice precision.
- A test of the applicability of staggered chiral perturbation theory (SχPT) [3, 4] for describing lattice data — both $N_f = 3$ and quenched.
- An extraction of the physical coefficients of the $\mathcal{O}(p^4)$ chiral Lagrangian.
- A determination of the extra, unphysical parameters that enter SχPT. This is important because these parameters also appear in staggered chiral forms for other physical quantities, *e.g.*, heavy-light decay constants and form factors [5]. Fixing the parameters from the light-light system reduces the systematic errors in heavy-light computations [6].

In our computations, we analyze two lattice spacings: $a \approx 0.12$ fm ("coarse") and $a \approx 0.086$ fm ("fine"). We call the valence quark masses are m_x and m_y; the dynamical quark masses (for $N_f = 3$) are $m'_u = m'_d \equiv \hat{m}'$, and m'_s. Here primes indicate simulation values; the corresponding masses without primes are the physical values. Table 1 shows the lattice parameters used.

The relative lattice scale is determined using the length r_1 [7] from the static quark potential. We reduce statistical fluctuations in r_1/a by fitting to a smooth function. The absolute lattice scale is obtained from the Υ 2S-1S splitting. Following the continuum extrapolation in Ref. [8], but using updated HPQCD results [9], rather than those in [10], we obtain $r_1 = 0.318(7)$ fm.

am_q / am_s	$10/g^2$	size	volume	number	a (fm)
0.03 / 0.05	6.81	$20^3 \times 64$	$(2.4\ \mathrm{fm})^3$	564	0.120
0.02 / 0.05	6.79	$20^3 \times 64$	$(2.4\ \mathrm{fm})^3$	484	0.120
0.01 / 0.05	6.76	$20^3 \times 64$	$(2.4\ \mathrm{fm})^3$	658	0.121
0.007 / 0.05	6.76	$20^3 \times 64$	$(2.4\ \mathrm{fm})^3$	493	0.121
0.005 / 0.05	6.76	$24^3 \times 64$	$(2.9\ \mathrm{fm})^3$	**400** (197)	0.120
0.03 / 0.03	**6.79**	$\mathbf{20^3 \times 64}$	$\mathbf{(2.4\ fm)^3}$	**350**	**0.120**
0.01 / 0.03	**6.75**	$\mathbf{20^3 \times 64}$	$\mathbf{(2.4\ fm)^3}$	**349**	**0.120**
quenched	**8.00**	$\mathbf{20^3 \times 64}$	$\mathbf{(2.4\ fm)^3}$	**408**	**0.119**
0.0124 / 0.031	7.11	$28^3 \times 96$	$(2.4\ \mathrm{fm})^3$	531	0.0863
0.0062 / 0.031	7.09	$28^3 \times 96$	$(2.4\ \mathrm{fm})^3$	583	0.0861
0.0031 / 0.031	**7.08**	$\mathbf{40^3 \times 96}$	$\mathbf{(3.4\ fm)^3}$	**210**	**0.0859**
quenched	**8.40**	$\mathbf{28^3 \times 96}$	$\mathbf{(2.4\ fm)^3}$	**396**	**0.0853**

Table 1: Lattice parameters. Runs and numbers of configuration in normal font were included in Ref. [1]; those in bold font are new. The lattice sets above the double line are "coarse;" those below are "fine."

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As in Ref. [1] we fit the partially quenched lattice data to SχPT forms [4]. The addition of new runs allows us (sometimes forces us) to change some details of the fits. To determine LO and NLO chiral parameters we fit only to the low quark-mass region. The cut on valence quark mass is the same as before: $am_x + am_y \leq 0.021 \approx 0.5am_s$ (coarse) and $am_x + am_y \leq 0.017 \approx 0.6am_s$ (fine). We now have enough data to cut on sea-quark masses, too: We remove the $0.03/0.05$, $0.02/0.05$, and $0.03/0.03$ sets for this fit. Because statistical errors are so small, (0.1% to 0.4% for decay constants, 0.1% to 0.7% for squared meson masses), we still require NNLO analytic terms in addition to complete NLO forms to get good fits. Such joint fits to decay constants and masses, including both coarse and fine lattices, have 26 free parameters:

- 2 LO parameters: f and μ (decay constant and condensate at tree-level).
- 8 NLO parameters: 4 physical and 2 taste-violating analytic terms, 2 taste-violating hairpins.
- 10 physical, NNLO analytic parameters.
- 6 tightly constrained parameters (prior width = 0.025): give variation of 2 LO and 4 NLO physical parameters with lattice spacing.

For interpolation around m_s, we must include higher quark masses. Once the LO and NLO parameters are determined, we fix them (up to statistical errors) and fit to all sea mass sets and wider ranges of valence masses. For central values we choose the range $am_x + am_y \leq 0.055 \approx 1.4m_s$ coarse, and $am_x + am_y \leq 0.0353 \approx 1.3m_s$ fine. This fit is called "Fit C." For systematic error tests, the range is widened to $am_x + am_y \leq 0.10 \approx 2.5m_s$ coarse, and $am_x + am_y \leq 0.062 \approx 2.2m_s$ fine. With either of these choices, we need to add in the NNNLO analytic terms (18 parameters, cubic in quark masses for f_π and $M_\pi^2/(m_x + m_y)$) to get good fits.

With our old data set, m_s' only changed with a, and m_s' was usually significantly larger than $\hat{m}'$, so the sea quark mass dependence and a dependence were difficult to disentangle. The new data, which includes coarse lattices with $am_s' = 0.03$ in addition to the previous value $am_s' = 0.05$, gives better control of the sea quark mass dependence and smaller a dependence of the LO, NLO, and NNLO parameters. Including the NNNLO terms and the a dependence of the NNLO terms gives 28 parameters more than the low-mass fits described above, for a total of 56 parameters. Twelve of these (LO, LO a dependence, and NLO parameters) are tightly constrained from the low-mass fits.

Figure 1 shows Fit C results for f_π and f_K. This fit has $\chi^2/\text{d.o.f.} = 0.99$ for 556 degrees of freedom (confidence level CL=0.59). In each plot, the maroon line is the "prediction" for the 0.0031/.031 fine run based on a second fit that leaves out that run; it should be compared with the solid black line that comes from Fit C. Since the time of the conference, we have accumulated about 25% more 0.0031/.031 configurations, and the effect of removing or including this run in the fit has decreased. Given that the CL decreases when the run is removed, we no longer see any reason to consider omitting the run. We note that the 0.0031/.031 run is still only about half finished, so there will probably be further noticeable shifts. In the f_K plot, two "experimental" points (shifted slightly to the left for clarity) are shown. Both points are based on the measured leptonic ($K \to \ell\nu$) rate, but $\ast$ assumes the PDG value $V_{us} = 0.2200(26)$ [11]; while $\times$ assumes the results of recent experiments $V_{us} = 0.2262(23)$ [12]. Both these values of V_{us} come from experimental determinations of the semileptonic ($K \to \pi\ell\nu$) rate and non-lattice theory for form factors.

Our preliminary results for decay constants are:

$$f_\pi = 128.1 \pm 0.5 \pm 2.8 \text{ MeV}, \qquad f_K = 153.5 \pm 0.5 \pm 2.9 \text{ MeV}, \qquad f_K/f_\pi = 1.198(3)(^{+16}_{-5}),$$

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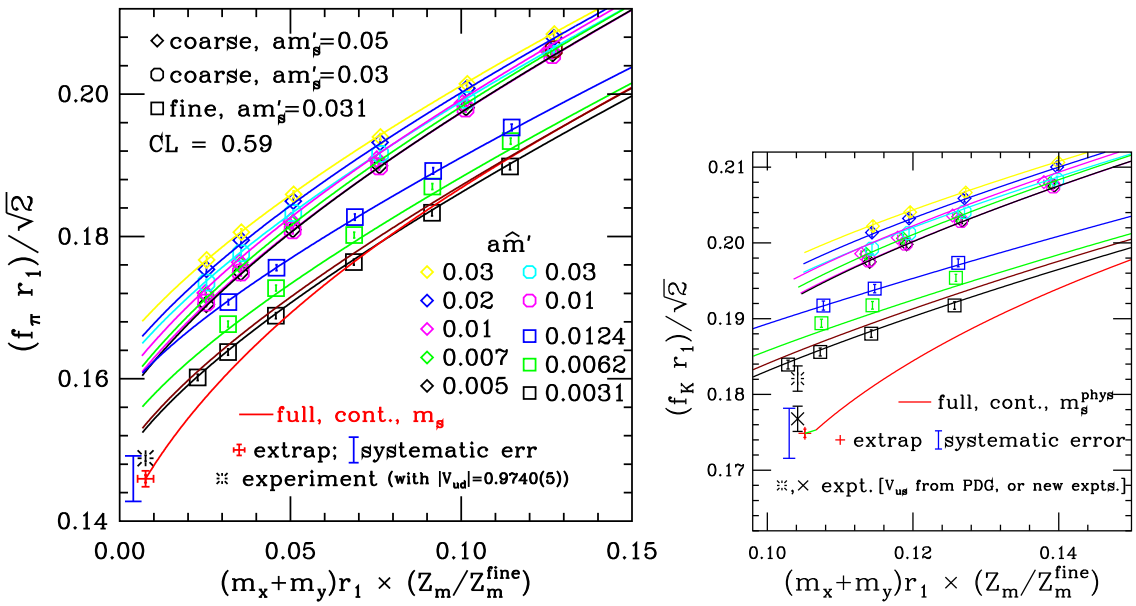

Figure 1: Left: Comparison of Fit C to partially quenched f_π data. Extrapolation to continuum, setting $m'_s = m_s$, and going to full QCD gives the red line. Red + shows the final result after extrapolation $m_x, m_y \to \hat{m}$. The maroon line (just barely visible above the black line) is the prediction for the 0.0031/0.031 fine run (black squares) based on the other data. Right: Same as left, but for f_K. The short green continuation of the red line keeps light sea masses fixed at the average physical value $\hat{m}$ and extrapolates $m_x \to m_u$.

where the errors are from statistics and lattice systematics. These results are consistent with our previous answers [1], with slightly smaller errors. The current $N_f = 3$ results for quark masses are little changed from those in Refs. [1, 13].

We extract V_{us} from our f_K/f_π result. This is probably safer than using f_K itself, because the ratio is largely free of scale errors. We obtain $|V_{us}| = 0.2242(^{+11}_{-31})$, which is consistent with world-average values, with comparably sized errors. From Fig. 1, one can deduce that using f_K alone would result in a somewhat higher value of V_{us}. The difference comes from the fact that our f_π result is slightly low compared with experiment, although consistent within errors. Runs planned for the near future, as well as those now in progress, should allow a further reduction in the errors.

We now turn to the quenched data. We fit to the same valence mass range as the $N_f = 3$ Fit C. Again, terms through NNNLO are needed; joint fits to decay constants and masses, including both coarse and fine lattices, have 34 free parameters:

- 3 LO parameters: f, μ, and the quenched chiral parameter δ [14]. We consider δ to be "LO" because its effects are not suppressed by powers of quark mass.
- 7 NLO parameters: 2 physical and 2 taste-violating analytic terms, 2 taste-violating hairpins, and the quenched chiral parameter α [14].
- 4 physical, NNLO analytic parameters.
- 4 physical, NNNLO analytic parameters.
- 14 tightly constrained parameters (prior width = 0.04): give variation of 2 LO, 4 NLO, 4 NNLO, and 4 NNNLO physical parameters with lattice spacing.
- 2 parameters to allow the r_1 scale on the coarse and fine lattices to vary within 1σ.

The parameter δ multiplies a function of the taste-singlet mass, which is large ($\gtrsim 500$ MeV) on coarse lattices because of taste splitting. The coarse lattices are therefore insensitive to quenched

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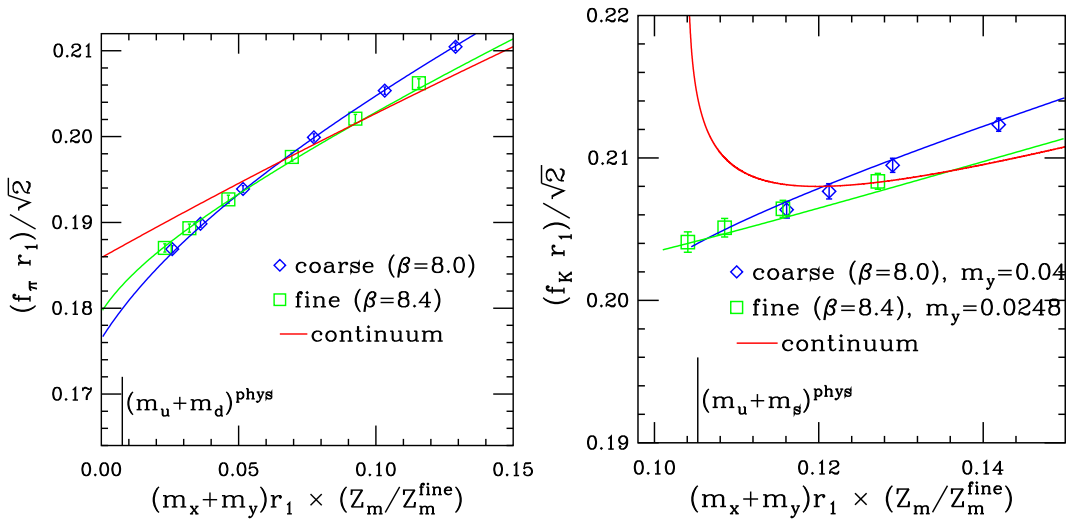

Figure 2: Left: Comparison of Fit Q to quenched f_π data. Continuum extrapolation gives the red curve. $(m_u+m_d)^{\rm phys}$ is from the $N_f=3$ analysis. Right: Same as left, but for quenched f_K.

chiral logs, and δ is poorly determined from coarse-lattice fits: $0.02 \lesssim \delta \lesssim 0.18$, depending on details of the fit and the higher order terms included. Currently, our preferred approach is to obtain δ from fits to the fine lattices alone, and hold it (as well as the fine lattice scale) fixed in the joint fits. The resulting joint fit, called Fit Q, has 32 parameters and $\chi^2/\text{d.o.f.} = 0.96$ for 98 degrees of freedom; CL=0.59. Comparison of this fit with the quenched f_π and f_K data is shown in Fig. 2.

The continuum-extrapolated curve (red line) blows up at low quark mass for f_K but not for f_π: the latter does not have a 1-loop quenched chiral log in the continuum. Taking into account SχPT, f_π increases significantly with the $a \to 0$ extrapolation. This makes makes our quenched f_π differ from experiment (or our $N_f=3$ results) by an even larger amount than was reported in Ref. [10] from coarse data alone. In fact, we find $f_\pi^{\rm quench}/f_\pi^{N_f=3} \approx 1.28$ (with the r_1 scale). Note, however, that the difference between the raw coarse and fine data for f_π is small, so it is not yet clear how seriously we should take the continuum extrapolation — data at smaller a is needed. This is an even more important issue for quenched f_K, where the continuum result blows up in the chiral limit, but the lattice data is smoothed out by SχPT effects. (This is an example of the lack of commutativity of chiral and continuum limits in SχPT for infrared sensitive quantities [15].)

Values of the taste-violating hairpin parameters in the quenched analysis are comparable to those for $N_f=3$. We find:

$$r_1^2 a^2 (\delta'_A)^{\rm quench} = -0.20(2)(^{+4}_{-8}) \qquad r_1^2 a^2 (\delta'_A)^{N_f=3} = -0.29(1)(4)$$
$$r_1^2 a^2 (\delta'_V)^{\rm quench} = 0.09(5)(^{+9}_{-7}) \qquad r_1^2 a^2 (\delta'_V)^{N_f=3} = -0.12(2)(^{+11}_{-5})$$

In both quenched and $N_f=3$, δ'_V is poorly determined and consistent with 0. For the quenched parameter δ we are finding: $\delta = 0.09(1)(5)$. This result is consistent with most quenched evaluations [16], which get $\delta \approx 0.1$, but not that of the Kentucky group [17], who obtain $\delta = 0.24(3)(4)$. (Results in Ref. [18] also tend toward higher values.) Assuming the analysis in Ref. [17], this difference makes sense. Despite the low quark masses in the current calculation, the large taste-violations in the taste-singlet sector puts us in a larger region of meson mass, where the effective δ from chiral fits is smaller. Modulo this issue, which needs further study, it seems that SχPT works reasonably well in the quenched theory, as it does in the $N_f=3$ case.

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