Probing the CP Properties of Higgs Boson through the process $e^- e^+ \to t\bar{t}\phi$ in a completely Model Independent Analysis

A Project Report
submitted in partial fulfilment of the
requirements for the Degree of
Master of Science
in the Faculty of Science

by
Paratma Sri Bhupal Dev

Department of Physics
Indian Institute of Science
Bangalore – 560 012, India

April 2007

Abstract

The study of CP structure of the Higgs sector is of great importance to current and future colliders. A pair of fermions can couple to CP-even and CP-odd Higgs states with comparable strength; hence reactions involving these interactions are the best places to study the CP properties of a neutral Higgs boson. The Higgs boson couplings may also be probed through the polarization of top quark, thus giving an additional handle on these couplings. Also, the effects of new physics on various observables can be enhanced by appropriately choosing the initial beam polarizations.

In this project work, we have studied the process $e^-e^+ \rightarrow t\bar{t}\phi$ in a completely model independent way in order to get some important information on the quantum numbers and the interactions of the Higgs boson. The effects of anomalous couplings of a Higgs boson to a $t\bar{t}$ pair and a pair of Z bosons have been investigated. The anomalous $\gamma t\bar{t}\phi$ and $Zt\bar{t}\phi$ vertices have also been derived. Top quark polarization asymmetries with polarized and unpolarized initial e^-/e^+ beams have been constructed to probe the non-standard Higgs boson couplings.

Chapter 1 is devoted to a brief description of the Standard Model (SM). After recalling the basic ingredients of the model, we discuss the mechanism of spontaneous symmetry breaking, a cornerstone of the SM, and the Glashow-Weinberg-Salam theory of electroweak interactions. Then we discuss the fermion and the Higgs sector of the SM. The phenomenon of CP violation and the CKM formalism are also described. Finally, we mention some of the reasons to look beyond the SM and the motivation for our work.

In Chapter 2, we derive the anomalous Higgs boson couplings. These effective vertices may come from contributions of loops including the effects of New Physics beyond the SM. The dominant anomalous contributions to the vertices come from the dimension-six operators, which have been written down in literature. Contributions from these dimension-six operators to the above mentioned vertices have been derived in this chapter.

Chapter 3 describes the process $e^-e^+ \rightarrow t\bar{t}\phi$ in detail. We calculate the individual helicity amplitudes for this process and also obtain the analytical expressions for the squared matrix elements. Then we study some aspects of the production cross section in the context of the SM in order to make comparisons with the known results and to ensure that our formalism is correct. We have also done consistency checks on our expressions for squared amplitudes by deriving them in two completely independent ways, namely the helicity method and the Bouchiat-Michel method.

In Chapter 4, we study the effect of the pseudo-scalar coupling parameter in the most general $t\bar{t}\phi$ Yukawa coupling on the cross section and the polarization asymmetry. The sensitivity of this pseudo-scalar coupling parameter to cross section and polarization asymmetry measurements has also been investigated.

The contents of Chapter 5 are not quite relevant in the context of the main project work. However, as we plan to include the top decay part and then to calculate the angular and energy distributions of the decay products which are known to be true probes of the non-standard effects in the production of t quark, we have included the work done on the decay width of a heavy quark in general. This gives us a feeling on the time scale of top quark decay.

Finally in Chapter 6, we summarize the progress made so far and anticipate some future directions of the ongoing work.

Acknowledgements

First of all, I would like to thank Prof. Rohini M. Godbole for her invaluable guidance, encouragement and support during my Master's Project work. I express my heart-felt gratitude to her for being an excellent guide, both academically and personally.

I gratefully acknowledge the collaboration with Prof. Saurabh D. Rindani (PRL, Ahmedabad) whose help and suggestions through e-mails were immensely beneficial for my project work.

I am very much thankful to Prof. Diptiman Sen, Prof. Apoorva Patel, Prof. B. Ananthanarayan, Prof. Sachindeo Vaidya and Prof. Sudhir Vempati, for being very kind and helpful during my stay at CHEP, IISc. I also thank all the instructors during my M.S. coursework who have helped me in building a solid foundation for a research career in Physics. Thanks are also due to Prof. B. P. Das (IIA) and Prof. C. S. Shukre (RRI) for their valuable suggestions and advice.

I thank the office staff in both CHEP and Physics, especially Keshava, Janardhana, Meena, Rakma and Srivatsa, for being very supportive in all official matters.

I have had a great time with all my friends and seniors at IISc who have made my life colorful. The time I have spent at CHEP with Naveen, Ravi, Amit, Pranaw, Alka and Sudhansu bhai has been quite inspirational; special thanks to Sudhansu bhai for being a very kind and helpful senior in my project work. Thanks are due to all the Oriya Sansad members, especially Hembram bhai, Debakanta bhai, Khatei bhai, Ramakanta bhai, Tapan bhai, Gopal bhai, Ganapati bhai, Alok bhai and Anant bhai, who have never let me feel away from home. I must also thank all my good friends at IISc – Shivaraj, Vaibhav, Sayantan, Saroj, Sayak, Subhro, Shaon, Nihal, Arunima, Tathagat, Girjesh, Ashish, Sakshath, Padmalekha, Prachi, Ved Prakash, Sangamesh and Viswanathan, just to name a few – who have added flavor to my life. Special thanks goes to Shiva for providing a very entertaining company all the time.

Last but not the least, I would like to thank my parents and my elder sister for their love, support and encouragement.

Paratma Sri Bhupal Dev

CONTENTS

List of Figures

CHAPTER 1

INTRODUCTION

Higgs boson is a massive spin-0 particle appearing in a local gauge theory, where the local gauge invariance is broken completely, or at least partially, by the mechanism of spontaneous symmetry breaking. The simplest way in which spontaneous breaking of a symmetry is achieved is by the introduction of elementary scalar fields in the theory. These ideas are used to write down the Standard Model (SM) of electromagnetic, weak and strong interactions which provides a unified mathematical framework to describe these three forces of Nature at the quantum level. Since the SM explains most of the experimentally observed phenomena with rather high accuracy, it now serves as *the* starting point for discussions of fundamental particles and the interactions among them. So in the following section, we present a brief introduction to the SM of elementary particle physics. For a detailed discussion, we refer to standard textbooks on quantum field theory [1] and elementary particle physics [2].

1.1 THE STANDARD MODEL

The electroweak theory, proposed by Glashow, Salam and Weinberg [3] to describe the electromagnetic and weak interactions between quarks and leptons, is a Yang-Mills theory [4] based on the symmetry group $SU(2)_L \otimes U(1)_Y$ of weak left-handed isospin and hypercharge. Combined with Quantum Chromo-Dynamics (QCD), the theory of strong interaction between colored quarks based on the symmetry group $SU(3)_C$ [5], it makes up the SM – a gauge theory with an $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge group.

The SM, before introducing the electroweak symmetry breaking mechanism, has two kinds of fields:

1. The **fermionic matter fields** corresponding to the three *generations* of quarks and leptons which transform according to left-handed (L) doublet and right-handed (R) singlet representations of $SU(2)_L$ to account for the $V - A$ nature of the

charged-current weak interactions:

$$\begin{aligned}
L_1 &= \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \quad e_{R_1} = e_R^- \,; \qquad Q_1 = \begin{pmatrix} u \\ d \end{pmatrix}_L, \quad u_{R_1} = u_R, \quad d_{R_1} = d_R \\
L_2 &= \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L, \quad e_{R_2} = \mu_R^- \,; \qquad Q_2 = \begin{pmatrix} c \\ s \end{pmatrix}_L, \quad u_{R_2} = c_R, \quad d_{R_2} = s_R \\
L_3 &= \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L, \quad e_{R_3} = \tau_R^- \,; \qquad Q_3 = \begin{pmatrix} t \\ b \end{pmatrix}_L, \quad u_{R_3} = t_R, \quad d_{R_3} = b_R
\end{aligned} \tag{1.1}$$

where the L,R fermion states are defined as $f_{L,R} = P_{L,R} f$ with $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$ being the *chirality projection operators*. It may be mentioned here that in the SM, the number of fermion generations is not fixed by any symmetry principle. However, anomaly cancellation requires that the number of lepton and quark families are the same, whatever may be the number of generations. Experimentally, there is strong evidence that there are only three generations.

The quarks (both L and R type) transform as color triplets of $SU(3)_C$, to account for the strong interaction of the quarks. The leptons are color singlets under $SU(3)_C$. The assignment of weak hypercharge Y_f corresponding to the $U(1)_Y$ group to the various $SU(2)_L$ and $SU(3)_C$ fermion multiplets is according to the charge formula:

$$Y_f = 2(Q_f - I_f^{3L}), \tag{1.2}$$

where Q_f is the electric charge in units of the proton charge $+e$ and I_f^{3L} the third component of weak isospin corresponding to $SU(2)_L$. For the various fermions listed in (1.1) $I_f^{3L,3R}$ takes values $\pm\frac{1}{2}$ or 0. Using Eq. (1.2) it is easy to see that

$$Y_{L_i} = -1, \;\; Y_{e_{R_i}} = -2, \;\; Y_{Q_i} = \frac{1}{3}, \;\; Y_{u_{R_i}} = \frac{4}{3}, \;\; Y_{d_{R_i}} = -\frac{2}{3} \tag{1.3}$$

Since no generator of $SU(3)_C$ appears in Eq. (1.2), electric charge is independent of color.

2. The **bosonic gauge fields** corresponding to the vector bosons which mediate the interactions. In the electroweak sector, we have the gauge field B_μ corresponding to the generator Y of the $U(1)_Y$ group and the three fields W_μ^a ($a = 1, 2, 3$) corresponding to the generators

$$T^a = \frac{1}{2}\tau^a \tag{1.4}$$

of the $SU(2)_L$ group where τ^a are the non-commuting 2×2 *Pauli matrices*. The commutation relations obeyed by these generators (1.4) are given by

$$\left[T^a, T^b\right] = i\epsilon^{abc} T^c \;\text{ and }\; [Y, Y] = 0 \tag{1.5}$$

where ϵ^{abc} is the antisymmetric Levi-Civita tensor. In the strong interaction sector, we have an octet of gluon fields W^a_μ $(a = 1, ..., 8)$ corresponding to the eight generators $T'^a = \frac{1}{2}\lambda^a$ of the $SU(3)_C$ group where λ^a are the 3×3 anti-commuting *Gell-Mann matrices*. These generators obey the relations

$$\left[T'^a, T'^b\right] = if^{abc}T'^c \;\text{ with }\; \text{Tr}\left[T'^a, T'^b\right] = \frac{1}{2}\delta^{ab} \tag{1.6}$$

where the tensor f^{abc} stands for the structure constants of the $SU(3)_C$ group.

The field strengths are given by

$$\begin{aligned} B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \\ W^a_{\mu\nu} &= \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_2\epsilon^{abc}W^b_\mu W^c_\nu \quad (a = 1, 2, 3) \\ G^a_{\mu\nu} &= \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc}G^b_\mu G^c_\nu \quad (a = 1, ..., 8) \end{aligned} \tag{1.7}$$

where g_s, g_2 and g_1 are, respectively, the coupling constants of $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$.

The matter fields ψ are minimally coupled to the gauge fields through the covariant derivative D_μ which is defined as

$$D_\mu\psi = \left(\partial_\mu - ig_2\frac{\tau^a}{2}W^a_\mu - ig_1\frac{Y}{2}B_\mu - ig_s\frac{\lambda^b}{2}G^b_\mu\right)\psi \tag{1.8}$$

where the sum over a is from 1 to 3 while that over b is from 1 to 8. D_μ is a differential operator and a matrix in $SU(2)_L$ space (2×2) as well as in $SU(3)_C$ color space (3×3).

The fermions interact with the gauge bosons through the minimal coupling:

$$\mathcal{L}_{\text{fermion}} = \sum \overline{\psi}_L i\not{D}\,\psi_L + \sum \overline{\psi}_R i\not{D}\,\psi_R \tag{1.9}$$

where the sum is over all fermion multiplets, quarks as well as leptons. D_μ is defined as in Eq. (1.8) appropriate to the representation:

$$D_\mu \equiv \begin{cases} \partial_\mu - i\frac{g_2}{2}\tau^a W^a_\mu - i\frac{g_1}{6}B_\mu - i\frac{g_s}{2}\lambda^b G^b_\mu & \text{for a left-handed quark} \\ \partial_\mu - i\frac{g_2}{2}\tau^a W^a_\mu + i\frac{g_1}{2}B_\mu & \text{for a left-handed lepton} \\ \partial_\mu - ig_1 Q_q B_\mu - i\frac{g_s}{2}\lambda^b G^b_\mu & \text{for a right-handed quark of charge } Q_q \\ \partial_\mu + ig_1 B_\mu & \text{for a right-handed electron with charge } -e \end{cases}$$

The use of covariant derivative in constructing the Lagrangian $\mathcal{L}_{\text{fermion}}$ guarantees that it is invariant under local $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge transformations of the fermion and gauge fields. The gauge transformations for the electroweak sector, for example, are given as

$$\psi_L(x) \to e^{i\alpha^a(x)T^a + i\beta(x)Y}\psi_L(x)\;,\; \psi_R(x) \to e^{i\beta(x)Y}\psi_R(x);$$

$$\vec{W}_\mu(x) \to \vec{W}_\mu(x) - \frac{1}{g_2}\partial_\mu \vec{\alpha}(x) - \vec{\alpha}(x) \times \vec{W}_\mu(x)\ ,\ B_\mu(x) \to B_\mu(x) - \frac{1}{g_1}\partial_\mu \beta(x)$$

The Lagrangian for gauge fields is given by

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{\mu\nu a} - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu a} \tag{1.10}$$

The gauge-invariant theory described above can not accommodate *masses* for any of the particles. Gauge boson mass terms are forbidden by $SU(2) \otimes U(1)$ local gauge invariance. Fermion mass terms are of the the form $m\overline{\psi}_L\psi_R + \text{H.c.}$, and are forbidden by even global gauge invariance. Thus the incorporation of mass terms by brute force leads to a breakdown of the local gauge invariance. Generation of masses for all the particles without violating the local gauge symmetry, and hence, without sacrificing the renormalizability of the theory is achieved by using the mechanism of *spontaneous symmetry breaking* (SSB).

1.2 Spontaneous Symmetry Breaking

A symmetry is said to be broken spontaneously if the Lagrangian of the theory is invariant under the symmetry, but the ground state (the vacuum state) is not. In this situation, many degenerate vacuum states are related to one another by the symmetry of the Lagrangian, and one of them is singled out to be the correct vacuum. Hence states constructed out of this vacuum reflect this bias, and the dynamics of the theory no longer shows the invariance.

The Lagrangian we described has a unique minimum energy state corresponding to all the fields taking the value zero. This vacuum is gauge invariant. Hence something has to be added to the theory to achieve spontaneous symmetry breaking. Moreover, if the vacuum expectation value of any but a spin-0 field is nonzero, even Lorentz invariance would be violated. To avoid this, one introduces non-zero *vacuum expectation value* (VEV) only for scalar fields. These scalar field(s) may be elementary or composite.

Next we discuss the phenomenon of SSB for three cases, viz. global symmetry (both discrete and continuous), and $U(1)$ abelian and $SU(2)$ non-abelian local gauge symmetries.

1.2.1 SSB OF A GLOBAL SYMMETRY

Let us consider the Lagrangian of a simple ϕ^4 theory:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) \ , \ \ V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4}\phi^4 \tag{1.11}$$

with $\lambda > 0$, which is needed to make the potential bounded from below. This Lagrangian is invariant under the discrete symmetry $\phi \to -\phi$. If the mass term $\mu^2 > 0$, the potential $V(\phi)$ is also positive and the minimum of the potential occurs for $\langle 0|\phi|0\rangle \equiv \phi_0 = 0$ which is the vacuum state as shown in Figure 1.1(a). On the other hand, if $\mu^2 < 0$, then $V(\phi)$ has two minima given by

$$\langle 0|\phi|0\rangle \equiv \phi_0 = \pm\sqrt{\frac{-\mu^2}{\lambda}} = \pm v \tag{1.12}$$

as shown in Figure 1.1(b). The quantity v is the VEV of the field ϕ.

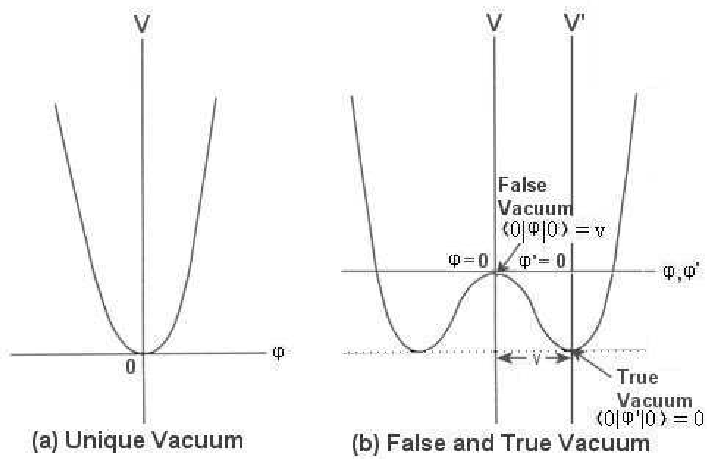


Figure 1.1: The scalar potential $V(\phi)$ (a) for $\mu^2 > 0$ and (b) for $\mu^2 < 0$.

In this case, $\mathcal{L}$ is no more the Lagrangian of a scalar particle with mass μ; to interpret this theory correctly, we have to define the field by expanding it around one of the minima:

$$\phi(x) = v + \sigma(x) \tag{1.13}$$

In terms of the new field $\sigma(x)$, the Lagrangian becomes

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \sigma)^2 - \frac{1}{2}(-2\mu^2)\sigma^2 - \sqrt{-\mu^2\lambda}\,\sigma^3 - \frac{\lambda}{4}\sigma^4 + \text{const.} \tag{1.14}$$

This Lagrangian describes a scalar field of mass $m = \sqrt{-2\mu^2}$, with σ^3 and σ^4 interactions. The cubic term breaks the reflection symmetry $\phi \to -\phi$; it is no longer apparent in $\mathcal{L}$. This is the simplest example of a spontaneously broken symmetry.

Now we generalize the ϕ^4 theory to a set of N real scalar fields $\phi_i(x)$ to illustrate the case of continuous symmetry rather than the discrete one. The Lagrangian in this case (called the *linear sigma model*) is given by

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi_i\partial^\mu\phi_i - \frac{1}{2}\mu^2(\phi_i\phi_i) - \frac{\lambda}{4}(\phi_i\phi_i)^2 \tag{1.15}$$

which is invariant under the $O(N)$ group of transformations $\phi_i(x) \to R_{ij}\phi_j(x)$ for any $N \times N$ orthogonal matrix R. Again, for $\mu^2 < 0$, the potential has a minimum at

$$\phi_0^2 = \frac{-\mu^2}{\lambda} \equiv v^2 \tag{1.16}$$

This condition determines only the length of the vector ϕ_0 and its direction is arbitrary. Without loss of generality, it is possible to choose coordinates so that ϕ_0 points in the Nth direction:

$$\phi_0 = (0, 0, ..., 0, v) \tag{1.17}$$

Now we define a set of shifted fields as

$$\phi_i(x) = (\pi_k(x), v + \sigma(x)), \qquad k = 1, ..., N-1 \tag{1.18}$$

Then the Lagrangian in terms of the new fields $\sigma(x)$ and $\pi_k(x)$ becomes

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}(\partial_\mu\sigma)^2 - \frac{1}{2}(-2\mu^2)\sigma^2 - \lambda v\sigma^3 - \frac{\lambda}{4}\sigma^4 \\ &+ \frac{1}{2}\partial_\mu\pi_k\partial^\mu\pi_k - \frac{\lambda}{4}(\pi_k\pi_k)^2 - \lambda v(\pi_k\pi_k)\sigma - \frac{\lambda}{2}(\pi_k\pi_k)\sigma^2\end{aligned} \tag{1.19}$$

We still obtain a massive σ field with $m = \sqrt{-2\mu^2}$ just as in Eq. (1.14), but also a set of $N-1$ massless π fields since all the bilinear $\pi_k\pi_k$ terms in the Lagrangian have vanished. The original $O(N)$ symmetry is hidden, leaving only the $O(N-1)$ subgroup, which rotates the π fields among themselves. As shown in Figure 1.2, the massive σ field describes oscillations of ϕ_i in the radial direction, in which the potential has a non vanishing second derivative. The massless π fields describe oscillations of ϕ_i in the tangential directions, along the trough of the potential. The trough is an $(N-1)$-dimensional surface, and all $N-1$ directions are equivalent, reflecting the unbroken $O(N-1)$ symmetry.

The appearance of massless particles when a continuous symmetry is spontaneously broken is a general result, known as the *Goldstone theorem* [6].

The Goldstone Theorem: For every spontaneously broken continuous symmetry, the theory must contain a massless particle. The massless fields arising through spontaneous symmetry breaking are called *Goldstone bosons* and the number of Goldstone bosons is equal to the number of broken generators. For an $O(N)$ continuous symmetry, for instance, there are $\frac{1}{2}N(N-1)$ generators; the residual unbroken symmetry $O(N-1)$ has $\frac{1}{2}(N-1)(N-2)$ generators, and therefore, the number of Goldstone bosons is the difference, $N-1$.

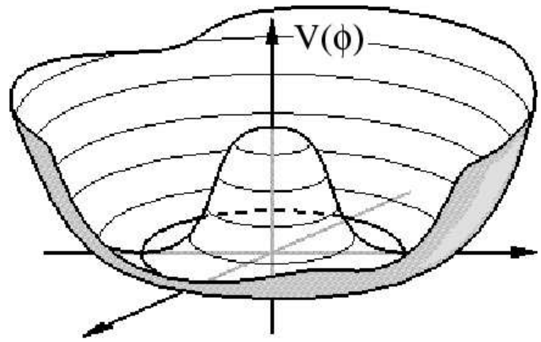


Figure 1.2: The scalar potential for spontaneous symmetry breaking of a continuous $O(N)$ symmetry, drawn for the case $N = 2$.

1.2.2 SSB OF AN ABELIAN LOCAL GAUGE SYMMETRY

Let us now consider a complex scalar field coupled both to itself and to an electromagnetic field A_μ:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + D_\mu\phi^* D^\mu\phi - V(\phi) \tag{1.20}$$

with $D_\mu = \partial_\mu - ieA_\mu$ and with the complex scalar potential

$$V(\phi) = \mu^2\phi^*\phi + \lambda(\phi^*\phi)^2 \tag{1.21}$$

This Lagrangian is invariant under the local $U(1)$ transformation

$$\phi(x) \to e^{i\alpha(x)}\phi(x)\,, \qquad A_\mu(x) \to A_\mu(x) - \frac{1}{e}\partial_\mu\alpha(x)$$

with $\alpha(x)$ real. For $\mu^2 > 0$, $\mathcal{L}$ is simply the QED Lagrangian for a charged scalar particle of mass μ and with ϕ^4 self-interactions. For $\mu^2 < 0$, the field $\phi(x)$ will acquire a VEV and the $U(1)$ local symmetry will be spontaneously broken. The minimum of this potential occurs at

$$\langle\phi\rangle \equiv \phi_0 = \sqrt{\frac{-\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}} \tag{1.22}$$

As before, we expand the Lagrangian about the vacuum state $\langle\phi\rangle$. The complex field $\phi(x)$ can be decomposed as

$$\phi(x) = \frac{1}{\sqrt{2}}[v + \phi_1(x) + i\phi_2(x)] \tag{1.23}$$

The Lagrangian then becomes (omitting the terms cubic and quartic in the fields A_μ, ϕ_1 and ϕ_2)

$$\mathcal{L} \;=\; -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (\partial^\mu + ieA^\mu)\phi^*(\partial_\mu - ieA_\mu)\phi - \mu^2\phi^*\phi - \lambda(\phi^*\phi)^2$$

$$= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_\mu\phi_1)^2 + \frac{1}{2}(\partial_\mu\phi_2)^2 - v^2\lambda\phi_1^2 + \frac{1}{2}e^2v^2A_\mu A^\mu - evA_\mu\partial^\mu\phi_2 \tag{1.24}$$

so that the field ϕ_1 acquires a mass $m = \sqrt{-2\mu^2}$ and ϕ_2 is the massless would-be Goldstone boson. Also, there is a photon mass term $\frac{1}{2}m_A^2 A_\mu A^\mu$ appearing in the Lagrangian with mass $m_A = ev = \frac{-\mu^2 e}{\lambda}$.

However, the following problem is now to be addressed: Before the symmetry was broken, we had four degrees of freedom in the theory, two for the complex scalar field ϕ and two for the massless electromagnetic field A_μ; but now we have apparently five degrees of freedom, one each for ϕ_1 and ϕ_2, and three for the massive photon A_μ. Therefore, there must be an unphysical field in the new Lagrangian which has to be eliminated. To do so, we notice that at first order, we have for the original field ϕ

$$\phi = \frac{1}{\sqrt{2}}(v + \phi_1 + i\phi_2) \equiv \frac{1}{\sqrt{2}}[v + \eta(x)]e^{i\zeta(x)/v} \tag{1.25}$$

By using the freedom of gauge transformation and also by performing the substitution

$$A_\mu \to A_\mu - \frac{1}{ev}\partial_\mu\zeta(x)$$

the $A_\mu\partial^\mu\zeta$ term, and in fact, all ζ terms disappear from the Lagrangian. This choice of gauge for which only the physical particles are left in the Lagrangian is called the *unitary gauge*. Thus the massless photon which had only two physical transverse polarization states has absorbed the Goldstone boson and becomes massive with three physical polarization states. The longitudinal polarization state is the extra degree of freedom that appears after canceling the unphysical contributions. The $U(1)$ gauge symmetry is no more apparent in the theory.

This mechanism, by which spontaneous symmetry breaking generates masses for gauge bosons, was explored and generalized to the non-Abelian case by Higgs, Englert, Brout, Guralnik, Hagen and Kibble [7], and is commonly known as the *Higgs mechanism*.

1.2.3 SSB of a non-Abelian Local Gauge Symmetry

Let us consider a model with an $SU(2)$ gauge field coupled to a scalar field Φ that transforms as a spinor of $SU(2)$. The Lagrangian is given by (with $a = 1, 2, 3$)

$$\mathcal{L} = -\frac{1}{4}W^a_{\mu\nu}W^{\mu\nu a} + (D^\mu\Phi)^\dagger(D_\mu\Phi) - V(\Phi) \tag{1.26}$$

with

$$D_\mu \equiv \partial_\mu - ig_2\frac{\tau^a}{2}W^a_\mu \text{ and } V(\Phi) = \mu^2\Phi^\dagger\Phi + \lambda\left(\Phi^\dagger\phi\right)^2 \tag{1.27}$$

For $\mu^2 < 0$, Φ acquires a VEV which, using the freedom of $SU(2)$ rotations, can be written as

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \tag{1.28}$$

Then the gauge boson masses arise from the scalar field kinetic energy term

$$|D_\mu \Phi|^2 = \frac{1}{8} g_2^2 \begin{pmatrix} 0 & v \end{pmatrix} \tau^a \tau^b \begin{pmatrix} 0 \\ v \end{pmatrix} W_\mu^a W^{\mu b} + ... \tag{1.29}$$

Symmetrizing the matrix product under the interchange of a and b, using $\{\tau^a, \tau^b\} = 2\delta^{ab}$, we find the mass term

$$\Delta \mathcal{L} = \frac{g_2^2 v^2}{8} W_\mu^a W^{\mu a} \tag{1.30}$$

Thus all three gauge bosons acquire the mass

$$m_W = \frac{g_2 v}{2} \tag{1.31}$$

implying that all three generators of $SU(2)$ are broken equally by (1.28).

1.3 THE GLASHOW-WEINBERG-SALAM THEORY OF ELECTROWEAK INTERACTION

The Glashow-Weinberg-Salam (GWS) theory [3] is a spontaneously broken gauge theory which provides a description of weak and electromagnetic interactions which has been verified to the loop level by experiments. In this unified picture, the three gauge bosons $W^\pm$ and Z^0 acquire mass from the broken $SU(2)$ sector while photon remains massless due to the unbroken $U(1)$ sector after spontaneous breaking of $SU(2) \otimes U(1)$ local gauge symmetry.

To break the $SU(2)$ gauge symmetry spontaneously, we introduce a scalar field in the spinor representation of $SU(2)$, as in Eq. (1.27). However, as we saw in §1.2.3, this theory will lead to a system with no massless gauge bosons. We therefore introduce an additional $U(1)$ gauge symmetry by assigning the scalar field a $U(1)$ charge $+\frac{1}{2}$ under this symmetry. The simplest choice is then a complex $SU(2)$ doublet of scalar fields Φ with $Y_\Phi = +1$ (corresponding to charge $\frac{Y}{2}$):

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \tag{1.32}$$

The Lagrangian of the theory is then given by

$$\mathcal{L} = \mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{scalar}} \tag{1.33}$$

with $\mathcal{L}_{\text{fermion}}$ and $\mathcal{L}_{\text{gauge}}$ defined in Eqs. (1.9) and (1.10) respectively, but excluding the strong interaction part which does not concern us here. $\mathcal{L}_{\text{scalar}}$ given by

$$\mathcal{L}_{\text{scalar}} = (D^\mu \Phi)^\dagger (D_\mu \Phi) - \mu^2 \Phi^\dagger \Phi - \lambda \left(\Phi^\dagger \Phi\right)^2 \tag{1.34}$$

For $\mu^2 < 0$, the minimum of the scalar potential occurs when the neutral component of the doublet field Φ has a non-zero VEV[1]

$$\langle \Phi \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \tag{1.35}$$

with $v = \sqrt{\frac{-\mu^2}{\lambda}}$. Now we write the doublet field Φ in terms of four fields $\theta_a(x)$ ($a = 1, 2, 3$) and $H(x)$ at first order:

$$\Phi(x) = \begin{pmatrix} \theta_2 + i\theta_1 \\ \frac{1}{\sqrt{2}}(v + H) - i\theta_3 \end{pmatrix} = e^{i\theta_a(x)\tau^a(x)/v} \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + H(x)) \end{pmatrix} \tag{1.36}$$

Then we make a gauge transformation on this field to move to the unitary gauge:

$$\Phi(x) \to e^{-i\theta_a(x)\tau^a(x)/v} \Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \tag{1.37}$$

The terms relevant for masses in the Lagrangian (1.33) are

$$\begin{aligned}
|D_\mu \Phi|^2 &= \left| \left(\partial_\mu - i g_2 \frac{\tau^a}{2} W^a_\mu - i g_1 \frac{1}{2} B_\mu \right) \Phi \right|^2 \\
&= \frac{1}{2} \left| \begin{pmatrix} \partial_\mu - \frac{i}{2}\left(g_2 W^3_\mu + g_1 B_\mu\right) & \frac{-ig_2}{2}\left(W^1_\mu - iW^2_\mu\right) \\ \frac{-ig_2}{2}\left(W^1_\mu + iW^2_\mu\right) & \partial_\mu + \frac{i}{2}\left(g_2 W^3_\mu - g_1 B_\mu\right) \end{pmatrix} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \right|^2 \\
&= \frac{1}{2} (\partial_\mu H)^2 + \frac{1}{8} (v + H)^2 \left[g_2^2 \left| W^1_\mu + iW^2_\mu \right|^2 + \left| g_2 W^3_\mu - g_1 B_\mu \right|^2 \right]
\end{aligned} \tag{1.38}$$

Defining new fields $W^\pm_\mu$ and Z_μ:

$$W^\pm_\mu = \frac{1}{\sqrt{2}} \left(W^1_\mu \mp iW^2_\mu \right), \quad Z_\mu = \frac{g_2 W^3_\mu - g_1 B_\mu}{\sqrt{g_2^2 + g_1^2}}, \quad A_\mu = \frac{g_1 W^3_\mu + g_2 B_\mu}{\sqrt{g_2^2 + g_1^2}} \tag{1.39}$$

with A_μ field orthogonal to Z_μ, we can pick up the terms from Eq. (1.38) which are bilinear in the fields $W^\pm_\mu$, Z_μ and A_μ:

$$\begin{aligned}
\Delta \mathcal{L} &= \frac{1}{4} v^2 g_2^2 W^+_\mu W^{-\mu} + \frac{1}{8} v^2 (g_2^2 + g_1^2) Z_\mu Z^\mu + \frac{1}{2}(0) A_\mu A^\mu \\
&= m_W^2 W^+_\mu W^{-\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu + \frac{1}{2} m_A^2 A_\mu A^\mu
\end{aligned} \tag{1.40}$$

[1] The VEV should not be in the charged direction to preserve $U(1)_{\text{QED}}$.

with

$$m_W = \frac{1}{2} v g_2 \,, \ \ m_Z = \frac{1}{2} v \sqrt{g_2^2 + g_1^2} \,, \ \ m_A = 0 \tag{1.41}$$

Thus we get three massive vector bosons $W^{\pm}$ and Z while the fourth vector field A_μ, orthogonal to Z_μ, remains massless. We identify this massless field with the usual electromagnetic field.

Thus by spontaneously breaking the local gauge symmetry $SU(2)_L \otimes U(1)_Y$ to $U(1)_Q$, the three Goldstone modes have been absorbed by the $W^{\pm}$ and Z bosons to form their longitudinal components and thus become massive. Since the $U(1)_Q$ symmetry is still unbroken, the photon which is its generator remains massless as it should be.

It is more convenient to write all expressions in terms of these mass eigenstates. Let us consider, for example, the coupling of the vector fields to fermions. The general form of the covariant derivative is given by Eq. (1.8):

$$D_\mu = \partial_\mu - i g_2 T^a W_\mu^a - i g_1 \frac{Y}{2} B_\mu \tag{1.42}$$

In terms of the mass eigenstate-fields, this becomes

$$\begin{aligned} D_\mu &= \partial_\mu - i\frac{g_2}{\sqrt{2}}(W_\mu^+ T^+ + W_\mu^- T^-) - i\frac{1}{\sqrt{g_2^2+g_1^2}} Z_\mu \left(g_2^2 T^3 - g_1^2 \frac{Y}{2} \right) \\ & \quad - i\frac{g_2 g_1}{\sqrt{g_2^2+g_1^2}} A_\mu \left(T^3 + \frac{Y}{2} \right) \end{aligned} \tag{1.43}$$

where $T^{\pm} = T^1 \pm iT^2$. The last term of Eq. (1.43) is identified as the electromagnetic interaction term with electric charge quantum number $Q = T^3 + \frac{Y}{2}$ as given in Eq. (1.2), where I^3 corresponds to the eigenvalue of the operator T^3. To put expression (1.43) into a more useful form, we should then identify the coefficient of the electromagnetic interaction as the proton charge e:

$$e = \frac{g_2 g_1}{\sqrt{g_2^2 + g_1^2}} = \frac{g_2 g_1}{g_Z} \tag{1.44}$$

with $g_Z = \sqrt{g_2^2 + g_1^2}$. Eqs. (1.39) for the field rotation which lead to the physical gauge bosons define the *electroweak mixing angle* (or *Weinberg angle*), θ_W:

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}, \tag{1.45}$$

that is,

$$\cos\theta_W \equiv c_W = \frac{g_2}{\sqrt{g_2^2+g_1^2}} \,, \ \ \sin\theta_W \equiv s_W = \frac{g_1}{\sqrt{g_2^2+g_1^2}} \tag{1.46}$$

Then, with the manipulation in the Z coupling

$$g_2^2 T^3 - g_1^2 \frac{Y}{2} = (g_2^2 + g_1^2) T^3 - g_1^2 Q$$

we can rewrite the covariant derivative (1.43) in the form

$$D_\mu = \partial_\mu - i\frac{g_2}{\sqrt{2}}(W_\mu^+ T^+ + W_\mu^- T^-) - i\frac{g_2}{c_W} Z_\mu (T^3 - s_W^2 Q) - ieQA_\mu, \tag{1.47}$$

where

$$g_2 = \frac{e}{s_W} \tag{1.48}$$

Thus we see here that the weak boson couplings are described by two parameters: e and θ_W. The couplings induced by W and Z exchange will also involve the masses of these particles. However, these masses are not independent, since it follows from Eqs. (1.41) that

$$m_W = c_W m_Z \tag{1.49}$$

So all effects of W and Z exchange processes, at least at tree level, can be written in terms of the three basic parameters e, θ_W, and m_Z.

Using the fermionic part of the SM Lagrangian, Eq. (1.33) written in terms of the new fields, and using the expression (1.47) for the covariant derivative, we the neutral-current (NC) and charged-current (CC) interactions

$$\begin{aligned}
\mathcal{L}_{\rm NC} &= eJ_\mu^A A^\mu + \frac{g_2}{c_W} J_\mu^Z Z^\mu \\
\mathcal{L}_{\rm CC} &= \frac{g_2}{\sqrt{2}}(J_\mu^+ W^{+\mu} + J_\mu^- W^{-\mu})
\end{aligned} \tag{1.50}$$

The currents J_μ are given by

$$\begin{aligned}
J_\mu^A &= Q_f \bar{f} \gamma_\mu f \\
J_\mu^Z &= \frac{1}{4}\bar{f}\gamma_\mu[(2I_f^3 - 4Q_f s_W^2) - \gamma_5(2I_f^3)]f \\
J_\mu^+ &= \frac{1}{2}\bar{f}_u \gamma_\mu (1-\gamma_5) f_d
\end{aligned} \tag{1.51}$$

where $f_u(f_d)$ is the up (down)-type fermion of isospin $I_f^3 = +(-)\frac{1}{2}$.

1.4 FERMION MASSES IN THE STANDARD MODEL

So far, we have discussed only the generation of gauge boson masses. We can also generate fermion masses using the same scalar field ϕ, with hypercharge $Y = 1$, and the isodoublet $\tilde{\Phi} = i\tau^2\Phi^*$ which has hypercharge $Y = -1$.

We introduce the $SU(2)_L \otimes U(1)_Y$ invariant Yukawa Lagrangian in an n_g-dimensional generation space involving the left-handed doublets

$$Q_L = \begin{pmatrix} p_L \\ n_L \end{pmatrix} , \; L_L = \begin{pmatrix} \nu_L \\ l_L \end{pmatrix}$$

and right-handed singlets p_R, n_R, l_R of fermions, and the Higgs doublet:

$$\begin{aligned}\mathcal{L}_{\text{Yukawa}} &= -\left(\overline{Q}_L \Gamma \Phi n_R + \overline{Q}_L \Delta \tilde{\Phi} p_R + \overline{L}_L \Pi \Phi l_R\right) + \text{H.c.} \\ &= -\left[\begin{pmatrix} \overline{p}_L & \overline{n}_L \end{pmatrix} \Gamma \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} n_R + \begin{pmatrix} \overline{p}_L & \overline{n}_L \end{pmatrix} \Delta \begin{pmatrix} \phi^{0\dagger} \\ -\phi^- \end{pmatrix} p_R \right. \\ &\quad \left. + \begin{pmatrix} \overline{\nu}_L & \overline{l}_L \end{pmatrix} \Pi \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} l_R \right] + \text{H.c.} \end{aligned} \tag{1.52}$$

where the couplings Γ, Δ and Π are $n_g \times n_g$ matrices and p_L, n_L, p_R, n_R are $n_g \times 1$ vectors in generation space. In the SM, for instance, $n_g = 3$ and Γ, Δ and Π are 3×3 matrices in generation space [2]. Gauge invariance does not constrain the flavor structure of the Yukawa interactions; as a result, Γ, Δ and Π are completely arbitrary [3].

If we substitute ϕ^0 by its VEV v, we obtain the mass terms

$$\mathcal{L}_{\text{mass}} = -\overline{n}_L M_n n_R - \overline{p}_L M_p p_R - \overline{l}_L M_l l_R + \text{H.c.}, \tag{1.53}$$

with fermion mass matrices

$$M_n = \frac{v\Gamma}{\sqrt{2}} , \; M_p = \frac{v\Delta}{\sqrt{2}} , \; M_l = \frac{v\Pi}{\sqrt{2}} \tag{1.54}$$

The interaction terms in Eq. (1.52) are given by

$$\begin{aligned}\mathcal{L}_{\text{int}} &= \mathcal{L}_{\text{Yukawa}} - \mathcal{L}_{\text{mass}} \\ &= -\overline{n}_L \frac{M_n}{\sqrt{2}v} H n_R - \overline{p}_L \frac{M_p}{\sqrt{2}v} H p_R - \overline{l}_L \frac{M_l}{\sqrt{2}v} H l_R - \overline{p}_L \frac{M_n}{\sqrt{2}v} \phi^+ n_R \\ &\quad + \overline{n}_L \frac{M_p}{\sqrt{2}v} \phi^- p_R - \overline{\nu}_L \frac{M_l}{\sqrt{2}v} \phi^+ l_R + \text{H.c.} + \ldots \end{aligned} \tag{1.55}$$

The Yukawa-coupling matrices are not necessarily Hermitian. In the quark sector, they may be diagonalized by *bi-unitary transformations*

$$\begin{aligned} p_L &= U_L^p u_L, \\ p_R &= U_R^p u_R, \\ n_L &= U_L^n d_L, \\ n_R &= U_R^n d_R, \end{aligned} \tag{1.56}$$

[2] Henceforth, we shall discuss only the SM case of $n_g = 3$ unless otherwise specified.

[3] This arbitrariness is actually responsible for most of the free parameters in the SM. This is the so-called *flavor problem*, one of the fundamental open questions in particle physics.

where $u_{L,R}$ and $d_{L,R}$ denote the 3×1 column matrices with the chiral components of the physical quark mass eigenstates. The 3×3 unitary matrices U_L^p and U_R^p are chosen such as to bi-diagonalize M_p (or, equivalently, Δ), while U_L^n and U_R^n bi-diagonalize M_n (or, equivalently, Γ):

$$\begin{aligned} U_L^{p\dagger} M_p U_R^p &= M_u = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \\ U_L^{n\dagger} M_n U_R^n &= M_d = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} \end{aligned} \tag{1.57}$$

The matrices M_u and M_d are, by definition, diagonal; their diagonal elements are real and non-negative.

In the leptonic sector, we bi-diagonalize M_l by performing unitary transformations of the fields, analogously to what is done in the quark sector. However, as the neutrinos are massless in the SM, we are free to transform them in such a way that a mixing matrix does not arise in the leptonic sector [4]:

$$\begin{aligned} \nu_L &= U_L^l \nu_L, \\ l_L &= U_L^l e_L, \\ l_R &= U_R^l e_R, \end{aligned} \tag{1.58}$$

where e and ν denote the mass eigenstates of the leptons. The unitary matrices U_L^l and U_R^l are chosen such that

$$U_L^{l\dagger} M_l U_R^l = M_e = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \tag{1.59}$$

If we define the Hermitian matrices

$$H_p \equiv M_p M_p^\dagger \,, \text{ and } H_n \equiv M_n M_n^\dagger \tag{1.60}$$

then we realize that the unitary matrices U_L^p and U_L^n diagonalize H_p and H_n:

$$U_L^{p\dagger} H_p U_L^p = M_u^2 \,, \text{ and } U_L^{n\dagger} H_n U_L^n = M_d^2 \tag{1.61}$$

In the quark sector, the NC interaction mediated by the Z boson and the photon is

$$\begin{aligned} \mathcal{L}_{\text{NC}}^{(q)} &= \frac{g_2}{2c_W} Z^\mu (\overline{p}_L \gamma_\mu p_L - \overline{n}_L \gamma_\mu n_L) + \left(eA^\mu - \frac{g_2 s_W^2}{c_W} Z^\mu \right) \\ &\times \left[\frac{2}{3} (\overline{p}_L \gamma_\mu p_L + \overline{p}_R \gamma_\mu p_R) - \frac{1}{3} (\overline{n}_L \gamma_\mu n_L + \overline{n}_R \gamma_\mu n_R) \right] \end{aligned} \tag{1.62}$$

[4] With the observation of non-zero neutrino masses and neutrino oscillations, this picture has to be modified to allow a mixing matrix for the neutrinos.

and the CC interaction mediated by the $W^{\pm}$ bosons is

$$\mathcal{L}_{\rm CC}^{(q)} = \frac{g_2}{\sqrt{2}}(W^{+\mu}\overline{p}_L\gamma_\mu n_L + W^{-\mu}\overline{n}_L\gamma_\mu p_L) \tag{1.63}$$

In the leptonic sector, these interactions are

$$\begin{aligned}\mathcal{L}_{\rm NC}^{(l)} &= \frac{g_2}{2c_W}Z^\mu(\overline{\nu}_L\gamma_\mu\nu_L - \bar{l}_L\gamma_\mu l_L) + \left(eA^\mu + \frac{g_2 s_W^2}{c_W}Z^\mu\right)(\bar{l}_L\gamma_\mu l_L + \bar{l}_R\gamma_\mu l_R) \\ \text{and } \mathcal{L}_{\rm CC}^{(l)} &= \frac{g_2}{\sqrt{2}}(W^{+\mu}\overline{\nu}_L\gamma_\mu l_L + W^{-\mu}\bar{l}_L\gamma_\mu\nu_L) \end{aligned} \tag{1.64}$$

Written in terms of the quark mass eigenstate, the CC interaction in Eq. (1.63) is

$$\mathcal{L}_{\rm CC}^{(q)} = \frac{g_2}{\sqrt{2}}((W^{+\mu}\overline{u}_L\gamma_\mu V d_L + W^{-\mu}\overline{d}_L\gamma_\mu V^\dagger u_L), \tag{1.65}$$

where

$$V = U_L^{p^\dagger} U_L^n \tag{1.66}$$

is the *Cabibbo-Kobayashi-Maskawa (CKM) matrix* [8, 9]. The appearance of a non-trivial CKM matrix in the CC reflects the fact that the Hermitian matrices H_p and H_n, defined by Eqs. (1.60), are in general diagonalized by different unitary matrices. Thus, if we start with u-type quarks being mass eigenstates, then in the d-type quark sector, the current eigenstates $|d'\rangle$ and the mass eigenstates $|d\rangle$ are connected by a unitary transformation

$$\begin{pmatrix} |d'\rangle \\ |s'\rangle \\ |b'\rangle \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} |d\rangle \\ |s\rangle \\ |b\rangle \end{pmatrix} \tag{1.67}$$

The NC Lagrangian preserves the form in Eq. (1.62), with the weak eigenstates $p_{L,R}$ and $n_{L,R}$ substituted by the mass eigenstates $u_{L,R}$ and $d_{L,R}$, respectively. This means that no mixing matrix analogous to V arises in the NC sector. This is the *GIM mechanism* [10] which ensures a natural absence of *flavor changing neutral currents* (FCNC) at tree-level in the SM.

In terms of the lepton mass eigenstates, both the CC and NC interactions in Eqs. (1.64) preserve their form, with the weak eigenstates $l_{L,R}$ and ν_L substituted by the mass eigenstates $e_{L,R}$ and ν_L, respectively. This is due to the fact that there is no mass matrix for the neutrinos in the SM if neutrinos are assumed to be massless.

1.5 MASS AND COUPLINGS OF THE HIGGS BOSON

The kinetic part of the Higgs field, $\frac{1}{2}(\partial_\mu H)^2$, comes from the covariant derivative-term cf. Eq. (1.38), while the mass and self-interaction parts come from the scalar potential

$$V_H(\Phi) = \mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2$$

$$= \frac{1}{2}\mu^2 \begin{pmatrix} 0 & v+H \end{pmatrix} \begin{pmatrix} 0 \\ v+H \end{pmatrix} + \frac{1}{4}\lambda \left| \begin{pmatrix} 0 & v+H \end{pmatrix} \begin{pmatrix} 0 \\ v+H \end{pmatrix} \right|^2$$
$$= -\frac{1}{2}\lambda v^2 (v+H)^2 + \frac{1}{4}\lambda (v+H)^4 \quad (1.68)$$

using the relation $v^2 = \frac{-\mu^2}{\lambda}$. The corresponding Lagrangian is given by

$$\mathcal{L}_{\text{Higgs}} = \frac{1}{2}(\partial_\mu H)^2 - V_H$$
$$= \frac{1}{2}(\partial_\mu H)^2 - \frac{1}{2}2\lambda v^2 H^2 - \lambda v H^3 - \frac{\lambda}{4}H^4 \quad (1.69)$$

From this Lagrangian, the Higgs boson mass simply reads

$$m_H = \sqrt{2\lambda}v = \sqrt{-2\mu^2} \quad (1.70)$$

The couplings of the Higgs boson to gauge bosons and fermions can be obtained from the relevant terms in the Lagrangian cf. Eqs. (1.38) and (1.55):

$$\mathcal{L}_{m_V} \sim m_V^2 \left(1 + \frac{H}{v}\right)^2 , \ \mathcal{L}_{m_f} \sim -m_f \left(1 + \frac{H}{v}\right) \quad (1.71)$$

The Feynman rules for Higgs boson couplings to gauge bosons and fermions are then given by

$$g_{Hff} = i\frac{m_f}{v} , \ g_{HVV} = -2i\frac{m_V^2}{v} \quad (1.72)$$

As with the gauge bosons, the Higgs boson couples to fermions with a strength proportional to their mass.

The VEV v is fixed in terms of the W boson mass m_W or the Fermi constant G_F determined from muon decay:

$$m_W = \frac{1}{2}g_2 v = \left(\frac{\sqrt{2}g_2^2}{8G_F}\right)^{1/2} \quad (1.73)$$

which implies that

$$v = \left(\sqrt{2}G_F\right)^{-1/2} \sim 246 \text{ GeV} \quad (1.74)$$

with $G_F = 1.16637 \times 10^{-5}\text{GeV}^{-2}$ [11].

The SM does not predict any value for the Higgs mass. However, the couplings of the Higgs boson to all the SM particles are known precisely and since the Higgs boson contributes to the radiative corrections to the high-precision electroweak observables, the electroweak precision measurements allow rather stringent constraints on m_H. There are also constraints from direct searches of the Higgs boson at colliders and in particular at LEP [12]. Taking into account all the precision electroweak data, one obtains the mass range of the SM Higgs boson at the 1σ level [12, 13]

$$m_H = 114^{+69}_{-45}\ \text{GeV} \tag{1.75}$$

leading to a 95% confidence level upper limit in the SM

$$m_H < 260\ \text{GeV} \tag{1.76}$$

Direct searches, on the other hand, put the lower bound

$$m_H \geq 114.4\ \text{GeV} \tag{1.77}$$

Thus it appears that the high-precision data clearly favor a light Higgs boson with a central value of mass that is very close to the present lower bound from direct searches. This is indeed very encouraging for the next-generation collider experiments.

1.6 CP VIOLATION

CP violation is an intriguing subject and our current knowledge of it is rather limited, both at the experimental and theoretical levels. Here we present a brief overview of the phenomenon of *CP* violation. For detailed discussion on the subject, we refer to [14].

Before discussing *CP* violation, let us first define the discrete symmetry transformations *P*, *T* and *C*:

- **Parity** (*P*) transformation consists of changing the sign of the space coordinates x, y, and z which changes the handedness of the system.
- **Time-reversal** (*T*) transformation consists of changing the sign of the time coordinate t while keeping the space coordinates unchanged.
- **Charge-conjugation** (*C*) transformation, contrary to *P* and *T*, does not have an analogue in classical physics. This symmetry is related to the existence of an antiparticle for every particle – a prediction of relativistic quantum field theory which has been brilliantly confirmed by experiment. The *C* transformation consists of changing the particle field ψ into a related field $\psi^\dagger$ which has opposite $U(1)$ charges – electric charge, baryon and lepton number, and flavor quantum numbers such as strangeness, the third component of isospin, and so on.

It turns out that the *P* and *C* symmetries are conserved in strong and electromagnetic interactions; but not in weak interaction. For the charged current of weak interaction, parity violation is maximal; the charged current only couples to left-handed fermions and right-handed antifermions. The neutral weak current is partially parity violating; it couples to left-handed and right-handed fermions and antifermions, but with different strengths. Similarly, charge conjugation is also not a symmetry of the weak interaction: when applied to a neutrino (which always comes as left-handed), C gives a left-handed anti-neutrino, which does not exist.

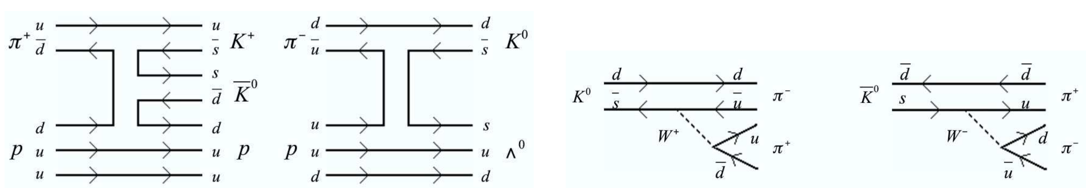
On the other hand, the combined CP symmetry is preserved in most weak interactions as cross sections and decay rates remain unchanged under the simultaneous C and P transformation. However in 1964, it was found that the CP symmetry is minimally violated (1 part in 10^3) in neutral kaon decay [15].

1.6.1 *CP* VIOLATION IN NEUTRAL KAON DECAY

Neutral kaons are typically produced via strong interactions, in eigenstates of strangeness (K^0 and $\overline{K}^0$):

$$\pi^+ p \to \overline{K}^0 K^+ p, \text{ and } \pi^- p \to K^0 \Lambda$$

as shown in Figure 1.3(a). But they decay via weak interactions into 2π and 3π modes shown in Figure 1.3(b) which are CP eigenstates with eigenvalues $+1$ and -1 respectively. The K^0 and $\overline{K}^0$ are not themselves CP eigenstates, but we can form CP eigen-



(a) (b)

Figure 1.3: (a) Production and (b) decay modes of neutral kaons K^0 and $\overline{K}^0$.

states by taking their linear combinations as follows:

$$|K_1\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle) \text{ and } |K_2\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle)$$

with CP eigenvalues $+1$ and -1 respectively. Assuming CP is conserved in weak interactions, K_1 can only decay into two pions while K_2 can only decay into three pions. Now the 2π decay is much faster, because the energy released is greater. Hence if we start with a beam of K^0's

$$|K^0\rangle = \frac{1}{\sqrt{2}}(|K_1\rangle + |K_2\rangle)$$

the K_1 component will quickly decay away, leaving only a beam of pure K_2's. Two such decay modes for K^0 with very different lifetimes were indeed observed [16], and experimentally, the two lifetimes were measured to be

$$\tau(K_S^0) = 0.89 \times 10^{-10}\text{sec} \;\; \text{, and} \;\; \tau(K_L^0) = 5.2 \times 10^{-8}\text{sec}$$

where the subscripts L and S stand for long and short due to their different lifetimes. By using a long enough beam, we can produce an arbitrarily pure sample of the long-lived species K_L. If CP were a perfect symmetry, then we would expect K_L to decay only into 3π modes, and not into 2π modes. However, it was found that a tiny fraction (roughly one in 500) of K_L decays into the 2π mode [15], but this is an unmistakable evidence of CP violation. Evidently the long-lived neutral kaon is not the perfect CP eigenstate K_2 after all, but contains a small admixture of K_1:

$$|K_L\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}(|K_2\rangle + \epsilon|K_1\rangle)$$

The coefficient ϵ is a measure of the CP violation; experimentally its magnitude is $(2.284 \pm 0.014) \times 10^{-3}$ [11].

By now, CP violation has also been experimentally confirmed in $B^0 \leftrightarrow \overline{B}^0$ system [17] and recently it was observed in $D^0 \leftrightarrow \overline{D}^0$ system.

1.6.2 THE CKM MODEL

With the success of gauge theories in explaining the fundamental interactions, the problem of constructing models which incorporate CP violation became more systematic and better defined. A pure gauge Lagrangian is necessarily CP-invariant [18]. The scalar potential of the SM, in which only one Higgs doublet exists, automatically conserves CP. As a result, CP violation can only arise from the simultaneous presence of Yukawa interactions and gauge interactions. The *CKM model* [8, 9] explicitly introduces complex coefficients in the Yukawa Lagrangian (1.52) in order to have CP violation.

The basic idea behind the CKM model is that the flavor eigenstates are not mass eigenstates for the down-type quarks. This idea was first used by Cabibbo [8] to explain the semi-leptonic hadron decays which yield a smaller value for the weak coupling than that obtained from muon decay life time. If a d-quark is transformed into a u-quark, as in the β-decay of neutron, the coupling constant appears to be about 4% smaller as compared to that in muon decay. In processes in which an s-quark is transformed into a u-quark, as in Λ^0 decay, it even appears to be 20 times smaller. Cabibbo proposed that quark transitions occur not only within a family but also, to a lesser degree, from one family to another. For charged currents, the "partner" of the flavor eigenstate $|u\rangle$ is therefore not the flavor eigenstate $|d\rangle$, but a linear combination of $|d\rangle$ and $|s\rangle$. We call this combination $|d'\rangle$. Similarly we denote the partner of the $|c\rangle$ state as $|s'\rangle$. The coefficients of these linear combinations can be written as the cosine and sine of an angle called the *Cabibbo angle* θ_C. The quark eigenstates $|d'\rangle$ and $|s'\rangle$ of W exchange are related to the flavor eigenstates $|d\rangle$ and $|s\rangle$ of the strong interaction, by a rotation through θ_C:

$$\begin{pmatrix} |d'\rangle \\ |s'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} |d\rangle \\ |s\rangle \end{pmatrix} \tag{1.78}$$

Experimentally, θ_C is determined by comparing the lifetimes and branching ratios of the semi-leptonic and hadronic decays of various particles. This yields [11]:

$$sin\theta_C \approx 0.22 \;\; , \quad \text{and} \quad \cos\theta_C \approx 0.98$$

The transitions $c \leftrightarrow d$ and $s \leftrightarrow u$, as compared to $c \leftrightarrow s$ and $d \leftrightarrow u$ respectively, are therefore suppressed by a factor of

$$\sin^2\theta_C : \cos^2\theta_C \approx 1 : 20$$

Now adding the third generation of quarks, the 2×2 matrix of (1.78) is replaced by the 3×3 matrix of (1.67). The CKM matrix V is complex, but some of the phases in it do not have physical meaning. In the SM with n_g generations V is an $n_g \times n_g$ unitary matrix, cf. Eq. (1.66). It would, therefore, in general, be parametrized by n_g^2 parameters. However, $(2n_g - 1)$ phases can be absorbed by rephasing all quark fields. Therefore, the number of physical parameters in V is

$$N_{\text{parameter}} = n_g^2 - (2n_g - 1) = (n_g - 1)^2 \tag{1.79}$$

An $n_g \times n_g$ orthogonal matrix is parametrized by $\frac{1}{2}n_g(n_g - 1)$ rotation angles, sometimes called the *Euler angles*. An unitary matrix is a complex extension of an orthogonal matrix; therefore, out of the $N_{\text{parameter}}$ parameters of V,

$$N_{\text{angle}} = \frac{1}{2}n_g(n_g - 1) \tag{1.80}$$

should be identified with rotation angles. The remaining

$$N_{\text{phase}} = N_{\text{parameter}} - N_{\text{angle}} = \frac{1}{2}(n_g - 1)(n_g - 2) \tag{1.81}$$

parameters of V are physical phases which can not be rotated away by redefinition. According to Eqs. (1.80) and (1.81), there are three rotation angles and one physical phase in V for $n_g = 3$. CP violation in the SM is attributed to the existence of this phase, as was first pointed out by Kobayashi and Maskawa [9]. They parametrized the

CKM matrix by means of three Euler angles – the angles of three successive rotations about different axes – and one phase:

$$\begin{aligned} V &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_2 & -s_2 \\ 0 & s_2 & c_2 \end{pmatrix} \begin{pmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & s_3 & -c_3 \end{pmatrix} \\ &= \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} \end{aligned} \tag{1.82}$$

where c_i and s_i are shorthands for $\cos\theta_i$ and $\sin\theta_i$, respectively ($i = 1,2,3$). The phase δ appears as a rephasing of the third generation; as the rephasing occurs in between two rotations involving that generation, it is impossible to identify δ with a rephasing of the quark fields.

Physically meaningful quantities must be invariant under a rephasing of the fields. Experimentally, the simplest rephasing-invariant functions of the CKM matrix that can be measured are the moduli of its matrix elements: $|V_{ij}|$. The allowed ranges of the magnitudes of these moduli are [11]

$$\left(|V_{ij}|\right) = \begin{pmatrix} 0.97383^{+0.00024}_{-0.00023} & 0.2272^{+0.0010}_{-0.0010} & (3.96^{+0.09}_{-0.09}) \times 10^{-3} \\ 0.2271^{+0.0010}_{-0.0010} & 0.97296^{+0.00024}_{-0.00024} & (42.21^{+0.10}_{-0.80}) \times 10^{-3} \\ (8.14^{+0.32}_{-0.64}) \times 10^{-3} & (41.61^{+0.12}_{-0.78}) \times 10^{-3} & 0.999100^{+0.000034}_{-0.000004} \end{pmatrix} \tag{1.83}$$

All the observed CP violation so far can be *completely* explained in the CKM picture [19].

1.7 PHYSICS BEYOND THE STANDARD MODEL

Except for the Higgs mass, all the parameters of the SM, viz. the three gauge coupling constants, the masses of weak vector bosons and fermions as well as the quark mixing angles, have been determined experimentally to an extremely high degree of accuracy over the last two decades or so, reaching its high point in the precision measurements at the CERN e^+e^- collider LEP [12]. However, there are strong conceptual as well as experimental indications for physics beyond the SM [20]. In this section, we discuss some of these points:

From the theoretical standpoint, CP violation can be incorporated in the three-generation SM by introducing the CKM mixing matrix. However, we lack a fundamental understanding of the origin of CP violation. This is all the more important, because CP violation is one of the crucial ingredients necessary to generate the observed matter-antimatter asymmetry of the Universe [21]. It is now believed that it is not possible to generate a baryon asymmetry of the observed size exclusively with the CP violation present in the SM [22]. A dynamically-generated matter-antimatter asymmetry of the universe requires additional sources of CP violation, and such sources naturally exist in the extensions of the SM.

One of the most crucial problems with the SM is the radiative instability of the Higgs mass [23]. The one loop corrections to Higgs mass are quadratically divergent, i.e. proportional to Λ^2, with Λ being the cut-off scale of the theory $\sim 10^{19}$ GeV (the *Planck scale*). The counter term necessary to cancel this divergence and to yield results of order 100 GeV (the electroweak scale), requires a fine tuning of the Higgs self-coupling to one part in 10^{34}. Two loop correction to the mass would require further fine tuning of the similar order. In other words, the self coupling of the Higgs boson has to be fine-tuned for the Higgs mass to be of order $\mathcal{O}(100 \text{ GeV})$, or else the Higgs mass would be raised to the cut-off scale by radiative corrections. This is called the *naturalness problem* of the SM Higgs boson. This can be solved if we have logarithmic divergences instead of quadratic ones in an extension of the SM.

It is considered highly implausible that the origin of the electroweak symmetry breaking can be explained by the standard Higgs mechanism, without accompanying any new phenomena. This conclusion follows from an extrapolation of the SM at very high energies. The computed behavior of the SM couplings with energy clearly points towards the unification of the electroweak and strong forces at scales of energy $M_{GUT} \sim 10^{15}$ GeV which are close to the scale of quantum gravity, $M_{Pl} \sim 10^{19}$ GeV [24]. It seems unlikely that the SM without new physics will be valid up to such large energies because the structure of the SM could not then naturally explain the relative smallness of the electroweak mass scale, set by the Higgs mechanism at $v \sim 246$ GeV, cf. Eq. (1.74), w.r.t. the unification mass scale $\sim 10^{15}$ GeV. This is the so-called *hierarchy problem* and is related to the naturalness problem discussed in the above paragraph.

There are certainly many more conceptual problems associated with the SM: the proliferation of parameters, the non-trivial pattern of fermion masses and so on. But while most of these problems can be postponed to the final theory that will take over at Planck scale, the hierarchy problem requires a solution at relatively low energies. Several models have been put forward to address these issues and to explore the new physics effects beyond the SM, albeit preserving all virtues of the SM. The most common one is the *Supersymmetric Standard Model* which apart from curing the hierarchy problem, also provides ready-made *cold dark matter candidates* [25].

1.8 MOTIVATION AND PLAN FOR THE WORK

Although the SM has been a phenomenal success, its bosonic sector is not yet completely verified. So far there has been no *direct* experimental evidence of the phenomenon of spontaneous symmetry breaking in the $SU(2) \otimes U(1)$ sector. With the Higgs mechanism being considered a cornerstone of the SM and its various extensions, search for the Higgs boson and study of its various properties are obviously the main aims for all the current and next generation colliders [26]. The Large Hadron Collider (LHC) is expected to be capable of searching for the Higgs boson in the entire mass range allowed theoretically. Once the Higgs is detected at the LHC, the next generation Linear Collider (LC) can provide a wealth of precise experimental information on its properties. One can study important synergistic effects arising from the interplay of LHC and LC [27].

1.8.1 IMPORTANCE OF STUDIES OF THE CP QUANTUM NUMBER OF THE HIGGS BOSON

Just the discovery of the Higgs boson at the LHC will not anyway be sufficient to validate the minimal SM. For one, the only fundamental neutral scalar in the SM is a $J^{CP} = 0^{++}$ state arising from a $SU(2)_L$ doublet with hypercharge 1, while its various extensions can have several Higgs bosons with different CP properties and $U(1)$ quantum numbers. The minimal supersymmetric standard model (MSSM), for example, has two CP-even states and a single CP-odd one [25]. Thus should a neutral spin-0 particle be detected at the LHC, a study of its CP-properties would be essential to establish it as *the* SM Higgs boson [28]. Calculating the CP eigenvalue(s) for the Higgs state(s) if CP is conserved, and measuring the mixing between the CP-even and CP-odd states if it is not is a major aim of collider physics experiments. CP violation in the Higgs sector is indeed an interesting option to generate CP violation beyond the SM [29] which has important implications for cosmology, for instance, by possibly helping to explain the observed baryon asymmetry in the universe.

1.8.2 IMPORTANCE OF TOP-QUARK IN STUDIES OF PROPERTIES OF THE HIGGS BOSON

Top-quark is the heaviest fundamental particle detected so far [30]. Its large mass ($m_t \sim 175$ GeV), being very close to the electroweak symmetry breaking scale ($v \sim 246$ GeV), is expected to provide a probe to understand the dynamics of electroweak symmetry breaking. Further, due to its large mass, and hence, large decay width ($\Gamma_t \sim 1.5$ GeV), its life time is much smaller than the typical hadronization time scale; hence its decay occurs much before hadronization and the decay process is not influenced by fragmentation effects [31]. Thus its polarization can be studied through the energy and angular distribution of its decay products. The angular distribution of the decay lepton in particular is independent of any non-standard effects in the decay vertex, and hence, is a true probe of these non-standard effects associated with the t-production [32].

Within the SM, the three generations of fermions are treated identically. Thus one expects the couplings of all the three generations of fermions to gauge bosons to be same at tree level. Any difference at higher order is due to the differences in the masses of the fermions. Due to the large difference between masses of the first two and the third generation of fermions (which is still an unsolved mystery), some theories of dynamical electroweak symmetry breaking treat the third generation fermions differently from the first two generations. Thus a precision study of t-quark mass and couplings can verify such models of dynamical symmetry breaking. Moreover, since the coupling of the Higgs boson to a $t\bar{t}$ pair is proportional to m_t, cf. Eq. (1.72), an experimental verification of this fact will serve as a very good test of the Higgs mechanism of SSB, whereas a deviation of the top quark Yukawa coupling from its SM value would be a signal for new physics.

It has been shown [33] that the Kobayashi-Maskawa mechanism of CP violation predicts a negligibly small effect for processes involving the top quark in the SM[5], and thus, the standard CP-violation effects in top quark production and decays will be unobservable in collider experiments. Therefore, the top quark system will be sensitive and may serve as a powerful probe to CP-violation due to the New Physics effects [34].

In the light of the above discussion, we re-state that our aim is to analyze the CP properties of the Higgs boson. A pair of fermions can couple to CP-even and -odd Higgs states with comparable strength, so can a pair of photons or gluons. Thus, reactions involving these interactions are the best place to study the CP properties of a Higgs boson. The case of massive weak bosons is, however, different. W and Z bosons couple to the CP-even Higgs state at tree level and to the CP-odd state only at one loop and higher level. Thus, reactions involving VVH couplings are less sensitive to the possible CP-mixing in the Higgs sector though they can be utilized to study possible anomalous couplings of Higgs boson [35]. Further, Higgs boson mixes the different chiralities of fermions, while vectorial interactions preserve it. Thus the presence of a Higgs boson can also be seen through the polarization of a heavy fermion like top quark. In addition, the effects of new physics on various observables can be enhanced by appropriately choosing the initial beam polarizations.

[5] This is primarily due to the fact that its large mass in comparison to other quarks renders the GIM [10] cancellation particularly effective.

CHAPTER 2

ANOMALOUS HIGGS COUPLINGS

In order to identify the CP nature of a Higgs boson, we must probe the structure of its couplings to known particles, in either its production or decay. The couplings of the SM Higgs boson with other SM particles are determined by the quantum numbers and masses of those particles. However, as we have discussed in §1.7, a first principle understanding of the phenomenon of CP violation and the stabilization of Higgs mass are some of the reasons which require us to look beyond the SM. Any such theory beyond the SM, which tries to stabilize the Higgs mass and/or explain the emergence of CP-violation, invariably introduces new particles. These new particles, if they have anything to do with the electroweak symmetry breaking, could have large couplings with the t-quark owing to its large mass.

Here we adopt a *model-independent approach* which entails uniting the most general Higgs boson interactions with other SM particles consistent with various invariance principles. For example, at tree level, the most general Lorentz invariant form of $f\bar{f}\phi$ coupling for a neutral Higgs boson ϕ is given by:

$$g_{f\bar{f}\phi} = -ig_2 \frac{m_f}{2m_W}(a + ib\gamma_5) \tag{2.1}$$

where $a,\ b$ give the Yukawa coupling strengths relative to that of a SM Higgs boson. In the SM, we have a purely CP-even Higgs with $a = 1$ and $b = 0$. For a purely CP-odd Higgs boson, $a = 0$ and $b \neq 0$, with the magnitude of b depending on the model. In CP-violating models, both a and b may be non-zero at tree level and may be of comparable strengths.

Similarly, the most general form for the coupling of a Higgs boson with a pair of gauge bosons can be expressed as

$$(g_{VV\phi})_{\mu\nu} = -ig_V \left[a_V g_{\mu\nu} + \frac{b_V}{\Lambda_V^2}(k_{1\mu}k_{2\nu} - g_{\mu\nu}k_1 \cdot k_2) + \frac{\tilde{b}_V}{\Lambda_V^2}\epsilon_{\mu\nu\alpha\beta}k_1^{\alpha}k_2^{\beta} \right], \tag{2.2}$$

where k_i denote the momenta of the two V's, Λ_V is some high energy scale which induces the non-standard $VV\phi$ couplings in the low energy effective Lagrangian, $g_W^{SM} =$

$e\cot\theta_W m_Z$ and $g_Z^{SM} = \frac{g_2 m_Z}{\cos\theta_W}$. In the context of the SM, at tree level, $a_W^{SM} = a_Z^{SM} = 1$ while all other couplings vanish identically. For the massless gauge bosons, viz. photon and gluon, the coupling a_V remains zero even at higher loops. The anomalous couplings b_V and $\tilde{b}_V$ usually appear either at higher order in perturbative expansion of a renormalizable theory [36] or even at tree level in some effective theory with higher dimensional operators [37].

2.1 DIMENSION-SIX OPERATORS

Collider experiments have been used to search for the new particles predicted by various new physics (NP) models, but no such direct signal has been observed so far. So, if NP indeed exists above the electroweak scale, it is very likely that the only observable effects at energies not too far above the electroweak scale could be in the form of new residual interactions affecting the couplings of the third-family quarks, and the untested sectors of the Higgs and gauge bosons. In this spirit, the NP effects can be expressed as non standard terms in an effective Lagrangian which contains the most general interactions compatible with the symmetries that we would like to impose on the model. Such interactions are of course non-renormalizable, but they must obey the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge symmetry as well as the Lorentz invariance.

Below the NP scale, the new residual interactions can be parametrized by the effective non-renormalizable $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge invariant Lagrangian [38]

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \sum_{n\geq 3}\sum_i \frac{C_i}{\Lambda^{2(n-2)}}\mathcal{O}_i^{(2n)} \tag{2.3}$$

where in addition to the SM piece, we have introduced the higher order operators $\mathcal{O}_i$ of dimension $2n$. The coefficients C_i are constants which represent the coupling strengths of $\mathcal{O}_i$, and are expected to be of order of 1. Assuming the NP energy scale to be larger than the accessible energy of the colliders, $\Lambda \geq \mathcal{O}$ (1 TeV), the series in Eq (2.3) can be truncated at $n = 3$ for energies in the vicinity of the electroweak symmetry breaking scale ($v \sim 246$ GeV). In this case, only the operators of dimension-6 need to be considered in practice [1]. These operators will not only contribute to the three-point $f\bar{f}\phi$ couplings, but will also induce new four-point couplings $Zf\bar{f}\phi$ and $\gamma f\bar{f}\phi$ which are absent in the SM at tree level. The contributions of these operators can be viewed as the corrections to the SM couplings.

We restrict ourselves to the case of t-quark only. Before electroweak symmetry breaking, the effective Lagrangian for Higgs couplings to a pair of t-quarks can

[1] The contributions of the operators with yet higher dimensions will be suppressed by additional powers of v^2/Λ^2.

be written as cf. Eq (2.3)

$$\mathcal{L}_{eff}(f\bar{f}\phi) = \sum_i \frac{C_i}{\Lambda^2}\mathcal{O}_i + \sum_i \frac{\overline{C}_i}{\Lambda^2}\overline{\mathcal{O}}_i \tag{2.4}$$

where $\mathcal{O}_i$ and $\overline{\mathcal{O}_i}$ are the CP-even and CP-odd operators respectively[2]. There are 14 such operators: seven $\mathcal{O}_i$ for CP-even [39] and seven $\bar{\mathcal{O}}_i$ for CP-odd [40], as listed below. For a complete list of all possible dimension-6 CP conserving operators, see [41].

$$\mathcal{O}_{t1} = \left(\phi^\dagger\phi - \frac{v^2}{2}\right)\left[\bar{q}_L t_R \tilde{\phi} + \tilde{\phi}^\dagger \bar{t}_R q_L\right] \tag{2.5}$$

$$\mathcal{O}_{t2} = i\left[\phi^\dagger D_\mu \phi - (D_\mu\phi)^\dagger \phi\right] \bar{t}_R \gamma^\mu t_R \tag{2.6}$$

$$\mathcal{O}_{Dt} = (\bar{q}_L D_\mu t_R)\, D^\mu \tilde{\phi} + \left(D^\mu \tilde{\phi}\right)^\dagger \left(\overline{D_\mu t_R} q_L\right) \tag{2.7}$$

$$\mathcal{O}_{tW\phi} = \left[\left(\bar{q}_L \sigma^{\mu\nu}\tau^I t_R\right)\tilde{\phi} + \tilde{\phi}^\dagger\left(\bar{t}_R\sigma^{\mu\nu}\tau^I q_L\right)\right] W^I_{\mu\nu} \tag{2.8}$$

$$\mathcal{O}_{tB\phi} = \left[(\bar{q}_L \sigma^{\mu\nu} t_R)\,\tilde{\phi} + \tilde{\phi}^\dagger\,(\bar{t}_R\sigma^{\mu\nu} q_L)\right] B_{\mu\nu} \tag{2.9}$$

$$\mathcal{O}^{(1)}_{\phi q} = i\left[\phi^\dagger D_\mu\phi - (D_\mu\phi)^\dagger\,\phi\right] \bar{q}_L\gamma^\mu q_L \tag{2.10}$$

$$\mathcal{O}^{(3)}_{\phi q} = i\left[\phi^\dagger \tau^I D_\mu\phi - (D_\mu\phi)^\dagger\,\tau^I\phi\right] \bar{q}_L\gamma^\mu \tau^I q_L \tag{2.11}$$

and

$$\overline{\mathcal{O}}_{t1} = i\left(\phi^\dagger\phi - \frac{v^2}{2}\right)\left[\bar{q}_L t_R \tilde{\phi} - \tilde{\phi}^\dagger \bar{t}_R q_L\right] \tag{2.12}$$

$$\overline{\mathcal{O}}_{t2} = \left[\phi^\dagger D_\mu \phi + (D_\mu\phi)^\dagger \phi\right] \bar{t}_R \gamma^\mu t_R \tag{2.13}$$

$$\overline{\mathcal{O}}_{Dt} = i\left[(\bar{q}_L D_\mu t_R)\, D^\mu \tilde{\phi} - \left(D^\mu \tilde{\phi}\right)^\dagger \left(\overline{D_\mu t_R} q_L\right)\right] \tag{2.14}$$

$$\overline{\mathcal{O}}_{tW\phi} = i\left[\left(\bar{q}_L \sigma^{\mu\nu}\tau^I t_R\right)\tilde{\phi} - \tilde{\phi}^\dagger\left(\bar{t}_R\sigma^{\mu\nu}\tau^I q_L\right)\right] W^I_{\mu\nu} \tag{2.15}$$

$$\overline{\mathcal{O}}_{tB\phi} = i\left[(\bar{q}_L \sigma^{\mu\nu} t_R)\,\tilde{\phi} - \tilde{\phi}^\dagger\,(\bar{t}_R\sigma^{\mu\nu} q_L)\right] B_{\mu\nu} \tag{2.16}$$

$$\overline{\mathcal{O}}^{(1)}_{\phi q} = \left[\phi^\dagger D_\mu\phi + (D_\mu\phi)^\dagger\,\phi\right] \bar{q}_L\gamma^\mu q_L \tag{2.17}$$

$$\overline{\mathcal{O}}^{(3)}_{\phi q} = \left[\phi^\dagger \tau^I D_\mu\phi + (D_\mu\phi)^\dagger\,\tau^I\phi\right] \bar{q}_L\gamma^\mu \tau^I q_L \tag{2.18}$$

where ϕ is the Higgs doublet field and $\tilde{\phi} = i\tau^2\phi^*$ its conjugate field. q_L is the left-handed third-family quark doublet:

$$q_L = \begin{pmatrix} t_L \\ b_L \end{pmatrix}, \quad \bar{q}_L = (\bar{t}_L,\, \bar{b}_L)$$

[2]For CP eigenstates, a pure Higgs scalar will be denoted by H and a pure pseudoscalar by A. Otherwise the generic notation ϕ will be used for a Higgs boson of indeterminate CP parity.

In order to shorten some of the expressions we shall use the following notation along with those given in §1.1:

$$W^{\pm,0}_{\mu\nu} = \partial_\mu W^{\pm,3}_\nu - \partial_\nu W^{\pm,3}_\mu \tag{2.19}$$

Using this notation, we can simplify Eqs. (2.5)-(2.18) to obtain

$$\mathcal{O}_{t1} = \frac{1}{2\sqrt{2}} h\,(h+2v)\,(h+v)\,(\bar{t}t) \tag{2.20}$$

$$\mathcal{O}_{t2} = -\frac{1}{2} g_Z\,(h+v)^2\,Z^\mu\,(\bar{t}_R\gamma_\mu t_R) \tag{2.21}$$

$$\begin{aligned}\mathcal{O}_{Dt} = {} & \frac{1}{2\sqrt{2}}(\partial^\mu h)\left[\partial_\mu(\bar{t}t) + \bar{t}\gamma_5(\partial_\mu t) - (\partial_\mu\bar{t})\gamma_5 t - i\frac{4}{3}g_1 B_\mu(\bar{t}\gamma_5 t)\right] \\ & - \frac{i}{4\sqrt{2}} g_Z\,(h+v)\,Z^\mu\left[\bar{t}(\partial_\mu t) - (\partial_\mu\bar{t})\,t + \partial_\mu(\bar{t}\gamma_5 t) - i\frac{4}{3}g_1 B_\mu(\bar{t}t)\right] \\ & - \frac{i}{2} g_2\,(h+v)\,W^-_\mu\left[\bar{b}_L(\partial^\mu t_R) - i\frac{2}{3}g_1 B^\mu(\bar{b}_L t_R)\right] \\ & + \frac{i}{2} g_2\,(h+v)\,W^+_\mu\left[(\partial^\mu\bar{t}_R)\,b_L + i\frac{2}{3}g_1 B^\mu(\bar{t}_R b_L)\right]\end{aligned} \tag{2.22}$$

$$\begin{aligned}\mathcal{O}_{tW\phi} = {} & \frac{1}{\sqrt{2}}(h+v)(\bar{t}\sigma^{\mu\nu}t)\left[W^0_{\mu\nu} - ig_2\left(W^+_\mu W^-_\nu - W^-_\mu W^+_\nu\right)\right] \\ & + (h+v)\left(\bar{b}_L\sigma^{\mu\nu}t_R\right)\left[W^-_{\mu\nu} - ig_2\left(W^-_\mu W^3_\nu - W^3_\mu W^-_\nu\right)\right] \\ & + (h+v)\left(\bar{t}_R\sigma^{\mu\nu}b_L\right)\left[W^+_{\mu\nu} - ig_2\left(W^3_\mu W^+_\nu - W^+_\mu W^3_\nu\right)\right]\end{aligned} \tag{2.23}$$

$$\mathcal{O}_{tB\phi} = \frac{1}{\sqrt{2}}(h+v)(\bar{t}\sigma^{\mu\nu}t)\,B_{\mu\nu} \tag{2.24}$$

$$\mathcal{O}^{(1)}_{\phi q} = -\frac{1}{2} g_Z\,(h+v)^2\,Z_\mu\left[\bar{t}_L\gamma^\mu t_L + \bar{b}_L\gamma^\mu b_L\right] \tag{2.25}$$

$$\begin{aligned}\mathcal{O}^{(3)}_{\phi q} = {} & \frac{1}{2} g_Z\,(h+v)^2\,Z_\mu\left[\bar{t}_L\gamma^\mu t_L - \bar{b}_L\gamma^\mu b_L\right] \\ & + \frac{1}{\sqrt{2}} g_2\,(h+v)^2\left[W^+_\mu(\bar{t}_L\gamma^\mu b_L) + W^-_\mu\left(\bar{b}_L\gamma^\mu t_L\right)\right]\end{aligned} \tag{2.26}$$

and

$$\overline{\mathcal{O}}_{t1} = \frac{i}{2\sqrt{2}} h\,(h+2v)\,(h+v)\,(\bar{t}\gamma_5 t) \tag{2.27}$$

$$\overline{\mathcal{O}}_{t2} = (h+v)\,(\partial^\mu h)\,(\bar{t}_R\gamma_\mu t_R) \tag{2.28}$$

$$\begin{aligned}\overline{\mathcal{O}}_{Dt} = {} & \frac{i}{2\sqrt{2}}(\partial^\mu h)\left[\bar{t}(\partial_\mu t) - (\partial_\mu\bar{t})\,t + \partial_\mu(\bar{t}\gamma_5 t) - i\frac{4}{3}g_1 B_\mu(\bar{t}t)\right] \\ & + \frac{1}{4\sqrt{2}} g_Z\,(h+v)\,Z^\mu\left[\partial_\mu(\bar{t}t) + \bar{t}\gamma_5(\partial_\mu t) - (\partial_\mu\bar{t})\gamma_5 t - i\frac{4}{3}g_1 B_\mu(\bar{t}\gamma_5 t)\right]\end{aligned}$$

$$+\frac{1}{2}g_2\,(h+v)\,W^-_\mu\left[\bar{b}_L\,(\partial^\mu t_R)-i\frac{2}{3}g_1B^\mu\,(\bar{b}_Lt_R)\right]$$
$$+\frac{1}{2}g_2\,(h+v)\,W^+_\mu\left[(\partial^\mu\bar{t}_R)\,b_L+i\frac{2}{3}g_1B^\mu\,(\bar{t}_Rb_L)\right] \quad (2.29)$$
$$\overline{\mathcal{O}}_{tW\phi} = \frac{i}{\sqrt{2}}\,(h+v)\,(\bar{t}\sigma^{\mu\nu}\gamma_5t)\left[W^0_{\mu\nu}-ig_2\left(W^+_\mu W^-_\nu-W^-_\mu W^+_\nu\right)\right]$$
$$+i\,(h+v)\,\left(\bar{b}_L\sigma^{\mu\nu}t_R\right)\left[W^-_{\mu\nu}-ig_2\left(W^-_\mu W^3_\nu-W^3_\mu W^-_\nu\right)\right]$$
$$-i\,(h+v)\,(\bar{t}_R\sigma^{\mu\nu}b_L)\left[W^+_{\mu\nu}-ig_2\left(W^3_\mu W^+_\nu-W^+_\mu W^3_\nu\right)\right] \quad (2.30)$$
$$\overline{\mathcal{O}}_{tB\phi} = \frac{i}{\sqrt{2}}\,(h+v)\,(\bar{t}\sigma^{\mu\nu}\gamma_5t)\,B_{\mu\nu} \quad (2.31)$$
$$\overline{\mathcal{O}}^{(1)}_{\phi q} = (h+v)\,(\partial_\mu h)\left[\bar{t}_L\gamma^\mu t_L+\bar{b}_L\gamma^\mu b_L\right] \quad (2.32)$$
$$\overline{\mathcal{O}}^{(3)}_{\phi q} = -\,(h+v)\,(\partial_\mu h)\left[\bar{t}_L\gamma^\mu t_L-\bar{b}_L\gamma^\mu b_L\right]$$
$$-\frac{i}{\sqrt{2}}g_2\,(h+v)^2\left(W^+_\mu\bar{t}_L\gamma^\mu b_L-W^-_\mu\bar{b}_L\gamma^\mu t_L\right) \quad (2.33)$$

The presence of derivatives induces an energy dependence of some of these couplings which is summarized in Table 2.1. Except for $\mathcal{O}_{t1}$, $\overline{\mathcal{O}}_{t1}$, $\mathcal{O}_{t2}$, $\mathcal{O}^{(1)}_{\phi Q}$ and $\mathcal{O}^{(3)}_{\phi Q}$, all the other operators are energy dependent. It must, however, be mentioned here that these energy dependences are not of so much importance for a given process as they do not give any information on the magnitudes of the anomalous couplings.

Operator	$t\bar{t}Z$	$t\bar{t}\gamma$	$t\bar{t}\phi$	$Zt\bar{t}\phi$	$\gamma t\bar{t}\phi$
$\mathcal{O}_{t1}$, $\overline{\mathcal{O}}_{t1}$			1		
$\mathcal{O}_{t2}$, $\mathcal{O}^{(1)}_{\phi Q}$, $\mathcal{O}^{(3)}_{\phi Q}$	1			$\frac{1}{v}$	
$\overline{\mathcal{O}}_{t2}$, $\overline{\mathcal{O}}^{(1)}_{\phi Q}$, $\overline{\mathcal{O}}^{(3)}_{\phi Q}$			$\frac{E}{v}$		
$\mathcal{O}_{Dt}$, $\overline{\mathcal{O}}_{Dt}$	$\frac{E}{v}$		$\frac{E^2}{v^2}$	$\frac{E}{v^2}$	$\frac{E}{v^2}$
$\mathcal{O}_{tW\phi}$, $\overline{\mathcal{O}}_{tW\phi}$, $\mathcal{O}_{tB\phi}$, $\overline{\mathcal{O}}_{tB\phi}$	$\frac{E}{v}$	$\frac{E}{v}$		$\frac{E}{v^2}$	$\frac{E}{v^2}$

Table 2.1: The energy dependence of the dimension-6 operators for the anomalous vertices. Here an overall normalization $\frac{v^2}{\Lambda^2}$ has been factored out

2.2 ANOMALOUS VERTICES

From the expressions (2.20) - (2.33) for the dimension-6 operators, the effective Lagrangians for the couplings $t\bar{t}Z$, $t\bar{t}\gamma$, $t\bar{t}\phi$, $Zt\bar{t}\phi$ and $\gamma t\bar{t}\phi$ can be derived using Eq.(2.4), as follows [3]: For CP-even operators, we have

$$\begin{aligned}
\mathcal{L}_{t\bar{t}Z} &= \frac{1}{\Lambda^2}\left[-C_{t2} v m_Z Z^\mu \bar{t}\gamma_\mu P_R t + C_{Dt}\left(\frac{-im_Z}{2\sqrt{2}}\right) Z^\mu \left[\partial_\mu\left(\bar{t}\gamma_5 t\right) + \bar{t}\partial_\mu t - (\partial_\mu \bar{t})t\right]\right. \\
&\quad + C_{tW\phi} v(\sqrt{2}\cos\theta_W)(\bar{t}\sigma^{\mu\nu}t)\partial_\mu Z_\nu - C_{tB\phi} v(\sqrt{2}\sin\theta_W)(\bar{t}\sigma^{\mu\nu}t)\partial_\mu Z_\nu \\
&\quad \left. - v m_Z \left(C^{(1)}_{\phi q} - C^{(3)}_{\phi q}\right) Z^\mu (\bar{t}\gamma_\mu P_L t)\right] \qquad (2.34)
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{t\bar{t}\gamma} &= \frac{1}{\Lambda^2}\left[C_{tW\phi} v(\sqrt{2}\sin\theta_W)(\bar{t}\sigma^{\mu\nu}t)\partial_\mu A_\nu\right. \\
&\quad \left. + C_{tB\phi} v(\sqrt{2}\cos\theta_W)(\bar{t}\sigma^{\mu\nu}t)\partial_\mu A_\nu\right] \qquad (2.35)
\end{aligned}$$

$$\mathcal{L}_{t\bar{t}H} = \frac{1}{\Lambda^2}\left[C_{t1}\frac{v^2}{\sqrt{2}} h\left(\bar{t}t\right) + C_{Dt}\frac{1}{2\sqrt{2}}\left(\partial^\mu h\right)\left[\partial_\mu\left(\bar{t}t\right) + \bar{t}\gamma_5\left(\partial_\mu t\right) - \left(\partial_\mu \bar{t}\right)\gamma_5 t\right]\right] \qquad (2.36)$$

$$\begin{aligned}
\mathcal{L}_{Zt\bar{t}H} &= \frac{1}{\Lambda^2}\left[-C_{t2} 2 m_Z h Z_\mu\left(\bar{t}P_L\gamma^\mu t\right) + C_{Dt}\frac{ig_Z}{12\sqrt{2}}\{8\sin^2\theta_W\left(\partial^\mu h\right) Z_\mu\left(\bar{t}\gamma_5 t\right) - 3hZ_\mu\right. \\
&\quad [\bar{t}(\partial^\mu t) - (\partial^\mu \bar{t})t + \partial^\mu(\bar{t}\gamma^5 t)]\} - C_{tW\phi}(\sqrt{2}\cos\theta_W) h\left(\bar{t}\sigma^{\mu\nu}t\right)\left(\partial_\nu Z_\mu\right) \\
&\quad + C_{tB\phi}(\sqrt{2}\sin\theta_W) h\left(\bar{t}\sigma^{\mu\nu}t\right)\left(\partial_\nu Z_\mu\right) - C^{(1)}_{\phi q} 2 m_Z h Z_\mu\left(\bar{t}\gamma^\mu P_L t\right) \\
&\quad \left. + C^{(3)}_{\phi q} 2 m_Z h Z_\mu\left(\bar{t}\gamma^\mu P_L t\right)\right] \qquad (2.37)
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{\gamma t\bar{t}H} &= \frac{1}{\Lambda^2}\left[C_{Dt}\left(\frac{-i\sqrt{2}}{3}\right) g_Z \sin\theta_W\cos\theta_W\left(\partial^\mu h\right) A_\mu\left(\bar{t}\gamma_5 t\right) - C_{tW\phi}\sqrt{2}h\left(\bar{t}\sigma^{\mu\nu}t\right)\right. \\
&\quad \left.\left(\sin\theta_W\partial_\nu A_\mu\right) - C_{tB\phi}\sqrt{2}h\left(\bar{t}\sigma^{\mu\nu}t\right)\left(\cos\theta_W\partial_\nu A_\mu\right)\right] \qquad (2.38)
\end{aligned}$$

while for CP-odd operators, we have

$$\begin{aligned}
\mathcal{L}_{t\bar{t}Z} &= \frac{1}{\Lambda^2}\left[\overline{C}_{Dt}\left(\frac{m_Z}{2\sqrt{2}}\right) Z^\mu\left[\partial_\mu(\bar{t}t) - (\partial_\mu\bar{t})\gamma_5 t + \bar{t}\gamma_5(\partial_\mu t)\right]\right. \\
&\quad + i\overline{C}_{tW\phi} v(\sqrt{2}\cos\theta_W)(\bar{t}\sigma^{\mu\nu}\gamma_5 t)\partial_\mu Z_\nu \\
&\quad \left. - i\overline{C}_{tB\phi} v(\sqrt{2}\sin\theta_W)(\bar{t}\sigma^{\mu\nu}\gamma_5 t)\partial_\mu Z_\nu\right] \qquad (2.39)
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{t\bar{t}\gamma} &= \frac{1}{\Lambda^2}\left[i\overline{C}_{tW\phi} v(\sqrt{2}\sin\theta_W)(\bar{t}\sigma^{\mu\nu}\gamma_5 t)\partial_\mu A_\nu\right. \\
&\quad \left. + i\overline{C}_{tB\phi} v(\sqrt{2}\cos\theta_W)(\bar{t}\sigma^{\mu\nu}\gamma_5 t)\partial_\mu A_\nu\right] \qquad (2.40)
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{t\bar{t}A} &= \frac{1}{\Lambda^2}\left[\overline{C}_{t1}\frac{iv^2}{\sqrt{2}} h\left(\bar{t}\gamma_5 t\right) + \overline{C}_{t2} v\left(\partial^\mu h\right)\left(\bar{t}\gamma_\mu P_R t\right) + \overline{C}_{Dt}\frac{i}{2\sqrt{2}}\left(\partial^\mu h\right)\left[\partial_\mu\left(\bar{t}\gamma_5 t\right)\right.\right. \\
&\quad \left. + \bar{t}\left(\partial_\mu t\right) - \left(\partial_\mu \bar{t}\right) t\right] + \left.\left(\overline{C}^{(1)}_{\phi q} - \overline{C}^{(3)}_{\phi q}\right) v\left(\partial^\mu h\right)\left(\bar{t}\gamma_\mu P_L t\right)\right] \qquad (2.41)
\end{aligned}$$

[3] Note that only those operators which contribute to the $t\bar{t}\phi$ and $t\bar{t}Z\phi$ vertex have been kept for $t\bar{t}Z$ and $t\bar{t}\gamma$, i.e. terms in $\mathcal{L}_{t\bar{t}\gamma}$ and $\mathcal{L}_{t\bar{t}Z}$ containing operators such as $\mathcal{O}_{qW}$ etc. are not included.

$$\begin{aligned}\mathcal{L}_{Zt\bar{t}A} &= \frac{1}{\Lambda^2}\Big[\overline{C}_{Dt}\frac{g_Z}{12\sqrt{2}}\{-8\sin^2\theta_W\left(\partial^\mu h\right)Z_\mu\left(\bar{t}t\right)+3hZ_\mu\left[\partial^\mu\left(t\bar{t}\right)+\bar{t}\gamma_5\left(\partial^\mu t\right)\right.\\ &\quad \left.-\left(\partial^\mu\bar{t}\right)\gamma_5 t\right]\}-\overline{C}_{tW\phi}i\sqrt{2}h\left(\bar{t}\sigma^{\mu\nu}\gamma_5 t\right)\left(\cos\theta_W\partial_\nu Z_\mu\right)\\ &\quad +\overline{C}_{tB\phi}i\sqrt{2}h\left(\bar{t}\sigma^{\mu\nu}\gamma_5 t\right)\left(\sin\theta_W\partial_\nu Z_\mu\right)\Big] \qquad (2.42)\\ \mathcal{L}_{\gamma t\bar{t}A} &= \frac{1}{\Lambda^2}\Big[\overline{C}_{Dt}\frac{\sqrt{2}}{3}g_Z\sin\theta_W\cos\theta_W\left(\partial^\mu h\right)A_\mu\left(\bar{t}t\right)-\overline{C}_{tW\phi}i\sqrt{2}h\left(\bar{t}\sigma^{\mu\nu}\gamma_5 t\right)\\ &\quad \left(\sin\theta_W\partial_\nu A_\mu\right)-\overline{C}_{tB\phi}i\sqrt{2}h\left(\bar{t}\sigma^{\mu\nu}\gamma_5 t\right)\left(\cos\theta_W\partial_\nu A_\mu\right)\Big] \qquad (2.43)\end{aligned}$$

From these Lagrangians, we can read off the Feynman rules for the various effective three and four-point vertices. We choose the following momentum convention: For the three-point vertices $\bar{t}(p_1)$-(p_2)-$\phi(p_3)$, $\bar{t}(p_1)$-$t(p_2)$-$Z(p_3)$ and $\bar{t}(p_1)$-$t(p_2)$-$\gamma(p_3)$, the momenta p_2 and p_3 are incoming and p_1 is outgoing so that $\partial_\mu\bar{t}=ip_{1_\mu}$, $\partial_\mu t=-ip_{2_\mu}$, and $\partial_\mu h=-ip_{3_\mu}$. For the four-point vertices $Z_\mu(p_4)$-$t(p_3)$-$\bar{t}(p_2)$-$\phi(p_1)$ and $\gamma_\mu(p_4)$-$t(p_3)$-$\bar{t}(p_2)$-$\phi(p_1)$, the momenta p_1, p_3, p_4 are incoming and p_2 is outgoing so that $\partial_\mu\bar{t}=ip_{2_\mu}$, $\partial_\mu t=-ip_{3_\mu}$, $\partial_\mu h=-ip_{1_\mu}$, $\sigma^{\mu\nu}\partial_\mu Z_\nu=-i\sigma^{\mu\nu}p_{4_\mu}$ and $\sigma^{\mu\nu}\partial_\mu\gamma_\nu=-i\sigma^{\mu\nu}p_{4_\mu}$. Thus we have the following Feynman rules for the anomalous vertices:

$$\begin{aligned}g_{t\bar{t}Z_\mu} &= \frac{i}{\Lambda^2}\Big[\frac{m_Z}{2\sqrt{2}}\left(C_{Dt}+i\gamma_5\overline{C}_{Dt}\right)\{(p_{1\mu}-p_{2\mu})\gamma_5-(p_{1\mu}+p_{2\mu})\}\\ &\quad -m_Z v\left(C_{t2}\gamma_\mu P_R+(C^{(1)}_{\phi q}-C^{(3)}_{\phi q})\gamma_\mu P_L\right)+i\sqrt{2}vp_3^\gamma\sigma_{\mu\nu}\\ &\quad \{(C_{tW\phi}+i\gamma_5\overline{C}_{tW\phi})\cos\theta_W-(C_{tB\phi}+i\gamma_5\overline{C}_{tB\phi})\sin\theta_W)\}\Big] \qquad (2.44)\\ g_{t\bar{t}\gamma_\mu} &= \frac{i}{\Lambda^2}i\sqrt{2}vp_3^\gamma\sigma_{\mu\nu}\left[(C_{tW\phi}+i\gamma_5\overline{C}_{tW\phi})\sin\theta_W+(C_{tB\phi}+i\gamma_5\overline{C}_{tB\phi})\cos\theta_W)\right] \qquad (2.45)\\ g_{t\bar{t}\phi} &= \frac{i}{\Lambda^2}\Big[\frac{v^2}{\sqrt{2}}\left(C_{t1}+i\gamma_5\overline{C}_{t1}\right)+\frac{1}{2\sqrt{2}}\left(C_{Dt}+i\gamma_5\overline{C}_{Dt}\right)(p_1\cdot p_3-p_2\cdot p_3-\gamma_5 p_2\cdot p_3\\ &\quad -\gamma_5 p_1\cdot p_3)-ivp_3^\mu\left\{\overline{C}_{t2}\gamma_\mu P_R+\left(\overline{C}^{(1)}_{\phi q}-\overline{C}^{(3)}_{\phi q}\right)\gamma_\mu P_L\right\}\Big] \qquad (2.46)\\ g_{Z_\mu t\bar{t}\phi} &= \frac{i}{\Lambda^2}\Big[\frac{g_Z}{12\sqrt{2}}\left(C_{Dt}+i\gamma_5\overline{C}_{Dt}\right)\left\{8\gamma_5 p_{1_\mu}\sin^2\theta_W-3(p_{2\mu}+p_{3\mu})+3\gamma_5(p_{2\mu}-p_{3\mu})\right\}\\ &\quad -2m_Z\left(C_{t2}\gamma_\mu P_R+(C^{(1)}_{\phi q}-C^{(3)}_{\phi q})\gamma_\mu P_L\right)+i\sqrt{2}\sigma_{\mu\nu}p_4^\gamma\\ &\quad \{(C_{tW\phi}+i\gamma_5\overline{C}_{tW\phi})\cos\theta_W-(C_{tB\phi}+i\gamma_5\overline{C}_{tB\phi})\sin\theta_W)\}\Big] \qquad (2.47)\\ g_{\gamma_\mu t\bar{t}\phi} &= \frac{i}{\Lambda^2}\Big[-\frac{\sqrt{2}}{3}g_Z\cos\theta_W\sin\theta_W\left(C_{Dt}+i\gamma_5\overline{C}_{Dt}\right)\gamma_5 p_{1\mu}+i\sqrt{2}\sigma_{\mu\nu}p_4^\gamma\\ &\quad \{(C_{tW\phi}+i\gamma_5\overline{C}_{tW\phi})\sin\theta_W+(C_{tB\phi}+i\gamma_5\overline{C}_{tB\phi})\cos\theta_W)\}\Big] \qquad (2.48)\end{aligned}$$

All the vertices given in Eqs. (2.56) - (2.59) were derived by us using the convention described in §1.1 which is consistent with standard textbooks and reviews [1, 23].

These expressions for anomalous vertices have been derived earlier by Han et al. [42]; however, we have figured out certain errors in their expressions. Whisnant et al. in Ref.[39] use the notation $\tau^I = \frac{\sigma^I}{2}$ for the $SU(2)$ generators, where σ's are the usual Pauli matrices; and then write their dimension-6 operators in terms of the τ's. As a result, the vertices derived from their operators and written down by Han et al., assuming that their notation is consistent, had factors of 2 and 4 wrong in the terms involving τ^I. Moreover, if we take the τ's in the operators written by Whisnant et al. to be actually the Pauli matrices, then the operators turn out to be consistent with those obtained by other authors like Gounaris et al. [41]. In that case, many of the discrepancies noted by us go away. In spite of this, there seem to be some errors in the vertices written down by Han et al. in the terms involving the operators $\mathcal{O}_{tW\phi}$ and $\mathcal{O}_{tB\phi}$, which couldn't be traced out. However, as we will see in the next section, we are not going to use these operators because their coefficients are constrained to be small.

2.3 PRESENT BOUNDS ON SOME OF THE ANOMALOUS COUPLINGS OF THE HIGGS BOSON

As mentioned earlier, the various operators contribute to the electroweak precision observables and are thus constrained by the present data [38]. The CP-even operators $\mathcal{O}_{\phi Q}^{(1)}$ and $\mathcal{O}_{\phi Q}^{(3)}$ enter the $Zb\bar{b}$ vectorial and axial couplings at the tree level and their coefficients are therefore constrained by precise measurements of the observable R_b at the Z pole which is calculated as [42]

$$R_b \equiv \frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to \text{hadrons})}$$

One obtains, at the 1σ level,

$$5 \times 10^{-5} \leq \frac{v^2}{\Lambda^2} C_{\phi q}^{(1)} \ \text{or}\ \frac{v^2}{\Lambda^2} C_{\phi q}^{(3)} \leq 3.9 \times 10^{-5} \tag{2.49}$$

The operators $\mathcal{O}_{t1}$, $\mathcal{O}_{t2}$, $\mathcal{O}_{Dt}$, $\mathcal{O}_{tW\phi}$ and $\mathcal{O}_{tb\phi}$ do not enter the $Zb\bar{b}$ vertex at the tree level, and hence, are not constrained by R_b. However, at one-loop level they contribute to gauge boson self-energies, and thus rather loose bounds exist [41] with significant uncertainties. The upper bounds obtained on the coefficients are as follows [41]:

$$|C_{t1}| \simeq \frac{16\pi}{3\sqrt{2}}\left(\frac{\Lambda}{v}\right), \qquad |C_{t2}| \simeq 8\pi\sqrt{3}, \tag{2.50}$$

$$C_{Dt} \simeq 10.4 \ \text{for}\ C_{Dt} > 0, \quad C_{Dt} \simeq -6.4 \ \text{for}\ C_{Dt} < 0, \tag{2.51}$$

$$|C_{tW\phi}| \simeq 2.5, \qquad |C_{tB\phi}| \simeq 2.5. \tag{2.52}$$

As to the NP scale, it is plausible to envision that $\Lambda \approx 1$ - 3 TeV, but we will keep $\frac{v^2}{\Lambda^2}$ as a free parameter. The ranges of the upper bounds on various coefficients for $\Lambda \simeq 3$ - 1 TeV are then given as follows:

$$|C_{t1}|\frac{v^2}{\Lambda^2} \simeq 1.0 \text{ - } 3.0, \qquad |C_{t2}|\frac{v^2}{\Lambda^2} \simeq 0.29 \text{ - } 2.6, \tag{2.53}$$

$$C_{Dt}\frac{v^2}{\Lambda^2} \simeq 0.07 \text{ - } 0.63 \text{ for } C_{Dt} > 0,\ C_{Dt}\frac{v^2}{\Lambda^2} \simeq -(0.04 \text{ - } 0.40) \text{ for } C_{Dt} < 0, \tag{2.54}$$

$$|C_{tW\phi}|\frac{v^2}{\Lambda^2} \simeq 0.02 \text{ - } 0.15, \qquad |C_{tB\phi}|\frac{v^2}{\Lambda^2} \simeq 0.02 \text{ - } 0.15. \tag{2.55}$$

Obviously, collider experiments have to reach a sensitivity on these couplings below this level to be useful.

Currently, there are no significant experimental constraints on the CP-odd couplings involving the t-quark sector.

Since it is impossible to include all dimension-6 operators simultaneously in the Feynman amplitudes containing the anomalous vertices, it is prudent to choose a subset for which the coefficients are not already constrained to be too small. As can be seen from Eqs. (2.54) - (2.55), for $\Lambda \simeq 1$ TeV, the coefficients $C_{tW\phi}$ and $C_{tB\phi}$ are about 1 order smaller in magnitude than $C_{t1},\ C_{t2}$ and C_{Dt}. Moreover, the operators $\mathcal{O}_{\phi q}^{(1)}$ and $\mathcal{O}_{\phi q}^{(3)}$ can be safely excluded from further discussion as their coefficients are sufficiently smaller than the rest, cf. Eq. (2.49). Hence, we have decided to use only the operators $\mathcal{O}_{t1},\ \mathcal{O}_{t2}$ and $\mathcal{O}_{Dt}$ (together with their CP-odd counterparts) for further analysis.

Thus we shall use the following effective anomalous vertices in our future analysis:

$$\begin{aligned} g^{\text{eff}}_{t\bar{t}Z_\mu} &= \frac{i}{\Lambda^2}\Big[\frac{m_Z}{2\sqrt{2}}\left(C_{Dt}+i\gamma_5\overline{C}_{Dt}\right)\{(p_{1\mu}-p_{2\mu})\gamma_5-(p_{1\mu}+p_{2\mu})\} \\ &\quad - m_Z v C_{t2}\gamma_\mu P_R\Big] \end{aligned} \tag{2.56}$$

$$\begin{aligned} g^{\text{eff}}_{t\bar{t}\phi} &= \frac{i}{\Lambda^2}\Big[\frac{v^2}{\sqrt{2}}\left(C_{t1}+i\gamma_5\overline{C}_{t1}\right)+\frac{1}{2\sqrt{2}}\left(C_{Dt}+i\gamma_5\overline{C}_{Dt}\right) \\ &\quad \times(p_1\cdot p_3-p_2\cdot p_3-\gamma_5 p_2\cdot p_3-\gamma_5 p_1\cdot p_3)-iv p_3^\mu\overline{C}_{t2}\gamma_\mu P_R\Big] \end{aligned} \tag{2.57}$$

$$\begin{aligned} g^{\text{eff}}_{Z_\mu t\bar{t}\phi} &= \frac{i}{\Lambda^2}\Big[\frac{g_Z}{12\sqrt{2}}\left(C_{Dt}+i\gamma_5\overline{C}_{Dt}\right)\Big\{8\gamma_5 p_{1_\mu}\sin^2\theta_W \\ &\quad -3(p_{2\mu}+p_{3\mu})+3\gamma_5(p_{2\mu}-p_{3\mu})\Big\}-2m_Z C_{t2}\gamma_\mu P_R\Big] \end{aligned} \tag{2.58}$$

$$g^{\text{eff}}_{\gamma_\mu t\bar{t}\phi} = \frac{i}{\Lambda^2}\left[-\frac{\sqrt{2}}{3}g_Z\cos\theta_W\sin\theta_W\left(C_{Dt}+i\gamma_5\overline{C}_{Dt}\right)\gamma_5 p_{1\mu}\right] \tag{2.59}$$

We note that all the terms in the anomalous vertex $t\bar{t}\gamma$ are small, and hence, can be dropped.

CHAPTER 3

THE PROCESS $e^-e^+ \to t\bar{t}\phi$

In §1.8.2, we have already discussed the importance of top-quark in Higgs studies. In connection with the t-quark sector, the most promising process to study will be the Higgs boson and t-quark pair associated production [43, 44]

$$e^-e^+ \to t\bar{t}\phi \tag{3.1}$$

which provides a direct way to determine the t-quark Yukawa coupling [45]. By scrutinizing this process in detail, one would hope to understand the nature of the Higgs boson interactions with t-quark and hopefully gain some insight for physics beyond the SM.

Here we take ϕ to be a neutral spin-zero boson of unknown CP property. In this chapter, We derive the expressions for the helicity amplitudes and the production cross section for individual helicity states. These results will be used later to calculate the polarization asymmetry of the top quark with both unpolarized and polarized initial beams, which will then be used as an observable to probe the CP property of the Higgs boson.

3.1 FEYNMAN DIAGRAMS FOR THE PROCESS

At the tree level, the process

$$e^-(p_1)e^+(p_2) \to t(p_3)\bar{t}(p_4)\phi(p_5) \tag{3.2}$$

receives contributions from six Feynman diagrams as shown in Figure 3.1. This process was discussed earlier in literature as a possible source of Higgs particles at low energies where only the photon exchange diagrams, Figures 3.1(a) and 3.1(b), had to be taken into account [43]. At high energies the Z exchange diagrams, Figures 3.1(c) and 3.1(d), lead to an axial vector contribution different from the vectorial γ amplitude.

An additional contribution also comes from the Higgs bremsstrahlung off the Z boson, as shown in Figure 3.1(e). We will see, however, that the last mentioned will however prove to be of minor importance [44] so that the Higgs-top Yukawa coupling can still be measured directly in the process (3.2). [1].

These first five diagrams add up to make the SM amplitude for the process (3.2) at the tree level and have been studied in detail [44]. QED corrections due to initial state radiation effects have also been studied [45]. The first order QCD corrections to this process, which turn out to be important near the $t\bar{t}$ threshold, have been computed including only γ exchange in Ref.[46] and with the complete γ and Z contributions in Ref.[47]. The electroweak corrections to this process are also important and have been computed by three groups [48] which are, incidentally, the first example of electroweak corrections for a $2 \to 3$ process.

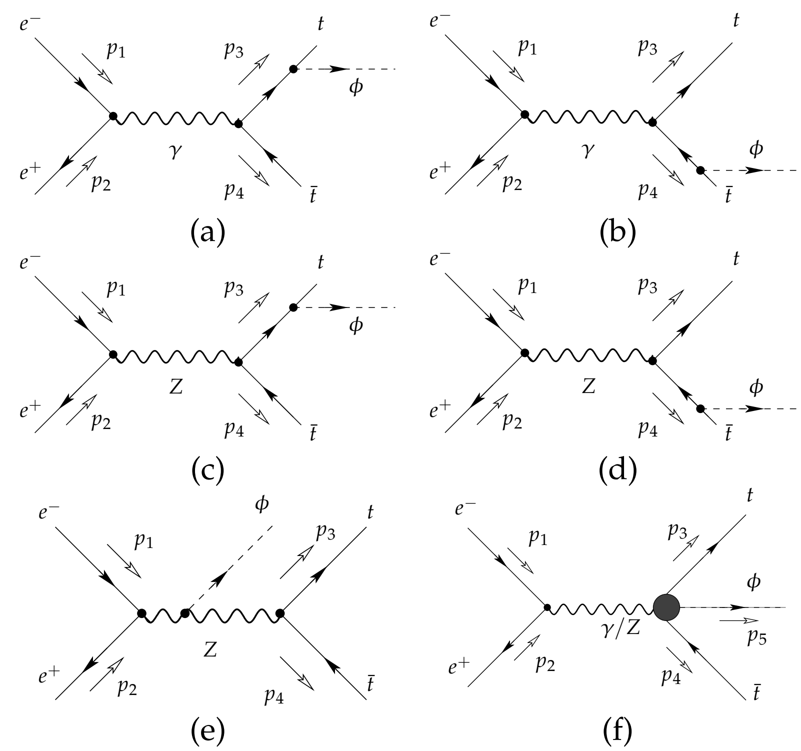


Figure 3.1: The Feynman diagrams for the process (3.2) at the tree level.

However, not much is known about the last diagram. The effective vertex in this diagram may be arising from contributions of loops including the effects of NP beyond the SM. If we allow ϕ to be a CP-non- eigenstate, we are already including the effects of NP beyond the SM. Hence, for consistency, all the six diagrams must be taken into account if we want to do the most general calculation of cross section at tree

[1] As we will see later, the effect of Z exchange is only a few percent correction, in particular at low energies, and hence the cross section is directly proportional to $g^2_{tt\phi}$.

level for the process (3.2). Here we take a model-independent approach to include the non-standard couplings derived in §2.2 for calculating the production cross section for the process (3.2). The calculation for total cross section has been done in Ref.[42]; but we make detailed studies in terms of the helicity structure by calculating the analytical expressions for individual helicity amplitudes and squared amplitudes, and then calculating the corresponding cross sections for individual helicity states. This approach has the advantage of being able to make analysis of the polarization asymmetry which can then be used to probe the non-standard effects.

A very interesting feature of the process (3.2) is that it exhibits a CP asymmetry at the tree level. Such an effect arises from interference of the Higgs emission from t or $\bar{t}$ leg with the Higgs emission from the Z boson. Being a tree level effect the resulting asymmetry is quite large. Moreover, this asymmetry may get enhanced due to addition of the non-standard coupling terms. Also, this asymmetry can be detected easily through a CP-odd, $\tilde{T}$-odd observable[2]. Such CP-violating observables have been constructed for the process $e^+e^- \to t\bar{t}H^0$ in the context of the THDM, for instance, in Ref.[49].

3.2 MATRIX ELEMENTS

Feynman rules used in writing down the matrix elements are summarized in Appendix A.1. At this stage, we introduce the general $t\bar{t}\phi$ and $ZZ\phi$ vertices only; hence we exclude the last diagram in Figure 3.1. This diagram along with the other anomalous couplings will be included in our analysis at a later stage.

While dealing with the massive propagators, we have to include their finite decay widths; thus the Z and t propagators will be modified to

$$\frac{-i}{q^2 - m_Z^2 + i m_Z \Gamma_Z + i\epsilon}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_Z^2 + i\epsilon}\right) \ , \ \text{and} \ \frac{i(\not{p} + m)}{p^2 - m_t^2 + i m_t \Gamma_t + i\epsilon} \ ,$$

respectively. In this particular case, however, we will deal with energies much higher than the Z and t-poles; hence it hardly makes any difference whether we add the decay width part or not.

We note that in the expression for the gauge-boson propagator, the term carrying the gauge index is proportional to $q_\mu q_\nu$; hence for the first four diagrams in which one end of the propagator is always connected to the e^-e^+ vertex, when inserted in the expression for the amplitude, it becomes proportional to m_e^2 (by Dirac equation) and hence can be neglected. However, this is not so for the second Z propagator in the

[2] $\tilde{T}$ is the naive time-reversal operator defined by replacing time with its negative without switching the initial and final states.

fifth diagram which is attached to the $t\bar{t}$ vertex; we must include the $q_\mu q_\nu$ term for this propagator. In the unitary gauge, the matrix elements corresponding to the Feynman diagrams in Figures 3.1(a)-(e) are as follows:

$$\begin{aligned} i\mathcal{M}_{(a)} &= [\overline{v}(p_2)(-ie\gamma^\mu)u(p_1)]\left(\frac{-ig_{\mu\nu}}{k^2+i\epsilon}\right)\left[\overline{u}(p_3)\left(\frac{-iem_t}{m_Z\sin 2\theta_W}(a+ib\gamma_5)\right)\right. \\ &\left.\frac{i(\not{k}-\not{p}_4+m_t)}{(k-p_4)^2-m_t^2+im_t\Gamma_t+i\epsilon}\left(\frac{2}{3}ie\gamma^\nu\right)v(p_4)\right] \end{aligned} \tag{3.3}$$

$$\begin{aligned} i\mathcal{M}_{(b)} &= [\overline{v}(p_2)(-ie\gamma^\mu)u(p_1)]\left(\frac{-ig_{\mu\nu}}{k^2+i\epsilon}\right)\left[\overline{u}(p_3)\left(\frac{2}{3}ie\gamma^\nu\right)\right. \\ &\left.\frac{i(\not{k}-\not{p}_3+m_t)}{(k-p_3)^2-m_t^2+im_t\Gamma_t+i\epsilon}\left(\frac{-iem_t}{m_Z\sin 2\theta_W}(a+ib\gamma_5)\right)v(p_4)\right] \end{aligned} \tag{3.4}$$

$$\begin{aligned} i\mathcal{M}_{(c)} &= \left[\overline{v}(p_2)\left(\frac{ie}{\sin 2\theta_W}\gamma^\mu(-P_L+2\sin^2\theta_W)\right)u(p_1)\right]\left(\frac{-ig_{\mu\nu}}{k^2-m_Z^2+im_Z\Gamma_Z+i\epsilon}\right) \\ &\left[\overline{u}(p_3)\left(\frac{-iem_t}{m_Z\sin 2\theta_W}(a+ib\gamma_5)\right)\frac{i(\not{k}-\not{p}_4+m_t)}{(k-p_4)^2-m_t^2+im_t\Gamma_t+i\epsilon}\right. \\ &\left.\left(\frac{ie}{\sin 2\theta_W}\gamma^\nu(P_L-\frac{4}{3}\sin^2\theta_W)\right)v(p_4)\right] \end{aligned} \tag{3.5}$$

$$\begin{aligned} i\mathcal{M}_{(d)} &= \left[\overline{v}(p_2)\left(\frac{ie}{\sin 2\theta_W}\gamma^\mu(-P_L+2\sin^2\theta_W)\right)u(p_1)\right]\left(\frac{-ig_{\mu\nu}}{k^2-m_Z^2+im_Z\Gamma_Z+i\epsilon}\right) \\ &\left[\overline{u}(p_3)\left(\frac{ie}{\sin 2\theta_W}\gamma^\nu(P_L-\frac{4}{3}\sin^2\theta_W)\right)\frac{i(\not{k}-\not{p}_3+m_t)}{(k-p_3)^2-m_t^2+im_t\Gamma_t+i\epsilon}\right. \\ &\left.\left(\frac{-iem_t}{m_Z\sin 2\theta_W}(a+ib\gamma_5)\right)v(p_4)\right] \end{aligned} \tag{3.6}$$

$$\begin{aligned} i\mathcal{M}_{(e)} &= \left[\overline{v}(p_2)\left(\frac{ie}{\sin 2\theta_W}\gamma^\mu\left(-P_L+2\sin^2\theta_W\right)\right)u(p_1)\right]\left(\frac{-ig_{\mu\alpha}}{k^2-m_Z^2+im_Z\Gamma_Z+i\epsilon}\right) \\ &\left(\frac{-icg_2m_Zg^{\alpha\beta}}{\cos\theta_W}\right)\left\{\frac{-i}{k'^2-m_Z^2+im_Z\Gamma_Z+i\epsilon}\left(g_{\beta\nu}-\frac{k'_\beta k'_\nu}{m_Z^2}\right)\right\} \\ &\left[\overline{u}(p_3)\left(\frac{ie}{\sin 2\theta_W}\gamma^\nu\left(P_L-\frac{4}{3}\sin^2\theta_W\right)\right)v(p_4)\right] \end{aligned} \tag{3.7}$$

with $k = p_1 + p_2$, $k' = p_3 + p_4$; p_1, p_2 being the initial four-momenta of e^-, e^+ and p_3, p_4 the final four-momenta of t, $\bar{t}$ respectively. Eqs. (3.3) - (3.7) can be written in a compact form if the coupling of gauge bosons to fermions is written in a general form: $e\gamma^\mu(a_cP_L + b_cP_R)$. Then the diagrams (a) & (c) and (b) & (d) in Figure 3.1 can be added as shown in Figure 3.2 with some appropriate weights. Then Eqs. (3.3) - (3.6) are of the form

$$\mathcal{M}_{(i)} \sim -[\overline{v}(p_2)(e\gamma^\mu)(a_cP_L+b_cP_R)u(p_1)]\frac{1}{\xi^2}\left[\overline{u}(p_3)\left(\frac{em_t}{\sin 2\theta_W m_Z}(a+ib\gamma_5)\right)\right.$$

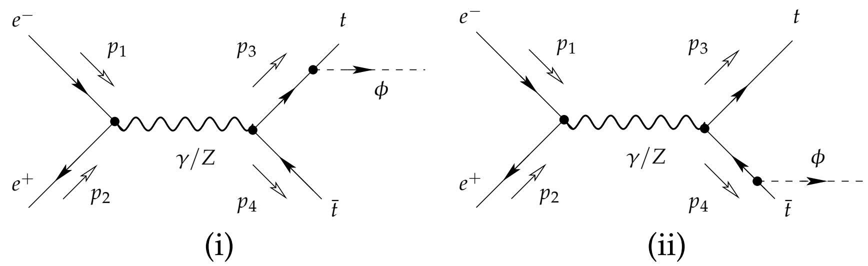


Figure 3.2: The combined diagrams (a) & (c) and (b) & (d).

$$\left. \frac{\not{p}_1 + \not{p}_2 - \not{p}_4 + m_t}{(p_1+p_2-p_4)^2 - m_t^2 + im_t\Gamma_t + i\epsilon} e\gamma_\mu (c_c P_L + d_c P_R) v(p_4) \right] \tag{3.8}$$

$$\mathcal{M}_{(ii)} \sim -\left[\overline{v}(p_2)(e\gamma^\mu)(a_c P_L + b_c P_R)u(p_1)\right] \frac{1}{\xi^2} \left[\overline{u}(p_3) e\gamma_\mu (c_c P_L + d_c P_R) \right.$$
$$\left. \frac{-\not{p}_1 - \not{p}_2 + \not{p}_3 + m_t}{(-p_1-p_2+p_3)^2 - m_t^2 + im_t\Gamma_t + i\epsilon} \left(\frac{em_t}{\sin 2\theta_W m_Z} (a + ib\gamma_5) \right) v(p_4) \right] \tag{3.9}$$

where we will denote the couplings with γ and Z and the propagators by putting the appropriate subscripts:

$$\xi_\gamma^2 = k^2 + i\epsilon, \qquad \xi_Z^2 = k^2 - m_Z^2 + im_Z\Gamma_Z + i\epsilon$$

$$a_\gamma = b_\gamma = -1, \qquad c_\gamma = d_\gamma = \frac{2}{3}$$

$$a_Z = \frac{-1 + 2\sin^2\theta_W}{\sin 2\theta_W}, \qquad b_Z = \frac{2\sin^2\theta_W}{\sin 2\theta_W}$$

$$c_Z = \frac{1 - \frac{4}{3}\sin^2\theta_W}{\sin 2\theta_W}, \qquad d_Z = \frac{-\frac{4}{3}\sin^2\theta_W}{\sin 2\theta_W}$$

With these identifications, the matrix elements in Eqs. (3.8) and (3.9) can be written as

$$\mathcal{M}_{(i)} = \frac{-e^3 m_t}{m_Z \sin 2\theta_W \left(q_1^2 - m_t^2 + im_t\Gamma_t + i\epsilon\right)}$$
$$\left[\left(\frac{a_\gamma c_\gamma}{\xi_\gamma^2} + \frac{a_Z c_Z}{\xi_Z^2} \right) \left[\overline{v}(p_2)\gamma^\mu P_L u(p_1)\right] g_{\mu\nu} \left[\overline{u}(p_3)(a + ib\gamma_5)(\not{q}_1 + m_t)\gamma^\nu P_L v(p_4)\right] \right.$$
$$+ \left(\frac{a_\gamma d_\gamma}{\xi_\gamma^2} + \frac{a_Z d_Z}{\xi_Z^2} \right) \left[\overline{v}(p_2)\gamma^\mu P_L u(p_1)\right] g_{\mu\nu} \left[\overline{u}(p_3)(a + ib\gamma_5)(\not{q}_1 + m_t)\gamma^\nu P_R v(p_4)\right]$$
$$+ \left(\frac{b_\gamma c_\gamma}{\xi_\gamma^2} + \frac{b_Z c_Z}{\xi_Z^2} \right) \left[\overline{v}(p_2)\gamma^\mu P_R u(p_1)\right] g_{\mu\nu} \left[\overline{u}(p_3)(a + ib\gamma_5)(\not{q}_1 + m_t)\gamma^\nu P_L v(p_4)\right]$$
$$\left. + \left(\frac{b_\gamma d_\gamma}{\xi_\gamma^2} + \frac{b_Z d_Z}{\xi_Z^2} \right) \left[\overline{v}(p_2)\gamma^\mu P_R u(p_1)\right] g_{\mu\nu} \left[\overline{u}(p_3)(a + ib\gamma_5)(\not{q}_1 + m_t)\gamma^\nu P_R v(p_4)\right] \right]$$

$$
\begin{aligned}
&\equiv A_1[Ac_{LL}F_{1LL} + Ac_{LR}F_{1LR} + Ac_{RL}F_{1RL} + Ac_{RR}F_{1RR}] \qquad (3.10)\\
\mathcal{M}_{(ii)} &= \frac{-e^3 m_t}{m_Z \sin 2\theta_W \left(q_2^2 - m_t^2 + i m_t \Gamma_t + i\epsilon\right)}\\
&\left[\left(\frac{a_\gamma c_\gamma}{\xi_\gamma^2} + \frac{a_Z c_Z}{\xi_Z^2}\right)[\overline{v}(p_2)\gamma^\mu P_L u(p_1)]\, g_{\mu\nu}\, [\overline{u}(p_3)\gamma^\nu P_L(\not{q}_2 + m_t)(a + ib\gamma_5)v(p_4)]\right.\\
&+\left(\frac{a_\gamma d_\gamma}{\xi_\gamma^2} + \frac{a_Z d_Z}{\xi_Z^2}\right)[\overline{v}(p_2)\gamma^\mu P_L u(p_1)]\, g_{\mu\nu}\, [\overline{u}(p_3)\gamma^\nu P_R(\not{q}_2 + m_t)(a + ib\gamma_5)v(p_4)]\\
&+\left(\frac{b_\gamma c_\gamma}{\xi_\gamma^2} + \frac{b_Z c_Z}{\xi_Z^2}\right)[\overline{v}(p_2)\gamma^\mu P_R u(p_1)]\, g_{\mu\nu}\, [\overline{u}(p_3)\gamma^\nu P_L(\not{q}_2 + m_t)(a + ib\gamma_5)v(p_4)]\\
&\left.+\left(\frac{b_\gamma d_\gamma}{\xi_\gamma^2} + \frac{b_Z d_Z}{\xi_Z^2}\right)[\overline{v}(p_2)\gamma^\mu P_R u(p_1)]\, g_{\mu\nu}\, [\overline{u}(p_3)\gamma^\nu P_R(\not{q}_2 + m_t)(a + ib\gamma_5)v(p_4)]\right]\\
&\equiv A_2[Ac_{LL}F_{2LL} + Ac_{LR}F_{2LR} + Ac_{RL}F_{2RL} + Ac_{RR}F_{2RR}] \qquad (3.11)
\end{aligned}
$$

with $q_1 = p_1 + p_2 - p_4$, $q_2 = -p_1 - p_2 + p_3$,

$$
A_1 = \frac{-e^3 m_t}{m_Z \sin 2\theta_W \left(q_1^2 - m_t^2 + i m_t \Gamma_t + i\epsilon\right)}\ ,\ A_2 = \frac{-e^3 m_t}{m_Z \sin 2\theta_W \left(q_2^2 - m_t^2 + i m_t \Gamma_t + i\epsilon\right)}, \qquad (3.12)
$$

$$
\begin{aligned}
Ac_{LL} &= \frac{a_\gamma c_\gamma}{\xi_\gamma^2} + \frac{a_Z c_Z}{\xi_Z^2} = Ag_{LL} + Az_{LL},\\
Ac_{LR} &= \frac{a_\gamma d_\gamma}{\xi_\gamma^2} + \frac{a_Z d_Z}{\xi_Z^2} = Ag_{LR} + Az_{LR},\\
Ac_{RL} &= \frac{b_\gamma c_\gamma}{\xi_\gamma^2} + \frac{b_Z c_Z}{\xi_Z^2} = Ag_{RL} + Az_{RL},\\
Ac_{RR} &= \frac{b_\gamma d_\gamma}{\xi_\gamma^2} + \frac{b_Z d_Z}{\xi_Z^2} = Ag_{RR} + Az_{RR}, \qquad (3.13)
\end{aligned}
$$

$$
F_{nab} = (Je_a)^\mu g_{\mu\nu} (Jt_{nb})^\nu \qquad (n = 1, 2;\ a, b = L, R), \qquad (3.14)
$$

$$
\begin{aligned}
(Je_a)^\mu &= \overline{v}(p_2)\gamma^\mu P_a u(p_1), \qquad (3.15)\\
(Jt_{1a})^\mu &= \overline{u}(p_3)(a + ib\gamma_5)(\not{q}_1 + m_t)\gamma^\mu P_a v(p_4), \qquad (3.16)\\
(Jt_{2a})^\mu &= \overline{u}(p_3)\gamma^\mu P_a(\not{q}_2 + m_t)(a + ib\gamma_5)v(p_4) \qquad (3.17)
\end{aligned}
$$

Similarly, Eq. (3.7) can be rewritten as follows:

$$
\mathcal{M}_{(e)} \equiv \mathcal{M}_{(iii)} = A_3\left[Az_{LL}F_{3LL} + Az_{LR}F_{3LR} + Az_{RL}F_{3RL} + Az_{RR}F_{3RR}\right] \qquad (3.18)
$$

where Az's are defined in Eq. (3.13). F_{3ab} is given by

$$
F_{3ab} = (Je_a)^\mu g_{\mu\alpha} g^{\alpha\beta} \left(g_{\beta\nu} - \frac{k'_\beta k'_\nu}{m_Z^2}\right) (Jt_{3b})^\nu \qquad (a, b = L, R), \qquad (3.19)
$$

where Je_a is given by Eq. (3.15) and Jt_{3a} is

$$(Jt_{3a})^\mu = \overline{u}(p_3)\gamma^\mu P_a v(p_4) \qquad (a = L, R) \tag{3.20}$$

$$A_3 = \frac{-ce^3 2m_Z}{\sin 2\theta_W \left(k'^2 - m_Z^2 + im_Z\Gamma_Z + i\epsilon\right)} \tag{3.21}$$

Thus the total Feynman amplitude is given by

$$\mathcal{M}_{\text{tot}} = \mathcal{M}_{(i)} + \mathcal{M}_{(ii)} + \mathcal{M}_{(iii)} \tag{3.22}$$

3.3 HELICITY AMPLITUDES

If we denote the helicity states of the spin-$\frac{1}{2}$ particles e^-, e^+, t and $\bar{t}$ as λ_1, λ_2, λ_3 and λ_4 respectively, then the Feynman amplitude for individual helicity states can be written as

$$\mathcal{M}_{\text{tot}}(h_1, h_2, h_3, h_4) = \mathcal{M}_{(i)}(h_1, h_2, h_3, h_4) + \mathcal{M}_{(ii)}(h_1, h_2, h_3, h_4) + \mathcal{M}_{(iii)}(h_1, h_2, h_3, h_4) \tag{3.23}$$

where $h_i = 2\lambda_i = \pm 1$ ($i = 1, ..4$) is twice the spin-$\frac{1}{2}$ particle helicity. In general, there are 16 individual helicity states for this process. However, we know that in the massless limit, the amplitude vanishes unless the electron and positron have opposite helicity, or equivalently, unless their spinors have the same helicity [3]. This can be seen easily from the current structure (3.15) and using the fact that the projection operators P_L and P_R are orthogonal to each other: $P_L P_R = 0 = P_R P_L$. Thus the current Je_a vanishes for the two combinations $(+\frac{1}{2}, -\frac{1}{2})$ and $(-\frac{1}{2}, +\frac{1}{2})$. This is, of course, true only for massless spinors for which the *helicity states* are equivalent to the *chirality states*. The helicity states for a massive spinor will be a mixture of left- and right- chirality states, and hence, all possible combinations of helicity states, viz. $(+\frac{1}{2}, +\frac{1}{2})$, $(+\frac{1}{2}, -\frac{1}{2})$, $(-\frac{1}{2}, +\frac{1}{2})$ and $(-\frac{1}{2}, -\frac{1}{2})$, contribute to the Feynman amplitude.

In our case, the energy scale involved is of the order of top-mass, m_t which is about 5 orders of magnitude larger than the electron mass, m_e. Hence, practically we can take m_e to be zero for our analysis. Then only 8 out of 16 helicity combinations will contribute to the Feynman amplitude (3.22).

In order to calculate the production cross section, we need the squared amplitude

$$|\mathcal{M}_{\text{tot}}|^2 = \mathcal{M}^*_{\text{tot}}\mathcal{M}_{\text{tot}} = \sum_{h_1, h_2, h_3, h_4 = \pm 1} |\mathcal{M}_{\text{tot}}(h_1, h_2, h_3, h_4)|^2 \tag{3.24}$$

We have calculated the squared amplitudes for individual helicity states in two completely independent ways.

[3] Note that an antiparticle state with helicity $+(-)\frac{1}{2}$ is denoted by the spinor state $v_L(v_R)$, *not* $v_R(v_L)$ unlike the case of a particle spinor where the helicity $+(-)\frac{1}{2}$ is denoted by $u_R(u_L)$.

3.3.1 HELICITY METHOD

This is the direct method in which we calculate the helicity amplitudes (3.23) by using the explicit forms for spinors in the Dirac representation (See Appendix 1) [50]:

$$
\begin{aligned}
u[\theta,\phi,p,E_p,h,m] &= \begin{pmatrix}
\sqrt{(E_p+m)}\, e^{\frac{-i\phi}{2}} \left[\frac{(1+h)}{2}\cos\left(\frac{\theta}{2}\right) - \frac{(1-h)}{2}\sin\left(\frac{\theta}{2}\right)\right] \\
\sqrt{(E_p+m)}\, e^{\frac{i\phi}{2}} \left[\frac{(1-h)}{2}\cos\left(\frac{\theta}{2}\right) + \frac{(1+h)}{2}\sin\left(\frac{\theta}{2}\right)\right] \\
\frac{h\ p}{\sqrt{E_p+m}} e^{\frac{-i\phi}{2}} \left[\frac{(1+h)}{2}\cos\left(\frac{\theta}{2}\right) - \frac{(1-h)}{2}\sin\left(\frac{\theta}{2}\right)\right] \\
\frac{h\ p}{\sqrt{E_p+m}} e^{\frac{i\phi}{2}} \left[\frac{(1-h)}{2}\cos\left(\frac{\theta}{2}\right) + \frac{(1+h)}{2}\sin\left(\frac{\theta}{2}\right)\right]
\end{pmatrix} \\
v[\theta,\phi,p,E_p,h,m] &= \begin{pmatrix}
\frac{-h\ p}{\sqrt{E_p+m}} e^{\frac{-i\phi}{2}} \left[\frac{(1-h)}{2}\cos\left(\frac{\theta}{2}\right) - \frac{(1+h)}{2}\sin\left(\frac{\theta}{2}\right)\right] \\
\frac{-h\ p}{\sqrt{E_p+m}} e^{\frac{i\phi}{2}} \left[\frac{(1+h)}{2}\cos\left(\frac{\theta}{2}\right) + \frac{(1-h)}{2}\sin\left(\frac{\theta}{2}\right)\right] \\
\sqrt{(E_p+m)}\, e^{\frac{-i\phi}{2}} \left[\frac{(1-h)}{2}\cos\left(\frac{\theta}{2}\right) - \frac{(1+h)}{2}\sin\left(\frac{\theta}{2}\right)\right] \\
\sqrt{(E_p+m)}\, e^{\frac{i\phi}{2}} \left[\frac{(1+h)}{2}\cos\left(\frac{\theta}{2}\right) + \frac{(1-h)}{2}\sin\left(\frac{\theta}{2}\right)\right]
\end{pmatrix}
\end{aligned} \tag{3.25}
$$

and for conjugate spinors: $\overline{u} = u^\dagger \gamma^0$, $\overline{v} = v^\dagger \gamma^0$.

The explicit expressions of the F's obtained in this way using the `MATHEMATICA` package are given in Appendix B. The matrix elements and squared matrix elements are then calculated using these expressions.

3.3.2 BOUCHIAT-MICHEL METHOD

This method, along with the *trace technique*, is used for evaluating the squared amplitudes and is well suited for scattering processes in which the initial state consists of two equal mass fermions. We introduce three four-vectors S^a_μ, $a = 1, 2, 3$ such that the S^a and the four-momentum $p = (E, \mathbf{p})$ form an orthonormal set of four-vectors [51]. That is,

$$
\begin{aligned}
p \cdot S^a &= 0, \\
S^a \cdot S^b &= -\delta^{ab}, \\
S^a_\mu S^a_\nu &= -g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}
\end{aligned} \tag{3.26}
$$

A convenient choice for the s^a is

$$
\begin{aligned}
S^{1\mu} &= (0;\ \cos\theta\cos\phi,\ \cos\theta\sin\phi,\ -\sin\theta), \\
S^{2\mu} &= (0;\ -\sin\phi,\ \cos\phi,\ 0),\ \text{and} \\
S^{3\mu} &= \frac{1}{m}(|\mathbf{p}|; E\hat{\mathbf{p}})
\end{aligned} \tag{3.27}
$$

in a coordinate system where $\hat{p} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$. S^3 is identified as the positive helicity spin vector S which is defined as

$$S^\mu = (2\lambda)\frac{1}{m}(|\mathbf{p}|; E\hat{\mathbf{p}}) \tag{3.28}$$

These helicity spinors satisfy

$$\begin{aligned} \gamma_5 \not{S}^a u(p,\lambda') &= \tau^a_{\lambda\lambda'} u(p,\lambda), \\ \gamma_5 \not{S}^a v(p,\lambda') &= \tau^a_{\lambda'\lambda} v(p,\lambda) \end{aligned} \tag{3.29}$$

where the τ^a are the Pauli matrices. For $a = 3$, we note that the helicity spinors satisfy the Dirac equation and are eigenstates of $\gamma_5 \not{S}$ with unit eigenvalue. That is, we have

$$\begin{aligned} \not{p}u(p,\lambda) &= mu(p,\lambda)\ , \qquad \gamma_5 \not{S}u(p,\lambda) = u(p,\lambda), \\ \not{p}v(p,\lambda) &= -mv(p,\lambda)\ , \qquad \gamma_5 \not{S}v(p,\lambda) = v(p,\lambda) \end{aligned} \tag{3.30}$$

Using Eqs. (3.29), one can derive the following formula first introduced by Bouchiat and Michel [52] for spin-$\frac{1}{2}$ particles of mass m:

$$\begin{aligned} u(p,\lambda')\overline{u}(p,\lambda) &= \frac{1}{2}[\delta_{\lambda\lambda'} + \gamma_5 \not{S}^a \tau^a_{\lambda\lambda'}](\not{p} + m), \\ v(p,\lambda')\overline{v}(p,\lambda) &= \frac{1}{2}[\delta_{\lambda'\lambda} + \gamma_5 \not{S}^a \tau^a_{\lambda'\lambda}](\not{p} - m) \end{aligned} \tag{3.31}$$

For $\lambda = \lambda'$ and using $2\lambda S^3 = S$, cf. Eq. (3.28), we can reduce Eqs. (3.31) to those for the *helicity projection operators*:

$$\begin{aligned} u(p,\lambda)\overline{u}(p,\lambda) &= \frac{1}{2}(1 + \gamma_5 \not{S})(\not{p} + m), \\ v(p,\lambda)\overline{v}(p,\lambda) &= \frac{1}{2}(1 + \gamma_5 \not{S})(\not{p} - m) \end{aligned} \tag{3.32}$$

We use Eqs. (3.32) in our calculation for squared amplitudes. To apply this formula to the massless case, we note from Eq. (3.28) that in the $m \to 0$ limit, $S = \frac{2\lambda p}{m} + \mathcal{O}\left(\frac{m}{E}\right)$. Inserting this result in Eqs. (3.30), it follows that the massless helicity spinors are eigenstates of γ_5:

$$\gamma_5 u(p,\lambda) = 2\lambda u(p,\lambda)\ ,\ \gamma_5 v(p,\lambda) = -2\lambda v(p,\lambda) \tag{3.33}$$

Applying the same limiting procedure to Eqs. (3.32) and using the mass-shell condition ($\not{p}\ \not{p} = p^2 = m^2$), we obtain the helicity projection operators for a massless spin-$\frac{1}{2}$ particle:

$$\begin{aligned} u(p,\lambda)\overline{u}(p,\lambda) &= \frac{1}{2}(1 + 2\lambda\gamma_5)\ \not{p}, \\ v(p,\lambda)\overline{v}(p,\lambda) &= \frac{1}{2}(1 - 2\lambda\gamma_5)\ \not{p} \end{aligned} \tag{3.34}$$

In order to evaluate the squared amplitudes in the trace method, we go back to Eqs. (3.3) - (3.7) and rewrite them as follows:

$$\begin{aligned}
\mathcal{M}_1 &= C_1 \left[\overline{v}(p_2) G^{\mu}_{e_1} u(p_1)\right] g_{\mu\nu} \left[\overline{u}(p_3)(a+ib\gamma_5)(\not{q}_1 + m_t) G^{\nu}_{t_1} v(p_4)\right] &(3.35)\\
\mathcal{M}_2 &= C_2 \left[\overline{v}(p_2) G^{\mu}_{e_2} u(p_1)\right] g_{\mu\nu} \left[\overline{u}(p_3) G^{\nu}_{t_2}(\not{q}_2 + m_t)(a+ib\gamma_5) v(p_4)\right] &(3.36)\\
\mathcal{M}_3 &= C_3 \left[\overline{v}(p_2) G^{\mu}_{e_3} u(p_1)\right] g_{\mu\nu} \left[\overline{u}(p_3)(a+ib\gamma_5)(\not{q}_1 + m_t) G^{\nu}_{t_3} v(p_4)\right] &(3.37)\\
\mathcal{M}_4 &= C_4 \left[\overline{v}(p_2) G^{\mu}_{e_4} u(p_1)\right] g_{\mu\nu} \left[\overline{u}(p_3) G^{\nu}_{t_4}(\not{q}_2 + m_t)(a+ib\gamma_5) v(p_4)\right] &(3.38)\\
\mathcal{M}_5 &= C_5 \left[\overline{v}(p_2) G^{\mu}_{e_5} u(p_1)\right] g_{\mu\alpha} g^{\alpha\beta} \left(g_{\beta\nu} - \frac{k'_\beta k'_\nu}{m_Z^2}\right) \left[\overline{u}(p_3) G^{\nu}_{t_5} v(p_4)\right] &(3.39)
\end{aligned}$$

and their conjugate amplitudes:

$$\begin{aligned}
\mathcal{M}^*_1 &= C^*_1 \left[\overline{u}(p_1) G^{\mu'}_{e_1} v(p_2)\right] g_{\mu'\nu'} \left[\overline{v}(p_4) G^{\nu'}_{t_1}(\not{q}_1 + m_t)(a+ib\gamma_5) u(p_3)\right] &(3.40)\\
\mathcal{M}^*_2 &= C^*_2 \left[\overline{u}(p_1) G^{\mu'}_{e_2} v(p_2)\right] g_{\mu'\nu'} \left[\overline{v}(p_4)(a+ib\gamma_5)(\not{q}_2 + m_t) G^{\nu'}_{t_2} u(p_3)\right] &(3.41)\\
\mathcal{M}^*_3 &= C^*_3 \left[\overline{u}(p_1) G^{\mu'}_{e_3} v(p_2)\right] g_{\mu'\nu'} \left[\overline{v}(p_4) G^{\nu'}_{t_3}(\not{q}_1 + m_t)(a+ib\gamma_5) u(p_3)\right] &(3.42)\\
\mathcal{M}^*_4 &= C^*_4 \left[\overline{u}(p_1) G^{\mu'}_{e_4} v(p_2)\right] g_{\mu'\nu'} \left[\overline{v}(p_4)(a+ib\gamma_5)(\not{q}_2 + m_t) G^{\nu'}_{t_4} u(p_3)\right] &(3.43)\\
\mathcal{M}^*_5 &= C^*_5 \left[\overline{u}(p_1) G^{\mu'}_{e_5} v(p_2)\right] g_{\mu'\alpha'} g^{\alpha'\beta'} \left(g_{\beta'\nu'} - \frac{k'_{\beta'} k'_{\nu'}}{m_Z^2}\right) \left[\overline{v}(p_4) G^{\nu'}_{t_5} u(p_3)\right] &(3.44)
\end{aligned}$$

where the vertices are given by (with $n = 1, ...5$)

$$G^{\mu}_{e_n} = \frac{1}{2}\gamma\left[l_{e_n}(1-\gamma_5) + r_{e_n}(1+\gamma_5)\right] \quad , \quad G^{\mu}_{t_n} = \frac{1}{2}\gamma\left[l_{t_n}(1-\gamma_5) + r_{t_n}(1+\gamma_5)\right] \qquad (3.45)$$

For our process (3.2),

$$\begin{aligned}
l_{e_1} &= -1 = l_{e_2} \ , \ r_{e_1} = -1 = r_{e_2},\\
l_{e_3} &= -1 + 2\sin^2\theta_W = l_{e_4} = l_{e_5} \ , \ r_{e_3} = 2\sin^2\theta_W = r_{e_4} = r_{e_5},\\
l_{t_1} &= \frac{2}{3} = l_{t_2} \ , \ r_{t_1} = \frac{2}{3} = r_{t_2},\\
l_{t_3} &= 1 - \frac{4}{3}\sin^2\theta_W = l_{t_4} = l_{t_5} \ , \ r_{t_3} = -\frac{4}{3}\sin^2\theta_W = r_{t_4} = r_{t_5}, &(3.46)
\end{aligned}$$

$$\begin{aligned}
C_1 &= \frac{e^3 m_t}{m_Z \sin(2\theta_W)(q_1^2 - m_t^2)s} \ , \ C_2 = \frac{e^3 m_t}{m_Z \sin(2\theta_W)(q_2^2 - m_t^2)s},\\
C_3 &= \frac{e^3 m_t}{m_Z \sin^3(2\theta_W)(q_1^2 - m_t^2)(s - m_Z^2)} \ , \ C_4 = \frac{e^3 m_t}{m_Z \sin^3(2\theta_W)(q_2^2 - m_t^2)(s - m_Z^2)},\\
C_5 &= \frac{2e^3 m_Z c}{\sin^3(2\theta_W)(k'^2 - m_Z^2)(s - m_Z^2)} &(3.47)
\end{aligned}$$

Here we have not included the decay width parts; hence, all the C's are real except C_5 which can be complex due to the parameter c of the anomalous $ZZ\phi$ vertex. For our parametrization described in Chapter 4, all the three parameters a, b and c are taken to be real; hence, all the C_i's are real in this case.

Now the squared amplitudes for individual helicity states are given by

$$\begin{aligned} |\mathcal{M}_{\rm tot}(h_1,h_2,h_3,h_4)|^2 &= \left|\sum_{n=1}^{5}\mathcal{M}_n(h_1,h_2,h_3,h_4)\right|^2 = \sum_{n=1}^{5}|\mathcal{M}_n|^2 + 2\cdot{\rm Re}\,[\mathcal{M}_1^*\mathcal{M}_2 \\ &\quad +\mathcal{M}_1^*\mathcal{M}_3+\mathcal{M}_1^*\mathcal{M}_4+\mathcal{M}_1^*\mathcal{M}_5+\mathcal{M}_2^*\mathcal{M}_3+\mathcal{M}_2^*\mathcal{M}_4 \\ &\quad +\mathcal{M}_2^*\mathcal{M}_5+\mathcal{M}_3^*\mathcal{M}_4+\mathcal{M}_3^*\mathcal{M}_5+\mathcal{M}_4^*\mathcal{M}_5] \end{aligned} \tag{3.48}$$

All the 15 terms in Eq. (3.48) are calculated, as functions of (h_1,h_2,h_3,h_4), by the trace method:

$$\begin{aligned}
\mathcal{M}_1^*\mathcal{M}_1 &= C_1^2\, g_{\mu\nu}\, g_{\mu'\nu'}\, {\rm Tr}\left[G_{e_1}^{\mu'}P_{e_v}(h_2)G_{e_1}^{\mu}P_{e_u}(h_1)\right]\cdot \\
&\quad {\rm Tr}\left[G_{t_1}^{\nu'}(\not{q}_1+m_t)(a+ib\gamma_5)P_{t_u}(h_3)(a+ib\gamma_5)(\not{q}_1+m_t)G_{t_1}^{\nu}P_{t_v}(h_4)\right] \\
\mathcal{M}_2^*\mathcal{M}_2 &= C_2^2\, g_{\mu\nu}\, g_{\mu'\nu'}\, {\rm Tr}\left[G_{e_2}^{\mu'}P_{e_v}(h_2)G_{e_2}^{\mu}P_{e_u}(h_1)\right]\cdot \\
&\quad {\rm Tr}\left[(a+ib\gamma_5)(\not{q}_2+m_t)G_{t_2}^{\nu'}P_{t_u}(h_3)G_{t_2}^{\nu}(\not{q}_2+m_t)(a+ib\gamma_5)P_{t_v}(h_4)\right] \\
\mathcal{M}_3^*\mathcal{M}_3 &= C_3^2\, g_{\mu\nu}\, g_{\mu'\nu'}\, {\rm Tr}\left[G_{e_3}^{\mu'}P_{e_v}(h_2)G_{e_3}^{\mu}P_{e_u}(h_1)\right]\cdot \\
&\quad {\rm Tr}\left[G_{t_3}^{\nu'}(\not{q}_1+m_t)(a+ib\gamma_5)P_{t_u}(h_3)(a+ib\gamma_5)(\not{q}_1+m_t)G_{t_3}^{\nu}P_{t_v}(h_4)\right] \\
\mathcal{M}_4^*\mathcal{M}_4 &= C_4^2\, g_{\mu\nu}\, g_{\mu'\nu'}\, {\rm Tr}\left[G_{e_4}^{\mu'}P_{e_v}(h_2)G_{e_4}^{\mu}P_{e_u}(h_1)\right]\cdot \\
&\quad {\rm Tr}\left[(a+ib\gamma_5)(\not{q}_2+m_t)G_{t_4}^{\nu'}P_{t_u}(h_3)G_{t_4}^{\nu}(\not{q}_2+m_t)(a+ib\gamma_5)P_{t_v}(h_4)\right] \\
\mathcal{M}_5^*\mathcal{M}_5 &= C_5^2\, g_{\mu\alpha}\, g_{\mu'\alpha'}\, g^{\alpha\beta}\, g^{\alpha'\beta'}\left(g_{\beta\nu}-\frac{k'_\beta k'_\nu}{m_Z^2}\right)\left(g_{\beta'\nu'}-\frac{k'_{\beta'}k'_{\nu'}}{m_Z^2}\right)\cdot \\
&\quad {\rm Tr}\left[G_{e_5}^{\mu'}P_{e_v}(h_2)G_{e_5}^{\mu}P_{e_u}(h_1)\right]\cdot{\rm Tr}\left[G_{t_5}^{\nu'}P_{t_u}(h_3)G_{t_5}^{\nu}P_{t_v}(h_4)\right] \\
\mathcal{M}_1^*\mathcal{M}_2 &= C_1C_2\, g_{\mu\nu}\, g_{\mu'\nu'}\, {\rm Tr}\left[G_{e_1}^{\mu'}P_{e_v}(h_2)G_{e_2}^{\mu}P_{e_u}(h_1)\right]\cdot \\
&\quad {\rm Tr}\left[G_{t_1}^{\nu'}(\not{q}_1+m_t)(a+ib\gamma_5)P_{t_u}(h_3)G_{t_2}^{\nu}(\not{q}_2+m_t)(a+ib\gamma_5)P_{t_v}(h_4)\right] \\
\mathcal{M}_1^*\mathcal{M}_3 &= C_1C_3\, g_{\mu\nu}\, g_{\mu'\nu'}\, {\rm Tr}\left[G_{e_1}^{\mu'}P_{e_v}(h_2)G_{e_3}^{\mu}P_{e_u}(h_1)\right]\cdot \\
&\quad {\rm Tr}\left[G_{t_1}^{\nu'}(\not{q}_1+m_t)(a+ib\gamma_5)P_{t_u}(h_3)(a+ib\gamma_5)(\not{q}_1+m_t)G_{t_3}^{\nu}P_{t_v}(h_4)\right] \\
\mathcal{M}_1^*\mathcal{M}_4 &= C_1C_4\, g_{\mu\nu}\, g_{\mu'\nu'}\, {\rm Tr}\left[G_{e_1}^{\mu'}P_{e_v}(h_2)G_{e_4}^{\mu}P_{e_u}(h_1)\right]\cdot \\
&\quad {\rm Tr}\left[G_{t_1}^{\nu'}(\not{q}_1+m_t)(a+ib\gamma_5)P_{t_u}(h_3)G_{t_4}^{\nu}(\not{q}_2+m_t)(a+ib\gamma_5)P_{t_v}(h_4)\right]
\end{aligned}$$

$$
\begin{aligned}
\mathcal{M}_1^*\mathcal{M}_5 &= C_1C_5\, g_{\mu\alpha}\, g^{\alpha\beta}\left(g_{\beta\nu} - \frac{k'_\beta k'_\nu}{m_Z^2}\right) g_{\mu'\nu'} \cdot \mathrm{Tr}\left[G_{e_1}^{\mu'} P_{e_v}(h_2) G_{e_5}^{\mu} P_{e_u}(h_1)\right] \cdot \\
&\quad \mathrm{Tr}\left[G_{t_1}^{\nu'}(\not{q}_1 + m_t)(a + ib\gamma_5) P_{t_u}(h_3) G_{t_5}^{\nu} P_{t_v}(h_4)\right] \\
\mathcal{M}_2^*\mathcal{M}_3 &= C_2C_3\, g_{\mu\nu}\, g_{\mu'\nu'}\, \mathrm{Tr}\left[G_{e_2}^{\mu'} P_{e_v}(h_2) G_{e_3}^{\mu} P_{e_u}(h_1)\right] \cdot \\
&\quad \mathrm{Tr}\left[(a + ib\gamma_5)(\not{q}_2 + m_t) G_{t_2}^{\nu'} P_{t_u}(h_3)(a + ib\gamma_5)(\not{q}_1 + m_t) G_{t_3}^{\nu} P_{t_v}(h_4)\right] \\
\mathcal{M}_2^*\mathcal{M}_4 &= C_2C_4\, g_{\mu\nu}\, g_{\mu'\nu'}\, \mathrm{Tr}\left[G_{e_2}^{\mu'} P_{e_v}(h_2) G_{e_4}^{\mu} P_{e_u}(h_1)\right] \cdot \\
&\quad \mathrm{Tr}\left[(a + ib\gamma_5)(\not{q}_2 + m_t) G_{t_2}^{\nu'} P_{t_u}(h_3) G_{t_4}^{\nu}(\not{q}_2 + m_t)(a + ib\gamma_5) P_{t_v}(h_4)\right] \\
\mathcal{M}_2^*\mathcal{M}_5 &= C_2C_5\, g_{\mu\alpha}\, g^{\alpha\beta}\left(g_{\beta\nu} - \frac{k'_\beta k'_\nu}{m_Z^2}\right) g_{\mu'\nu'} \cdot \mathrm{Tr}\left[G_{e_2}^{\mu'} P_{e_v}(h_2) G_{e_5}^{\mu} P_{e_u}(h_1)\right] \cdot \\
&\quad \mathrm{Tr}\left[(a + ib\gamma_5)(\not{q}_2 + m_t) G_{t_2}^{\nu'} P_{t_u}(h_3) G_{t_5}^{\nu} P_{t_v}(h_4)\right] \\
\mathcal{M}_3^*\mathcal{M}_4 &= C_3C_4\, g_{\mu\nu}\, g_{\mu'\nu'}\, \mathrm{Tr}\left[G_{e_3}^{\mu'} P_{e_v}(h_2) G_{e_4}^{\mu} P_{e_u}(h_1)\right] \cdot \\
&\quad \mathrm{Tr}\left[G_{t_3}^{\nu'}(\not{q}_1 + m_t)(a + ib\gamma_5) P_{t_u}(h_3) G_{t_4}^{\nu}(\not{q}_2 + m_t)(a + ib\gamma_5) P_{t_v}(h_4)\right] \\
\mathcal{M}_3^*\mathcal{M}_5 &= C_3C_5\, g_{\mu\alpha}\, g^{\alpha\beta}\left(g_{\beta\nu} - \frac{k'_\beta k'_\nu}{m_Z^2}\right) g_{\mu'\nu'} \cdot \mathrm{Tr}\left[G_{e_3}^{\mu'} P_{e_v}(h_2) G_{e_5}^{\mu} P_{e_u}(h_1)\right] \cdot \\
&\quad \mathrm{Tr}\left[G_{t_3}^{\nu'}(\not{q}_1 + m_t)(a + ib\gamma_5) P_{t_u}(h_3) G_{t_5}^{\nu} P_{t_v}(h_4)\right] \\
\mathcal{M}_4^*\mathcal{M}_5 &= C_4C_5\, g_{\mu\alpha}\, g^{\alpha\beta}\left(g_{\beta\nu} - \frac{k'_\beta k'_\nu}{m_Z^2}\right) g_{\mu'\nu'} \cdot \mathrm{Tr}\left[G_{e_4}^{\mu'} P_{e_v}(h_2) G_{e_5}^{\mu} P_{e_u}(h_1)\right] \cdot \\
&\quad \mathrm{Tr}\left[(a + ib\gamma_5)(\not{q}_2 + m_t) G_{t_4}^{\nu'} P_{t_u}(h_3) G_{t_5}^{\nu} P_{t_v}(h_4)\right] \qquad (3.49)
\end{aligned}
$$

where

$$
\begin{aligned}
P_{e_u}(h_1) &= \frac{1}{2}(1 + h_1\gamma_5)\ \not{p}_1\ ,\ P_{e_v}(h_2) = \frac{1}{2}(1 - h_2\gamma_5)\ \not{p}_2\ , \\
P_{t_u}(h_3) &= \frac{1}{2}(1 + h_3\gamma_5\ \not{S}'_3)(\not{p}_3 + m_t),\ P_{t_v}(h_4) = \frac{1}{2}(1 + h_4\gamma_5\ \not{S}'_4)(\not{p}_4 - m_t) \qquad (3.50)
\end{aligned}
$$

from Eqs. (3.32) and (3.34). The four-vectors S'_3 and S'_4 are defined as in Eq (3.28), but omitting the helicity term which has already been factored out in Eqs. (3.50):

$$
S'_3 = \frac{1}{m_t}\left(|\mathbf{p}_3|; \frac{E_3\mathbf{p}_3}{|\mathbf{p}_3|}\right)\ \ ,\ \ S'_4 = \frac{1}{m_t}\left(|\mathbf{p}_4|; \frac{E_4\mathbf{p}_4}{|\mathbf{p}_4|}\right) \qquad (3.51)
$$

The analytical expressions for the helicity states (3.48) have been obtained using a combination of the packages `FORM` and `MATHEMATICA`; however, the expressions are too lengthy to be included here.

3.4 THE PRODUCTION CROSS SECTION

In this section, we calculate the differential and total production cross section for the process (3.2). The analytical expressions for the SM case were first obtained in Ref.[44]. We extend the calculation to the general case, including the anomalous couplings. Then the calculation becomes quite involved due to the complicated structure of amplitude. Hence, we do the integration numerically using the *Monte Carlo method*.

3.4.1 THE DIFFERENTIAL CROSS SECTION

Differential cross section of the 3-body final state (3.2) is given by [53]

$$d\sigma = \frac{(2\pi)^4}{2I} |\overline{\mathcal{M}}_{fi}|^2 dR_3(P; p_3, p_4, p_5) \tag{3.52}$$

where

$$I = \sqrt{[s-(m_1+m_2)^2]\,[s-(m_1-m_2)^2]} = s$$

as $m_1 = m_2 = m_e \to 0$ and

$$|\overline{\mathcal{M}}_{fi}|^2 = \frac{3}{4} |\mathcal{M}_{\text{tot}}|^2$$

($\frac{1}{4}$ for average over initial beam polarizations and 3 for the color factor of the quarks).

Thus in the center-of-mass (c.o.m.) frame, we have

$$d\sigma = \frac{(2\pi)^4}{2s} |\overline{\mathcal{M}}_{fi}|^2 dR_3(P; p_3, p_4, p_5) \tag{3.53}$$

where $P = (\sqrt{s}; 0, 0, 0)$ and

$$dR_3(P; p_3, p_4, p_5) = \frac{d^3\mathbf{p}_3}{(2\pi)^3} \frac{1}{2E_3} \frac{d^3\mathbf{p}_4}{(2\pi)^3} \frac{1}{2E_4} \frac{d^3\mathbf{p}_5}{(2\pi)^3} \frac{1}{2E_5} \delta^{(4)}(P - p_3 - p_4 - p_5) \tag{3.54}$$

where $p_1 + p_2 = P = p_3 + p_4 + p_5$ in the c.o.m. frame.

In order to express Eq. (3.54) in a simple form, we split the 3-body decay into a set of two 2-body decays [54] as shown in Figure 3.3. Then we can do the 3-body kinematics in terms of two 2-body problems: one in the c.o.m. frame with momenta p_3 and p_K while the other in the K-rest frame with p'_4 and p'_5. Thus we have

$$\begin{aligned} P &= (\sqrt{s}; 0, 0, 0) = p_3 + p_K \quad \text{(in the c.o.m. frame)} \\ p_K &= (m_K; 0, 0, 0) = p'_4 + p'_5 \quad \text{(in the } K\text{-rest frame)} \end{aligned}$$

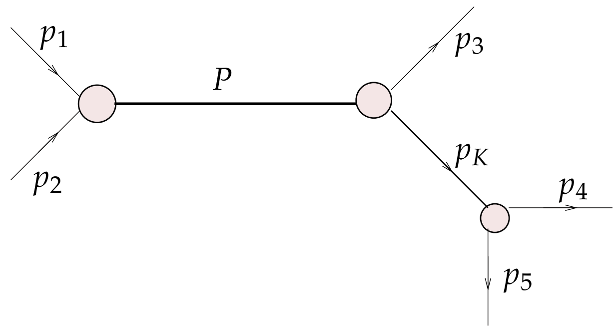


Figure 3.3: A 3-body decay split into a set of two 2-body decays in the c.o.m. frame.

p'_4 and p'_5 are then to be boosted and rotated into the c.o.m. frame to yield the values p_4 and p_5 respectively[4].

Separating these two parts and inserting the identity

$$1 = \int dK^2 \int \frac{d^3\mathbf{p}_K}{2E_K} \delta^{(4)}(p_K - p'_4 - p'_5)$$

where $E_K^2 = \mathbf{p}_K^2 + m_K^2$ and $m_K^2 = K^2$. We can write Eq. (3.54) as

$$\begin{aligned} dR_3 &= (2\pi)^3 dK^2 \overbrace{\frac{d^3\mathbf{p}_K}{(2\pi)^3}\frac{1}{2E_K}\frac{d^3\mathbf{p}_3}{(2\pi)^3}\frac{1}{2E_3}\delta^{(4)}(P - p_K - p_3)}^{\text{c.o.m. frame}} \cdot \\ & \underbrace{\frac{d^3\mathbf{p}_4}{(2\pi)^3}\frac{1}{2E_4}\frac{d^3\mathbf{p}_5}{(2\pi)^3}\frac{1}{2E_5}\delta^{(4)}(p_K - p'_4 - p'_5)}_{K-\text{rest frame}} \end{aligned} \tag{3.55}$$

It can be easily shown that

$$\begin{aligned} \frac{d^3\mathbf{p}_K}{(2\pi)^3}\frac{1}{2E_K}\frac{d^3\mathbf{p}_3}{(2\pi)^3}\frac{1}{2E_3}\delta^{(4)}(P - p_K - p_3) &= \frac{1}{4(2\pi)^6} d(\cos\theta_3) d\phi_3 \frac{|\mathbf{p}_3|}{E_{\text{cm}}}, \\ \frac{d^3\mathbf{p}_4}{(2\pi)^3}\frac{1}{2E_4}\frac{d^3\mathbf{p}_5}{(2\pi)^3}\frac{1}{2E_5}\delta^{(4)}(p_K - p'_4 - p'_5) &= \frac{1}{4(2\pi)^6} d(\cos\theta'_4) d\phi'_4 \frac{|\mathbf{p}'_4|}{m_K} \end{aligned}$$

Without any loss of generality, we can assume $\phi_3 = 0$ so that we can replace $\int d\phi_3$ by 2π straightaway. Then Eq. (3.53) becomes

$$d\sigma = \frac{1}{2^9\pi^4}\, d\phi'_4\, d(\cos\theta_3)\, d(\cos\theta'_4) d(K^2) \left(\frac{b_s}{s}\right) |\overline{\mathcal{M}}_{fi}|^2 \tag{3.56}$$

[4]In practice, we need to boost and rotate only one of the two momenta as the other one will be fixed by four-momentum conservation.

where

$$b_s = \frac{|\mathbf{p}_3|}{\sqrt{s}} \frac{|\mathbf{p}_4'|}{m_K} \tag{3.57}$$

is the phase-space volume.

3.4.2 Rotation and Boost

In the K-rest frame, the four vector p_4' is defined as

$$p_4' = (E_4'; |\mathbf{p}_4'| \sin\theta_4' \cos\phi_4', |\mathbf{p}_4'| \sin\theta_4' \sin\phi_4', |\mathbf{p}_4'| \cos\theta_4') \tag{3.58}$$

First it has to be boosted along the z-axis by the boost operator

$$B_z = \begin{pmatrix} \gamma & 0 & 0 & \gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma\beta & 0 & 0 & \gamma \end{pmatrix} \tag{3.59}$$

where $\gamma = \frac{E_K}{m_K}$ and $\gamma\beta = \frac{|\mathbf{p}_K|}{m_K}$. Then (3.58) has to be rotated about the y-axis by an angle $\theta' = \pi - \theta_3$ followed by a rotation about the z-axis by an angle $\phi' = \pi - \phi_3 = \pi$ (as we have chosen $\phi_3 = 0$) by the rotation operators given by

$$R_y = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta' & 0 & \sin\theta' \\ 0 & 0 & 1 & 0 \\ 0 & -\sin\theta' & 0 & \cos\theta' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\cos\theta_3 & 0 & \sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & -\sin\theta_3 & 0 & -\cos\theta_3 \end{pmatrix} \tag{3.60}$$

and

$$R_z = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\phi' & -\sin\phi' & 0 \\ 0 & \sin\phi' & \cos\phi' & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3.61}$$

respectively. Thus the total transformation matrix is given by

$$T = R_z \cdot R_y \cdot B_z = \begin{pmatrix} \gamma & 0 & 0 & \gamma\beta \\ -\gamma\beta\sin\theta_3 & \cos\theta_3 & 0 & -\gamma\sin\theta_3 \\ 0 & 0 & -1 & 0 \\ -\gamma\beta\cos\theta_3 & -\sin\theta_3 & 0 & -\gamma\cos\theta_3 \end{pmatrix} \tag{3.62}$$

Thus the four-vector p_4 in the c.o.m. frame is given by

$$p_4 = (E_4; |\mathbf{p}_4| \sin\theta_4 \cos\phi_4, |\mathbf{p}_4| \sin\theta_4 \sin\phi_4, |\mathbf{p}_4| \cos\theta_4) = T \cdot p_4' \tag{3.63}$$

We define p_3 as

$$p_3 = (E_3; |\mathbf{p}_3| \sin\theta_3, , |\mathbf{p}_3| \cos\theta_3) \tag{3.64}$$

Then p_5 becomes fixed by four-momentum conservation:

$$p_5 = (E_{\rm cm} - E_3 - E_4; -\mathbf{p}_3 - \mathbf{p}_4) \tag{3.65}$$

The following relations will be useful in the numerical integration:

$$\begin{aligned} E_3 &= \frac{1}{2E_{\rm cm}}(E_{\rm cm}^2 + m_t^2 - m_K^2)\,, \\ E_4' &= \frac{1}{2m_K}(m_K^2 + m_t^2 - m_H^2) \end{aligned} \tag{3.66}$$

3.4.3 THE TOTAL CROSS SECTION

The total production cross section is obtained by integrating the expression (3.56):

$$\sigma_{\rm tot} = \frac{1}{2^9\pi^4}\int_0^{2\pi} d\phi_4' \int_{-1}^{+1} d(\cos\theta_3) \int_{-1}^{+1} d(\cos\theta_4') \int_{(m_H+m_t)^2}^{(s-m_t)^2} d(K^2)\left(\frac{b_s}{s}\right)|\overline{\mathcal{M}}_{fi}|^2 \tag{3.67}$$

Before proceeding to analyze the general case including the non-standard parts, we first verify the known results for the SM case in which $a = 1 = c$ and $b = 0$ in the $t\bar{t}\phi$ and $ZZ\phi$ couplings. The integrated cross sections are shown in Figure 3.4 for various cases [5]. As already observed in Ref.[44], the dominant mode is the Higgs radiation off the top quarks. The contribution from the Higgs emission off Z boson is always less than a few percent in the whole range of energy shown here. Hence, this process provides a chance for direct measurement of the $tt\phi$ Yukawa coupling [45]. Also we note that the Z exchange contribution for the Higgs emission off t is considerably smaller (roughly by a factor of $\sin^2\theta_W$) as compared to that for γ exchange, as already noted in [46].

In Figure 3.5, we show the variations of cross section with c.o.m. energy for some representative values of Higgs mass and with Higgs mass for some fixed c.o.m. energy values. From Figure 3.5(a), we see that at a given c.o.m. energy, the cross section decreases with increase in the Higgs mass; this is due to the reduction of available phase space. On the other hand, it can be seen from Figure 3.5(b) that with increase in the c.o.m. energy, the cross section decreases slightly for small Higgs masses, a consequence of scaling, while they fall off less steeply for large Higgs masses.

[5] We use the *CompHEP* values (See Appendix E) for all the SM parameters used in our calculation unless otherwise specified, in order to be able to make a direct and authentic check of our results.

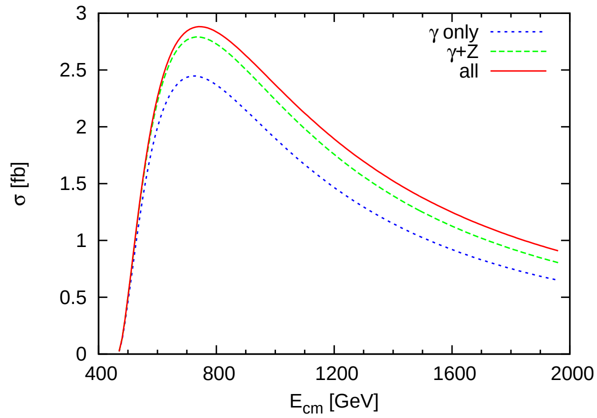
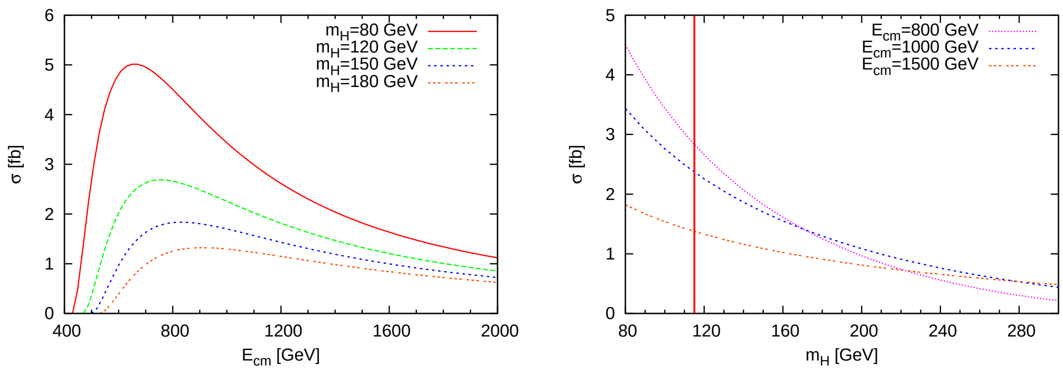


Figure 3.4: Total cross section for $t\bar{t}\phi$ for $m_t = 174.3$ GeV and $m_H = 115$ GeV, as a function of the c.o.m. energy. In (a) the contribution coming only from the photon exchange is shown. In (b) both the γ and Z exchange contributions for Higgs emission off t is shown. The total SM cross section, including the contribution coming from the Higgs emission off Z, is illustrated in (c).

We can also obtain the polarized cross sections for individual helicity states by plugging in the appropriate squared matrix element in the expression (3.67):

$$\sigma(h_1,h_2,h_3,h_4) = \frac{1}{2^9\pi^4}\int_0^{2\pi} d\phi_4' \int_{-1}^{+1} d(\cos\theta_3) \int_{-1}^{+1} d(\cos\theta_4') \int_{(m_H+m_t)^2}^{(s-m_t)^2} d(K^2)\left(\frac{b_s}{s}\right)\cdot |\overline{\mathcal{M}}_{fi}(h_1,h_2,h_3,h_4)|^2 \tag{3.68}$$

where $|\overline{\mathcal{M}}_{fi}(h_1,h_2,h_3,h_4)|^2 = 3|\mathcal{M}_{\rm tot}(h_1,h_2,h_3,h_4)|^2$. Note that here we do not have the $\frac{1}{4}$ factor as we have fixed the initial helicity states. The cross sections for all the 8 helicity combinations are shown in Figure 3.6 as a function of the c.o.m. energy. We observe that the cross section values are the same when both the t and $\bar{t}$ are produced with the same helicity for a given initial state. We will discuss more about the polarized cross sections in Chapter 4 while constructing the polarization asymmetries both with and without initial beam polarizations.



(a) (b)

Figure 3.5: Total cross section (a) as a function of the c.o.m. energy for four values of the Higgs mass m_H = 80, 120, 150, and 180 GeV, and (b) as a function of the Higgs boson mass for three energy values $E_{\rm cm}$ = 800, 1000 and 1500 GeV, with the SM Higgs mass shown as a vertical line at 115 GeV.

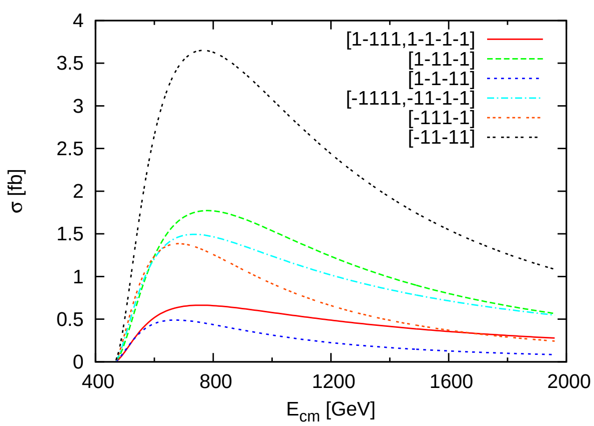


Figure 3.6: The polarized cross sections for all the 8 helicity combinations as functions of the c.o.m. energy with m_H = 115 GeV. The unpolarized total cross section given in Figure 3.4(c) will be obtained by adding these 8 polarized cross sections and then averaging over the initial states (by multiplying a factor of $\frac{1}{4}$).

CHAPTER 4

PROBE OF THE HIGGS PSEUDO-SCALAR COUPLING PARAMETER

CP violation allows simultaneous existence of terms in the interaction Lagrangian with opposite CP transformation properties. The neutral Higgs sector is unique in the sense that a single Higgs boson coupling to a massive fermion is enough to manifest CP violation as long as the Yukawa coupling contains both scalar and pseudo-scalar components. Therefore, this CP-violating aspect is the most interesting part of the Higgs physics beyond the SM once a neutral Higgs boson is identified.

In Chapter 2, we have introduced the most general Lorentz invariant form of the $t\bar{t}\phi$ Yukawa coupling (2.1):

$$g_{t\bar{t}\phi} = ig_2 \frac{m_t}{2m_W}(a + i\gamma_5 b)$$

We parametrize this coupling with $|a|^2 + |b|^2 = 1$, keeping in mind that for the SM case, we have $a = 1,\ b = 0$. Similarly, the $ZZ\phi$ coupling (2.2):

$$g_{ZZ\phi} = c\frac{g_2 m_Z}{c_W} g_{\mu\nu}$$

can be parametrized with $c = a$ so that for the SM case $c = 1$. Here we have taken the three parameters $a,\ b,\ c$ to be real and to be related to each other in a natural model-independent way, because a CP-mixed state of the Higgs is expected to reduce the scalar coupling from the SM value by exactly the same amount for both the vertices. Due to this particular way of parametrization (in which we have only one independent real parameter), the only possible CP violating term in the squared matrix element will be ab. However, if we treat $a,\ b$, and c as independent parameters [55], then we will have an additional CP-violating term bc. In principle, one may indeed have $a,\ b$ and c unrelated to each other in a particular model, e.g. in THDM [49]. Thus the parametrization we have adopted, though model-independent, is not the most general.

In this chapter, we study the sensitivity of the Higgs pseudo-scalar coupling parameter b to both cross section and polarization asymmetry measurements. In the following two sections, we study the variation of cross section and polarization asymmetry with b, which we take to be the independent parameter.

4.1 UNPOLARIZED INITIAL BEAMS

We define the final state polarization asymmetry P_t as

$$P_t = \frac{\sigma(t_L) - \sigma(t_R)}{\sigma(t_L) + \sigma(t_R)}, \tag{4.1}$$

where $\sigma(t_{L,R})$ are the cross sections for producing t with left- and right-handed helicities, respectively.

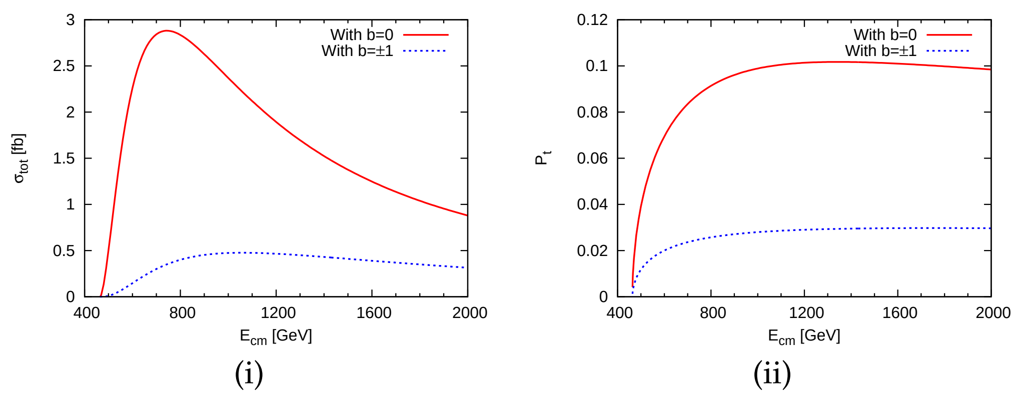
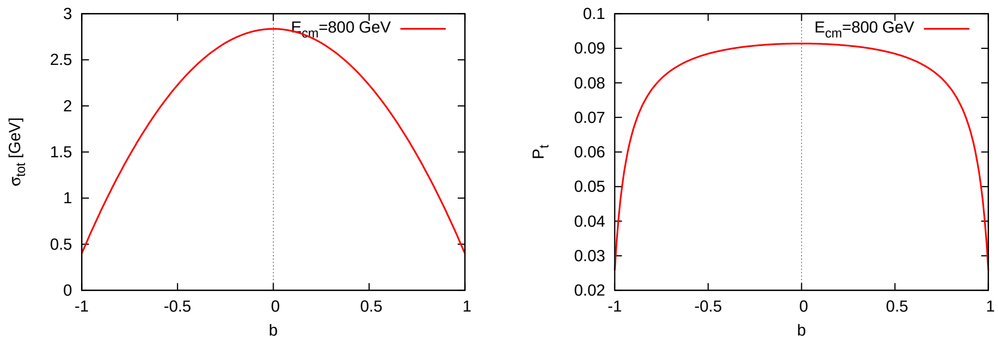


Figure 4.1: Variation of (i) σ_{tot} and (ii) P_t with E_{cm} for $b = 0$ and $b = 1$.

Figure 4.1 (i) shows the variation of the total production cross section, $\sigma_{\text{tot}} = \sigma(t_L) + \sigma(t_R)$, with the c.o.m. energy for a scalar ($b = 0$) as well as a pseudo-scalar ($b = 1$) Higgs boson. Figure 4.1 (ii) shows the variation of top polarization asymmetry P_t with the c.o.m. energy. It is clear from these figures that the two cases ($b = 0$ and $b = 1$) yield remarkably different values for cross section as well as the top polarization asymmetry. Hence, in principle, both these measurements can be used for determination of the CP property of the Higgs boson. However, in practice, the cross section values can change due to higher order radiative corrections coming from various sectors while the polarization asymmetry is insensitive to these corrections. Therefore, the polarization asymmetry can be a very useful observable for CP studies of the Higgs boson.

Figures 4.2 (i) and (ii) show the variations of σ_{tot} and P_t respectively with the pseudo-scalar coupling parameter b for a fixed c.o.m. energy $E_{\text{cm}} = 800$ GeV. As we can see from this Figure, though the asymmetry values are quite different for the



(i) (ii)

Figure 4.2: (i) Variation of (i) $\sigma_{\rm tot}$ and (ii) P_t with b for $E_{\rm cm} = 800$ GeV.

two cases [$P_t(b = 0) \simeq 3.5 P_t(b = \pm 1)$], and hence, can be clearly distinguished from each other by measurements, it is not so sensitive to the variation of b unless b is very close to 1. Therefore, it may not be a good observable to determine a CP-mixed state of the Higgs boson, as we will see later.

As can be seen from Figure 4.2(i), the unpolarized cross section (after being integrated over the whole phase space) varies quadratically with b:

$$\sigma_{\rm tot} = [x_t - y_t b^2]{\rm fb} \tag{4.2}$$

At $E_{\rm cm} = 800$ GeV, $x_t = 2.8355$ and $y_t = 2.4354$. Similarly for all the helicity states,we find

$$\begin{aligned}
\sigma_1 &\equiv \sigma[1,-1,1,1] = [0.6591 - 0.4613\, b^2]{\rm fb}\,, \\
\sigma_2 &\equiv \sigma[1,-1,1,-1] = [1.7695 - 1.6904\, b^2]{\rm fb}\,, \\
\sigma_3 &\equiv \sigma[1,-1,-1,1] = [0.4364 - 0.4122\, b^2]{\rm fb}\,, \\
\sigma_4 &\equiv \sigma[1,-1,-1,-1] = [0.6591 - 0.4613\, b^2]{\rm fb}\,, \\
\sigma_5 &\equiv \sigma[-1,1,1,1] = [1.4658 - 1.0276\, b^2]{\rm fb}\,, \\
\sigma_6 &\equiv \sigma[-1,1,1,-1] = [1.2584 - 1.1941\, b^2]{\rm fb}\,, \\
\sigma_7 &\equiv \sigma[-1,1,-1,1] = [3.6279 - 3.4675\, b^2]{\rm fb}\,, \\
\sigma_8 &\equiv \sigma[-1,1,-1,-1] = [1.4658 - 1.0276\, b^2]{\rm fb}
\end{aligned} \tag{4.3}$$

Now $\sigma(t_L)$ and $\sigma(t_R)$ are respectively given by

$$\begin{aligned}
\sigma(t_L) &= \frac{1}{4}[\sigma_3 + \sigma_4 + \sigma_7 + \sigma_8] = [1.5473 - 1.3421\, b^2]{\rm fb}, \\
\sigma(t_R) &= \frac{1}{4}[\sigma_1 + \sigma_2 + \sigma_5 + \sigma_6] = [1.2882 - 1.0933\, b^2]{\rm fb}
\end{aligned} \tag{4.4}$$

The variation of P_t with b is given by cf. Eq. (4.1)

$$P_t = \frac{x_{lr} - y_{lr} b^2}{x_t - y_t b^2} \tag{4.5}$$

where $x_{lr} = 0.2591$ and $y_{lr} = 0.2488$ at $E_{\rm cm} = 800$ GeV.

Here we make one important observation: Numerically, the coefficient of the CP-odd term ab vanishes altogether for individual helicity states when integrated over the whole phase space. Of course, this is expected to be zero in the total cross section as it is an CP-even function. However, as we are interested in the CP-odd term ab, we want to have a partial cross section in which this term is non-zero. We have figured out that there is an underlying symmetry w.r.t. the azimuthal angle ϕ'_4, i.e. *up-down symmetry* due to which the ab term vanishes when integrated over the whole range of ϕ'_4 from 0 to 2π. In other words, the contribution for ϕ'_4 from 0 to π is equal in magnitude and opposite in sign to that for ϕ'_4 from π to 2π. As in our parametrization, the only CP-odd term is ab, neither the cross section nor the polarization asymmetry can be a good observable to probe the CP-mixed state of the Higgs boson. This observation will be justified when we make sensitivity studies later in this chapter. An up-down asymmetry of the $\bar{t}$ production w.r.t. the e^--t plane, on the other hand, will provide an excellent probe of CP violating combination ab.

4.2 LONGITUDINALLY POLARIZED INITIAL BEAMS

If we have longitudinally polarized electron beams, the polarized cross section can be written as [56]

$$\begin{aligned}\sigma_{P_{e^-}P_{e^+}} &= \frac{1}{4}\{(1+P_{e^-})(1+P_{e^+})\sigma_{RR} + (1-P_{e^-})(1-P_{e^+})\sigma_{LL} \\ &\quad +(1+P_{e^-})(1-P_{e^+})\sigma_{RL} + (1-P_{e^-})(1+P_{e^+})\sigma_{LR}\}\,,\end{aligned} \tag{4.6}$$

where σ_{RL} stands for the cross section of the process when both the electron and the positron beam are 100% polarized in right-handed e^- and left-handed e^+; σ_{LL}, σ_{RR} and σ_{LR} are defined analogously. $P_{e^\pm}$ denote the longitudinal polarizations of the initial $e^\pm$ beams and their values range between -1 and +1; we use the right-handed helicity basis, so that $P_{e^\pm} < 0$ means that the beam is left-handed polarized.

In the case of e^+e^- annihilation into a vector particle (in our case γ/Z^0) with total angular momentum $J = 1$ only the two configurations σ_{RL} and σ_{LR} contribute. The cross section for arbitrary beam polarization is then given by

$$\sigma_{P_{e^-}P_{e^+}} \equiv \sigma_t^e = \frac{1+P_{e^-}}{2}\,\frac{1-P_{e^+}}{2}\,\sigma_{RL} + \frac{1-P_{e^-}}{2}\,\frac{1+P_{e^+}}{2}\,\sigma_{LR}$$

$$
\begin{aligned}
&= (1 - P_{e^-}P_{e^+}) \, \frac{\sigma_{LR} + \sigma_{RL}}{4} \left[1 - \frac{P_{e^-} - P_{e^+}}{1 - P_{e^-}P_{e^+}} \, \frac{\sigma_{LR} - \sigma_{RL}}{\sigma_{LR} + \sigma_{RL}} \right] \\
&= (1 - P_{e^-}P_{e^+}) \, \sigma_0 \, [1 - P_{eff} A_{LR}], \qquad (4.7)
\end{aligned}
$$

$$
\begin{aligned}
\text{with the unpolarized cross section} : \sigma_0 &= \frac{\sigma_{LR} + \sigma_{RL}}{4}, \qquad (4.8) \\
\text{the left} - \text{right asymmetry} : A_{LR} &= \frac{\sigma_{LR} - \sigma_{RL}}{\sigma_{LR} + \sigma_{RL}}, \qquad (4.9) \\
\text{and the effective polarization} : P_{eff} &= \frac{P_{e^-} - P_{e^+}}{1 - P_{e^-}P_{e^+}} \qquad (4.10)
\end{aligned}
$$

The collision cross sections can be enhanced if both beams are polarized and if P_{e^-} and P_{e^+} have opposite sign, cf. Eq (4.7). The realistic values of P_{e^-} and P_{e^+} for which the effective luminosity (i.e. the fraction of interacting particles) is maximum, are -0.8 and +0.6, respectively [56]. Hence, we have chosen these values for our analysis with polarized initial beams.

We can define the final state polarization asymmetry P_t exactly as in Eq. 4.1:

$$
P_t^e = \frac{\sigma_t^e(t_L) - \sigma_t^e(t_R)}{\sigma_t^e(t_L) + \sigma_t^e(t_R)}, \qquad (4.11)
$$

In Figure 4.3 we have shown the variation of $|P_t^e|$ with $E_{\rm cm}$ for three representative sets of initial beam configurations: one with the electron beam completely left-handed ($P_{e^-} = -1$) and the positron beam right-handed ($P_{e^+} = +1$), the second with the spin directions reversed and the third one with realistic values: $P_{e^-} = -0.8$ and $P_{e^+} = +0.6$. In the second case, P_t^e turns out to be -ve, so we have taken its absolute value in the plot. For comparison, the unpolarized case is also shown here. As expected, the polarization asymmetry gets enhanced due to initial beam polarizations. We also note that the polarization asymmetry is maximum if the electron beam completely right-handed and the positron beam left-handed.

As a function of b, σ_t^e and P_t^e at $E_{\rm cm} = 800$ GeV are calculated as follows:

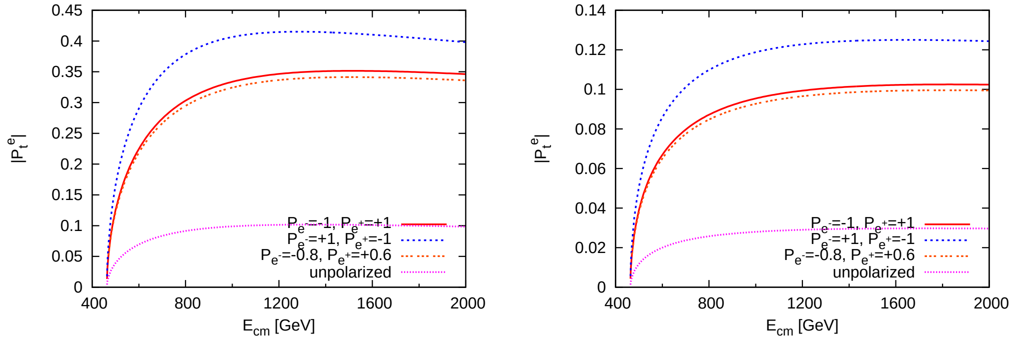
$$
\begin{aligned}
\sigma_{LR} &= \sigma_5 + \sigma_6 + \sigma_7 + \sigma_8 = [7.8179 - 6.7167\, b^2]\text{fb} \, , \\
\sigma_{RL} &= \sigma_1 + \sigma_2 + \sigma_3 + \sigma_4 = [3.5242 - 3.0251\, b^2]\text{fb} \qquad (4.12)
\end{aligned}
$$

Hence using Eq. (4.7) we get for $P_{e^-} = -0.8$ and $P_{e^+} = 0.6$,

$$
\sigma_t^e = [x_t^e - y_t^e \, b^2]\text{fb} \qquad (4.13)
$$

where $x_t^e = 5.6994$ and $y_t^e = 4.8966$. Further,

$$
\begin{aligned}
\sigma_{LR}(t_L) &= \sigma_7 + \sigma_8 = [5.0937 - 4.4951\, b^2]\text{fb} \\
\sigma_{LR}(t_R) &= \sigma_5 + \sigma_6 = [2.7242 - 2.2216\, b^2]\text{fb} \\
\sigma_{RL}(t_L) &= \sigma_3 + \sigma_4 = [1.0956 - 0.8734\, b^2]\text{fb} \\
\sigma_{RL}(t_R) &= \sigma_1 + \sigma_2 = [2.4286 - 2.1517\, b^2]\text{fb} \qquad (4.14)
\end{aligned}
$$



(i) (ii)

Figure 4.3: (i) Variation of $|P_t^e|$ with E_{cm} for (i) $b = 0$ and (ii) $b = 1$.

Then Eq. (4.13) can be split into two parts:

$$\begin{aligned} \sigma_t^e(t_L) &= [3.6894 - 3.2539\, b^2]\text{fb} \\ \sigma_t^e(t_R) &= [2.0100 - 1.6426\, b^2]\text{fb} \end{aligned} \tag{4.15}$$

Then using Eq. (4.11) we get

$$P_t^e = \frac{x_{lr}^e - y_{lr}^e\, b^2}{x_t^e - y_t^e\, b^2} \tag{4.16}$$

where $x_{lr}^e = 1.6794$ and $y_{lr}^e = 1.6113$.

4.3 SENSITIVITY

If one wishes to test the hypothesis that the value $b = b_0$ is the real value of b, one needs to demand that the change $\Delta(b)$ in an observable O as b_0 is changed to $b_0 \pm \Delta b_0$ is more than the statistical fluctuation in $O(b)$, i.e. we would say that Δb is the sensitivity at $b = b_0$ if

$$|O(b) - O(b_0)| = \Delta O(b_0) \tag{4.17}$$

for $|b - b_0| < \Delta b$. Now applying this condition to our observables, namely the cross section and polarization asymmetry, we must have

$$|\sigma(b) - \sigma(b_0)| = \Delta\sigma(b_0) \tag{4.18}$$

$$|P_t(b) - P_t(b_0)| = \Delta P_t(b_0) \tag{4.19}$$

to say that $b = b_0$ is sensitive to these measurements. The sensitivity limits can be obtained by solving these equations for Δb.

At a luminosity L, the statistical fluctuations in the measurements of cross section and polarization asymmetry at a confidence level f are respectively given by

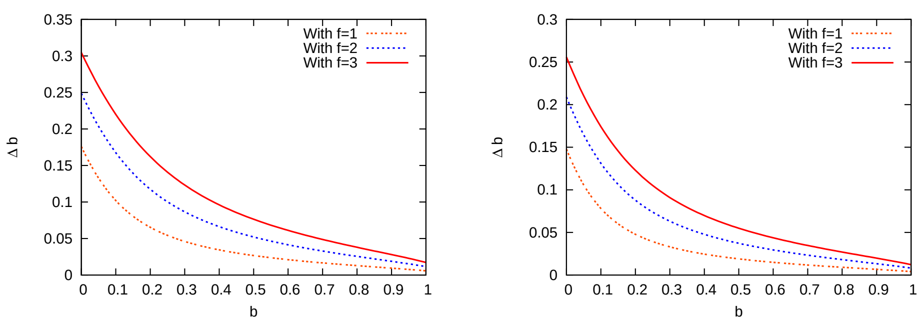
$$\Delta\sigma = f\sqrt{\frac{\sigma}{L}} \tag{4.20}$$

$$\Delta P_t = \frac{f}{\sqrt{\sigma L}}\sqrt{1-P_t^2} \tag{4.21}$$

For unpolarized initial beams, the only physically acceptable solution for $|\Delta b|$, as obtained from Eq. (4.18) using Eq. (4.2), is

$$\Delta b = -b_0 + \sqrt{b_0^2 + c_f} \tag{4.22}$$

where $c_f = \frac{\Delta\sigma_t(b_0)}{y_t}$. This is plotted in Figure 4.4(a) for 1σ, 2σ and 3σ C.L. Here we have taken the luminosity L to be 500fb^{-1}. Similarly, for polarized initial beams, using Eq. (4.13) we get the solution as given by Eq. (4.22), but with $c_f = \frac{\Delta\sigma_t^e(b_0)}{y_t^e}$. This is plotted in Figure 4.4(b) for 1σ, 2σ and 3σ C.L.



(a) (b)

Figure 4.4: Sensitivity plots for b using cross section for (a) unpolarized initial beams and (b) polarized beams with $P_{e^-} = -0.8$, $P_{e^+} = 0.6$.

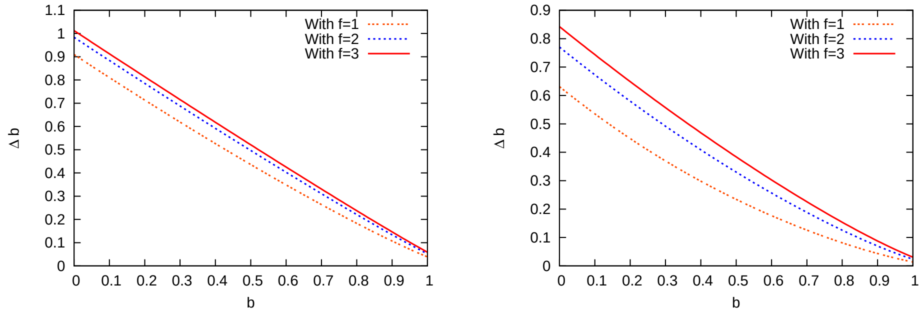
Similarly, using polarization asymmetry as the observable, we obtain the following acceptable solution from Eq. (4.19) using Eq. (4.5) for unpolarized initial beam:

$$\Delta b = -b_0 + \sqrt{\frac{\Delta P_t(b_0)x_t(x_t - b_0^2 y_t) + b_0^2(x_t y_{lr} - x_{lr} y_t)}{(x_t(y_{lr} + \Delta P_t(b_0)y_t) - y_t(x_{lr} + b_0^2 \Delta P_t(b_0)y_t))}} \tag{4.23}$$

This is plotted in Figure 4.5(a) for 1σ, 2σ and 3σ C.L.. For polarized initial beams, we use Eq. (4.16) to obtain the solution

$$\Delta b = -b_0 + \sqrt{\frac{\Delta P_t^e(b_0)x_t^e(x_t^e - b_0^2 y_t^e) + b_0^2(x_t^e y_{lr}^e - x_{lr}^e y_t^e)}{(x_t^e(y_{lr}^e + \Delta P_t^e(b_0)y_t^e) - y_t^e(x_{lr}^e + b_0^2 \Delta P_t^e(b_0)y_t^e))}} \tag{4.24}$$

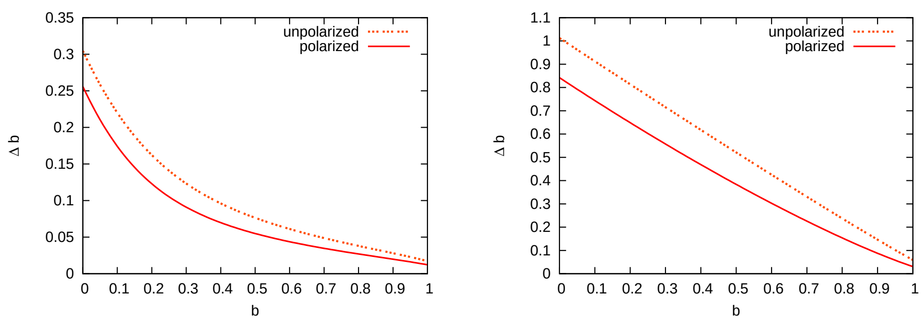
This is plotted in Figure 4.5(b) for 1σ, 2σ and 3σ C.L..



(a) (b)

Figure 4.5: Sensitivity plots for b using polarization asymmetry of the top quark for (a) unpolarized initial beams and (b) polarized beams with $P_{e^-} = -0.8$, $P_{e^+} = 0.6$.

We thus find that neither the cross section nor the polarization asymmetry measurement is very sensitive to b except for b values close to 1, although initial beam polarization enhances the sensitivity by some amount, as shown in Figure 4.6.



(a) (b)

Figure 4.6: Comparison of sensitivity plots at 3σ C.L. for unpolarized and polarized initial beams with $P_{e^-} = -0.8$, $P_{e^+} = 0.6$: (a) in cross section measurement and (b) in polarization asymmetry measurements.

Nevertheless, the polarization asymmetry is a very good observable to distinguish a purely CP-even state of the Higgs from a purely CP-odd state. This is, anyway, useful because in a model in which both these states have the same mass, the CP-odd state can not be determined in other conventional ways such as the Higgs decay to W or Z bosons [57] because a pure CP-odd state does not couple to vector bosons. In this case, for instance, the polarization asymmetry measurement can instead be used to determine the pure CP-odd state of the Higgs.

CHAPTER 5

HEAVY QUARK DECAY

A direct consequence of weak interactions is the decay of the heavier leptons and quarks into lighter particles. Decays are important not only as a test of the SM model but also as a means to detect new physics effects in experiments. In this chapter, we discuss the two possible decay modes of a heavy quark depending on its mass. This work is not directly relevant for the main project work discussed earlier. However, we plan to study, at a later stage, the energy and angular distributions of the top quark which are extremely useful probes of non-standard effects, as we have already discussed in §1.8.2; hence it is instructive to know the details of a heavy quark decay in general. In particular, this work was done to get a feel for the time-scale involved in top decay and to make a comparison with the hadronization time scale to see that the former is indeed much smaller than the latter.

We have studied the decay width of a heavy-quark as a function of its mass. The situation for charged-current heavy-quark decay depends on whether $m_Q < m_W$ or $m_Q > m_W$. In the former case, Q decays in the usual 3-body manner through a *virtual* W; in the latter case, it decays into a *real* W and a lighter quark. Because of the nearly-diagonal character of quark mixing matrix (1.83), the most favored route for a heavy quark charged-current decay is either to the same generation (e.g. $t \to b$) or – if this is kinematically impossible – to the nearest generation (e.g. $b \to c$). As a result, heavy quark decays go preferentially via a cascade:

$$\begin{aligned} c &\to s\,, \\ b &\to c \to s\,, \\ t &\to b \to c \to s\,, \end{aligned}$$

with real or virtual W emission at each stage. These cascade decays have many interesting experimental consequences [58].

5.1 QUARK DECAYS TO REAL W BOSONS

A heavy quark with mass $m_Q > m_W + m_q$ will decay into a real W boson and a lighter quark q, as illustrated in Figure 5.1. Denoting the four-momenta by particle labels, the

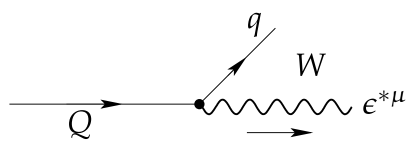


Figure 5.1: Charged-current weak decay of a heavy quark Q for $m_Q > m_W$.

matrix element for this decay is

$$\mathcal{M} = -i\frac{g_2}{2\sqrt{2}} V_{Qq} \left[\overline{u}(q)\gamma_\mu(1-\gamma_5)u(Q)\right] \epsilon_W^{*\mu} \tag{5.1}$$

where V_{Qq} is the CKM mixing element for QqW vertex, and ϵ_W^μ is the W polarization vector. The spin-averaged matrix element squared is given by

$$\begin{aligned}
|\overline{\mathcal{M}}|^2 &= \frac{1}{2}\sum|\mathcal{M}|^2 \\
&= \frac{g_2^2}{16}|V_{Qq}|^2 \sum_{\text{spins}} \left[\overline{u}(q)\gamma_\mu(1-\gamma_5)u(Q)\right]\left[\overline{u}(Q)\gamma_\nu(1-\gamma_5)u(q)\right] \sum_{\text{polarizations}} \epsilon_W^{*\mu}\epsilon_W^\nu \\
&= \frac{g_2^2}{16}|V_{Qq}|^2 \cdot \mathrm{Tr}\left[(\not{q}+m_q)\gamma_\mu(1-\gamma_5)(\not{Q}+m_Q)\gamma_\nu(1-\gamma_5)\right]\left(-g^{\mu\nu}+\frac{W^\mu W^\nu}{m_W^2}\right) \\
&= \frac{g_2^2}{16}|V_{Qq}|^2 \times 8\left[q_\mu Q_\nu + Q_\mu q_\nu - g_{\mu\nu}(q\cdot Q) - i\epsilon_{\rho\mu\lambda\nu}q^\rho Q^\lambda\right]\left(-g^{\mu\nu}+\frac{W^\mu W^\nu}{m_W^2}\right) \\
&= \frac{g_2^2}{2}|V_{Qq}|^2\left[q\cdot Q + \frac{2(q\cdot W)(Q\cdot W)}{m_W^2}\right]
\end{aligned} \tag{5.2}$$

The decay rate for this process is given by

$$\Gamma_2(Q\to qW) = \frac{1}{2E_Q}\int \frac{d^3\mathbf{q}}{(2\pi)^3 2E_q}\frac{d^3\mathbf{W}}{(2\pi)^3 2E_W}(2\pi)^4\delta^4(Q-W-q)\times|\overline{\mathcal{M}}|^2 \tag{5.3}$$

Using the delta function (i.e. four-momentum conservation), the dot products in Eq. (5.2) can be expressed in terms of the masses of the on-shell particles:

$$q\cdot Q = \frac{1}{2}\left(m_Q^2+m_q^2-m_W^2\right),\ q\cdot W = \frac{1}{2}\left(m_Q^2-m_q^2-m_W^2\right),\ Q\cdot W = \frac{1}{2}\left(m_Q^2-m_q^2+m_W^2\right)$$

Thus Eq. (5.2) becomes

$$|\overline{\mathcal{M}}|^2 = \frac{g_2^2}{4m_W^2}|V_{Qq}|^2\left[(m_Q^2 - m_q^2)^2 + m_W^2(m_Q^2 + m_q^2) - 2m_W^4\right] \tag{5.4}$$

To take into account the running of couplings with the mass scale, which is m_W in this case, we introduce $\alpha(m_W^2)$:

$$G = \frac{g_2^2\sqrt{2}}{8m_W^2} = \frac{\pi\alpha(m_W^2)}{\sqrt{2}x_W(m_W^2)m_W^2)} \tag{5.5}$$

with the numerical values $\alpha(m_W^2) \approx \frac{1}{128}$ and $x_W(m_W^2) \approx 0.23$ [11]. Thus Eq. (5.4) becomes

$$|\overline{\mathcal{M}}|^2 = G\sqrt{2}|V_{Qq}|^2\left[(m_Q^2 - m_q^2)^2 + m_W^2(m_Q^2 + m_q^2) - 2m_W^4\right] \tag{5.6}$$

Now Eq. (5.3) can be written as

$$\Gamma_2 = \frac{1}{2E_Q}|\overline{\mathcal{M}}|^2(2\pi)^4 R_2 \tag{5.7}$$

where the Lorentz invariant phase space (LIPS) is given by [1] [54]

$$R_2 = \frac{1}{(2\pi)^6}\frac{\pi}{2}\lambda^{\frac{1}{2}}\left(1, \frac{m_W^2}{m_Q^2}, \frac{m_q^2}{m_Q^2}\right) \tag{5.8}$$

where $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ca$. In the Q rest frame we obtain from Eq. (5.7)

$$\Gamma_2 = \frac{Gm_Q^3}{8\pi\sqrt{2}}|V_{Qq}|^2\lambda^{\frac{1}{2}}\left(1, \frac{m_W^2}{m_Q^2}, \frac{m_q^2}{m_Q^2}\right)\left[\left(1 - \frac{m_q^2}{m_Q^2}\right)^2 + \frac{m_W^2}{m_Q^2}\left(1 + \frac{m_q^2}{m_Q^2}\right) - 2\frac{m_W^4}{m_Q^4}\right] \tag{5.9}$$

For $m_q^2 \ll m_Q^2$, this simplifies to

$$\Gamma_2 = \frac{Gm_Q^3}{8\pi\sqrt{2}}|V_{Qq}|^2\left(1 - \frac{m_W^2}{m_Q^2}\right)^2\left(1 + \frac{2m_W^2}{m_Q^2}\right) \tag{5.10}$$

from which we conclude that $\Gamma_2 \sim m_Q^3$ for $m_Q \gg m_W$. this results from the longitudinal part of the W polarization tensor [59].

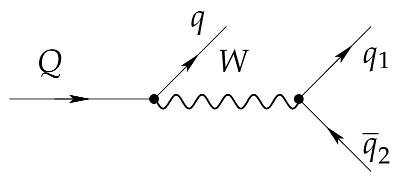


Figure 5.2: Charged-current weak decay of a heavy quark Q for $m_Q < m_W$.

5.2 QUARK DECAYS TO VIRTUAL W BOSONS

The matrix element for this process is given by

$$\begin{aligned}\mathcal{M} &= \left(\frac{-ig_2}{2\sqrt{2}}\right)^2 V_{Qq}\left[\overline{u}(q)\gamma_\mu(1-\gamma_5)u(Q)\right]\left\{\frac{-i}{W^2-m_W^2}\left(g^{\mu\nu}-\frac{k^\mu k^\nu}{m_W^2}\right)\right\}\cdot \\ &\quad \left[\overline{u}(q_1)\gamma_\nu(1-\gamma_5)u(q_2)\right] \qquad (5.11)\end{aligned}$$

where $k = Q - q$. Hence the spin-averaged matrix element squared is given by

$$\begin{aligned}|\overline{\mathcal{M}}|^2 &= \frac{1}{2}\sum|\mathcal{M}|^2 = \frac{1}{2}\left(\frac{g_2^2}{8}\right)^2\frac{|V_{Qq}|^2}{(W^2-m_W^2)^2}\sum_{spins}\left[\overline{u}(q)\gamma_\mu(1-\gamma_5)u(Q)\right] \\ &\quad \left[\bar{u}(q_1)\gamma_{\mu'}(1-\gamma_5)u(q_2)\right]\left[\bar{u}(Q)\gamma_\nu(1-\gamma_5)u(q)\right]\left[\bar{u}(q_2)\gamma_{\nu'}(1-\gamma_5)u(q_1)\right] \\ &\quad \left(g^{\mu\mu'}-\frac{k^\mu k^{\mu'}}{m_W^2}\right)\left(g^{\nu\nu'}-\frac{k^\nu k^{\nu'}}{m_W^2}\right) \\ &= \frac{1}{2}\left(\frac{Gm_W^2}{\sqrt{2}}\right)^2\frac{|V_{Qq}|^2}{(W^2-m_W^2)^2}\cdot \mathrm{Tr}\left[(\not{q}+m_q)\gamma_\mu(1-\gamma_5)(\not{Q}+m_Q)\gamma_\nu(1-\gamma_5)\right]\cdot \\ &\quad \mathrm{Tr}\left[\not{q}_1\gamma_{\mu'}(1-\gamma_5)\ \not{q}_2\gamma_{\nu'}(1-\gamma_5)\right]\left(g^{\mu\mu'}-\frac{k^\mu k^{\mu'}}{m_W^2}\right)\left(g^{\nu\nu'}-\frac{k^\nu k^{\nu'}}{m_W^2}\right) \\ &= \frac{1}{2}\left(\frac{Gm_W^2}{\sqrt{2}}\right)^2\frac{|V_{Qq}|^2}{(W^2-m_W^2)^2}\times 256(Q\cdot q_1)(q\cdot q_2) \\ &= \frac{64G^2|V_{Qq}|^2}{\left(1-\frac{W^2}{m_W^2}\right)^2}(Q\cdot q_1)(q\cdot q_2) \qquad (5.12)\end{aligned}$$

where we have assumed that the masses of q_1, q_2 are much smaller than m_W and hence negligible. In the rest frame of Q, we have

$$Q\cdot q_1 = m_Q E_1 \ \text{, and}\ \ q\cdot q_2 = \frac{1}{2}(m_Q^2 - m_q^2 - 2m_Q E_1) \qquad (5.13)$$

[1]The matrix element does not have any momentum dependence in this case; hence we are able to take it outside the phase space integral.

Hence Eq. (5.12) becomes

$$|\overline{\mathcal{M}}|^2 = \frac{32G^2 m_Q |V_{Qq}|^2}{\left(1-\frac{W^2}{m_W^2}\right)^2} E_1[m_Q^2 - m_q^2 - 2m_Q E_1] \tag{5.14}$$

The differential decay rate for the process is given by

$$d\Gamma_3 = \frac{1}{2E_Q} |\overline{\mathcal{M}}|^2 (2\pi)^4 dR_3 \,, \tag{5.15}$$

where the differential LIPS is given by

$$\begin{aligned} dR_3 &= \frac{d^3\mathbf{q}}{(2\pi)^3 2E_q} \frac{d^3\mathbf{q}_1}{(2\pi)^3 2E_1} \frac{d^3\mathbf{q}_2}{(2\pi)^3 2E_2} \delta^4(Q - q - q_1 - q_2) \\ &= \frac{1}{8(2\pi)^8} \frac{dE_1\, d^3\mathbf{q}}{E_q\, |\mathbf{q}|} I \end{aligned} \tag{5.16}$$

where

$$I = \int_{p_-}^{p_+} \delta(m_Q - p - E_q - E_1) dp, \tag{5.17}$$

and $p_\pm = |\mathbf{q}| \pm |\mathbf{q}_1|$. Hence

$$I = \begin{cases} 1 & \text{if } p_- < m_Q - E_q - E_1 < p_+ \\ 0 & \text{otherwise} \end{cases} \tag{5.18}$$

This also defines the range of the E_1 integral:

$$E_\pm = \frac{m_Q^2 + m_q^2 - 2m_Q E_q}{2(m_Q - E_q \mp |\mathbf{q}|)} \tag{5.19}$$

Now Eq. (5.15) becomes

$$d\Gamma_3 = \frac{1}{(2\pi)^4} \frac{2G^2}{\left(1-\frac{W^2}{m_W^2}\right)^2} \frac{d^3\mathbf{q}}{E_q |\mathbf{q}|} J(E_q) \tag{5.20}$$

where using Eq. (5.19), we get

$$\begin{aligned} J(E_q) &= \int_{E_-}^{E_+} dE_1\, E_1 \left[m_Q^2 - m_q^2 - 2m_Q E_1\right] \\ &= \frac{|\mathbf{q}|}{6} \left(3m_Q^2 E_q + 3m_q^2 E_q - 4m_Q E_q^2 - 2m_Q m_q^2\right) \end{aligned} \tag{5.21}$$

Now using $d^3\mathbf{q} = |\mathbf{q}|E_q dE_q d\Omega$, Eq. (5.20) can be written as

$$d\Gamma_3 = \frac{|V_{Qq}|^2}{(2\pi)^3} \frac{2G^2}{\left(1 - \frac{W^2}{m_W^2}\right)^2} \frac{|\mathbf{q}|}{6} \left(3m_Q^2 E_q + 3m_q^2 E_q - 4m_Q E_q^2 - 2m_Q m_q^2\right) dE_q \tag{5.22}$$

Introducing the variables $x \equiv 2E_q/m_Q$ and $a \equiv (m_Q/m_W)^2$, we rewrite (5.22) in the form given in Ref.[59]

$$\frac{d\Gamma_3}{dx} = \left(\frac{G^2 m_Q^5}{192\pi^3}\right) |V_{Qq}|^2 \frac{2x^2(3-2x)}{[(1-a)+ax]^2} \tag{5.23}$$

We have assumed that $q_1\bar{q}_2$ are leptons; if they are quarks, then Eq (5.23) is multiplied by a color factor of 3.

Integrating Eq (5.23) over the interval $0 \leq x \leq 1$ we get

$$\frac{\Gamma_3}{\Gamma_0} = F(a) = -\frac{2}{a} + \frac{6}{a^4}\left[2a - a^2 + 2(1-a)\ln(1-a)\right] \tag{5.24}$$

where $\Gamma_0 = \frac{G^2 m_Q^5}{192\pi^3}|V_{Qq}|^2$. For small enough a (or, equivalently, $m_Q \ll m_W$), we have

$$F(a) \simeq 1 + \frac{3}{5}a + \frac{2}{5}a^2 + ... \tag{5.25}$$

Thus we conclude that $\Gamma_3 \sim m_Q^5$ for $m_Q \ll m_W$.

5.3 THE GENERAL CASE

Near W threshold the expression for decay width has to be modified due to the finite W width which provides a smooth transition to the conventional weak 3-body decay below W threshold. To derive an expression for the total decay width of a heavy quark as a function of its mass, we consider the sequential decay where the W may be either

$$\begin{array}{l} Q \longrightarrow W + q \\ \qquad\quad \lfloor\!\longrightarrow F\bar{f} \end{array}$$

real or virtual. The matrix element for this transition is given by the four-fermion effective interaction, multiplied by a W propagation factor [58]

$$\mathcal{M} = \mathcal{M}(m_W \longrightarrow \infty) \frac{-m_W^2}{W^2 - m_W^2 + i m_W \Gamma_W} \tag{5.26}$$

where W is the virtual mass and Γ_W is the finite width of the W decay. Denoting the four-momenta by particle labels as before, we get

$$\mathcal{M} = -i\frac{V_{Qq}m_W^2}{W^2 - m_W^2 + im_W\Gamma_W}\frac{G}{\sqrt{2}}\left[\overline{u}(q)\gamma_\mu(1-\gamma_5)u(Q)\right]\cdot \\ \left[\overline{u}(F)\gamma_\nu(1-\gamma_5)v(f)\right]\left(-g^{\mu\nu} + \frac{W^\mu W^\nu}{m_W^2}\right) \tag{5.27}$$

where

$$G = \frac{\pi\alpha(W^2)}{\sqrt{2}x_W(W^2)m_W^2} \tag{5.28}$$

Squaring Eq (5.27) and summing over the spin states, we get

$$\begin{aligned} |\overline{\mathcal{M}}|^2 &= \frac{1}{2}\sum|\mathcal{M}|^2 = \frac{|V_{Qq}|^2 m_W^4}{(W^2-m_W^2)^2 + m_W^2\Gamma_W^2}\frac{G^2}{4}\times 64 \\ &\times \Bigg[4(F\cdot q)(f\cdot Q) + \frac{2}{m_W^2}\left[q^2(F^2(Q\cdot f) + f^2(Q\cdot F) - Q^2(F^2(q\cdot f) + f^2(q\cdot F))\right] \\ &+ \frac{1}{m_W^4}\left[(F^2+f^2)(F\cdot f) + 2F^2f^2\right]\left[(Q^2+q^2)(Q\cdot q) - 2q^2Q^2\right]\Bigg] \end{aligned} \tag{5.29}$$

The differential decay rate is given by

$$d\Gamma = \frac{1}{2m_Q}|\overline{\mathcal{M}}|^2\frac{1}{(2\pi)^5}\delta^4(Q-q-F-f)\prod_i\frac{d^3\mathbf{p}_i}{2E_i} \tag{5.30}$$

We define the following dimensionless quantities [60]:

$$x = \frac{W^2}{Q^2},\ w = \frac{m_W^2}{Q^2},\ \gamma = \frac{\Gamma_W^2}{Q^2},\ \alpha = \frac{F^2}{Q^2},\ \beta = \frac{f^2}{Q^2},\ \delta = \frac{q^2}{Q^2} \tag{5.31}$$

Integrating Eq (5.30) first over the $F, \overline{f}$ phase space and then over the q, W phase space, we obtain the differential decay rate

$$\begin{aligned} \frac{d\Gamma}{dx} &= \Gamma_0\frac{w^2}{(x-w)^2 + w\gamma}\lambda^{\frac{1}{2}}(1,\delta,x)\lambda^{\frac{1}{2}}(x,\alpha,\beta) \\ &\times \Bigg[\frac{2}{x^2}\lambda(x,\alpha,\beta)(1+\delta-x) + \frac{2}{x^3}\left[x(x+\alpha+\beta) - 2(\alpha-\beta)^2\right](1-\delta-x)(1+x-\delta) \\ &+ \frac{3}{x^2w^2}(x-2w)\left[(\alpha+\beta)x - (\alpha-\beta)^2\right]\left[(1-\delta)^2 - (1+\delta)x\right]\Bigg] \end{aligned} \tag{5.32}$$

where $\lambda(a,b,c)$ and Γ_0 are defined earlier. The limits on the variable x are

$$(\sqrt{\alpha}+\sqrt{\beta})^2 \leq x \leq (1-\sqrt{\delta})^2 \tag{5.33}$$

In the limiting case $\alpha=\beta=\gamma=0$, Eq (5.32) reproduces the results stated earlier.

We are interested in the effects of finite Γ_W; hence we keep the γ term in Eq (5.32). However, for simplicity and without any loss of generality, we can put $\alpha=\beta=0$. Then Eq (5.32) can be written as [53]

$$\Gamma = \Gamma_0 F\left(\frac{m_Q^2}{m_W^2}, \frac{m_q^2}{m_Q^2}, \frac{\Gamma_W^2}{m_W^2}\right) \tag{5.34}$$

where

$$F(a,b,c) = 2\int_0^{(1-\sqrt{b})^2} dx \frac{f_Q(b,x)\sqrt{1+b^2+x^2-2(b+bx+x)}}{[(1-ax)^2+c]} \tag{5.35}$$

and

$$f_Q(x,y) = (1-x)^2+(1+x)y-2y^2 \tag{5.36}$$

Putting $m_q=0$, we get

$$\begin{aligned} F(a,0,c) &= 2\int_0^1 dx \frac{f_t(0,x)\sqrt{1+x^2-2x}}{[(1-ax)^2+c]} \\ &= 2\int_0^1 dx \frac{1-3x^2+2x^3}{X} \end{aligned} \tag{5.37}$$

where $X=(1-ax)^2+c$. Eq (5.37) contains some standard integrals involving $X = a_0x^2+b_0x+c_0$ and can be evaluated analytically [61]. Substituting the expressions for these integrals, we get

$$F(a,0,c) = \frac{2}{a^4}\left[A[c-3(1-a)]+2a(2-a)-Ba\left[3c(2-a)-(2+a)(1-a)^2\right]\right] \tag{5.38}$$

where

$$A = \ln\frac{c+1}{c+(1-a)^2}, \quad B = \frac{1}{a\sqrt{c}}\left[\tan^{-1}\left(\frac{1}{\sqrt{c}}\right)-\tan^{-1}\left(\frac{1-a}{\sqrt{c}}\right)\right] \tag{5.39}$$

Hence the decay width (5.34) is given by

$$\Gamma = \Gamma_0\frac{2}{a^4}\left[A[c-3(1-a)]+2a(2-a)-Ba\left[3c(2-a)-(2+a)(1-a)^2\right]\right] \tag{5.40}$$

with $a=\frac{m_Q^2}{m_W^2}$, $c=\frac{\Gamma_W^2}{m_W^2}$. This function is plotted in Figure 5.3. We have taken $G=G_F=$

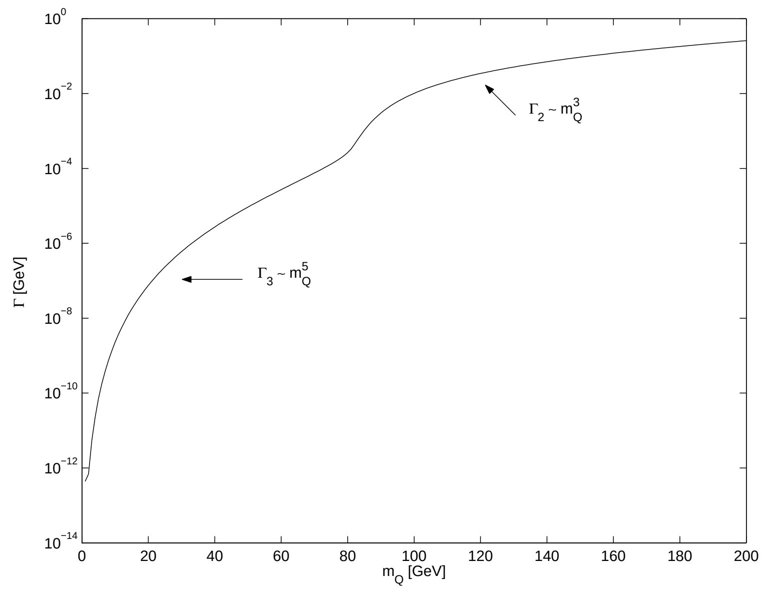


Figure 5.3: Dependence of partial decay width for $Q \to q + W^*$ on m_Q.

1.16×10^{-5} GeV^{-2}, $m_W = 80.4$ GeV and $\Gamma_W = 2.1$ GeV [11]. The change from the m_Q^5 dependence for $m_Q < m_W + m_q$ to the m_Q^3 dependence for $m_Q > m_W + m_q$ is evident.

Now as the mass of top-quark $m_t = 174.3$ GeV is much larger than the W-boson mass $m_W = 80.4$ GeV, the decay mode of the top-quark will be a 2-body decay to a real W-boson along with another down-type quark. The most dominant decay mode is $t \longrightarrow W + b$ as $|V_{tb}|^2 \simeq 1$. Plugging in the mass values in Eq. 5.10 we get

$$\Gamma_{t \to Wb} \sim 1.5 \text{ GeV} \tag{5.41}$$

The life-time of the top-quark is then given by $\Gamma^{-1}_{t \to Wb}$ which is of the order of 10^{-25} sec. This is about 2 orders of magnitude smaller than the typical hadronization time-scale ($\sim 10^{-23}$ sec). Hence, the top-quark decay occurs much before hadronization.

CHAPTER 6

CONCLUSIONS AND FUTURE PROSPECTS

The major part of our work was calculating the analytical expressions of the individual helicity amplitudes as well as squared amplitudes for general $t\bar{t}\phi$ and $ZZ\phi$ vertices. With these expressions at our hand, we can do a lot of interesting physics studies of the anomalous Higgs couplings. So far we have been able to calculate the polarization asymmetry with the hope that it would be a good observable to probe the CP structure of the Higgs boson.

As we have seen from the sensitivity studies in Chapter 4, the polarization asymmetry is indeed a good observable to distinguish a purely CP-even Higgs state from a purely CP-odd one. It is, however, not sensitive to CP-mixing as it is not influenced by the CP-violating term ab. Hence, in order to probe a CP-mixed state of Higgs, we have to think of some other asymmetry which depends on the ab term. We have thought of one such asymmetry, namely the up-down asymmetry constructed out of the azimuthal angle ϕ_4. This angle can be related directly to some cross product of the various momenta which can be measured directly in experiments. Our next aim is to do the sensitivity studies with this asymmetry. We have already noted one interesting feature that this asymmetry is non-zero only for the cases in which the final states have the same helicity, i.e. $(\bar{t}_L, t_L)$ and $(\bar{t}_R, t_R)$. So we can enhance this asymmetry by suitably choosing the initial beam polarizations. Later we also plan to construct other CP-odd observables out of the cross products of various momenta[1].

We can introduce another CP-odd term into the analysis by choosing a different parametrization of the coupling parameters $a,\ b$ and c. As the contribution coming from the $ZZ\phi$ diagram is fairly small, by choosing only c to be different we may not get a much improved sensitivity. Hence, we plan to choose all the three parameters $a,\ b$ and c to be independent, as in Ref.[55], and then to probe the sensitivities of all the parameters to our observables. Also, there are specific predictions for $a,\ b$ and c in the

[1] A similar approach was taken by Bernreuther et al. [62] to investigate the top-quark spin correlations at hadron collider. However, experimentally it is very difficult to measure the spins of the particles unlike their momenta.

framework of *CP*-violating MSSM [63]; we wish to evaluate numerical results for this case.

We can also include the other anomalous couplings derived in Chapter 2 which have momentum dependence in them. The coefficients of these additional terms in the anomalous vertices are expected to be sensitive enough for our observables.

From practical point of view, the most important study would be to include the top decay part and then to calculate the angular distributions of the decay products which are known to be true probes of the non-standard effects in the production of t quark.

BIBLIOGRAPHY

[1] See for instance: T. P. Cheng and L. F. Li, *Gauge theory of elementary particle physics*, Clarendon Press (1984); C. Itzykson and J. B. Zuber, *Quantum Field Theory*, McGraw-Hill (1985); M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory*, Addison-Wesley (1995).

[2] See for instance: F. Halzen and A. D. Martin, *Quarks and Leptons: An Introductory Course in Modern Particle Physics*, John Wiley (1984); D. J. Griffiths, *Introduction to Elementary Particles*, John Wiley (1987); D. H. Perkins, *Introduction to High Energy Physics*, 4th Ed., Cambridge University Press (2000).

[3] S. Glashow, *Nucl. Phys.* **22**, 579 (1961); S. Weinberg, *Phys. Rev. Lett.* **19**, 1264 (1967); A. Salam, in *Elementary Particle Theory*, ed. N. Svartholm, Almqvist and Wiksells, Stockholm (1969) p. 367.

[4] C. N. Yang and R. Mills, *Phys. Rev.* **96**, 191 (1954).

[5] M. Gell-Mann, *Phys. Lett.* **8**, 214 (1964).

[6] J. Goldstone, *Nuovo Cim.* **19**, 154 (1961).

[7] P. W. Higgs, *Phys. Rev. Lett.* **13**, 508 (1964); *ibid. Phys. Rev.* **145**, 1156 (1966); F. Englert and R. Brout, *Phys. Rev. Lett.* **13**, 321 (1964); G. S. Guralnik, C. R. Hagen and T. Kibble, *Phys. Rev. Lett.* **13**, 585 (1964); T. Kibble, *Phys. Rev.* **155**, 1554 (1967).

[8] N. Cabibbo, *Phys. Rev. Lett.* **10**, 531 (1963).

[9] M. Kobayashi and T. Maskawa, *Prog. Theor. Phys.* **49**, 652 (1973).

[10] S. Glashow, J. Illiopoulos and L. Maiani, *Phys. Rev.* **D 2**, 1285 (1970).

[11] W.-M. Yao et al., *J. Phys. G: Nucl. Part. Phys.* **33**, 1 (2006); The Particle Data Group (`http://pdg.lbl.gov/`).

[12] The LEP Electroweak Working Group (`http://lepewwg.web.cern.ch/LEPEWWG/`).

[13] The LEP Collab., `arXiv:hep-ex/0412015` (2004).

[14] See for instance: G. C. Branco, L. Lavoura, and J. P. Silva, *CP Violation*, Clarendon Press (1999); I. I. Bigi and A. I. Sanda, *CP Violation*, Cambridge University Press (2000).

[15] J. H. Christenson et al., *Phys. Rev. Lett.* **13**, 138 (1964).

[16] K. Lande et al., *Phys. Rev.* **103**, 1901 (1956).

[17] K. Abe et al., (Belle Collab.), arXiv:hep-ex/0308034 (2003); B. Aubert et al. (BABAR Collab.), *Phys. Rev.* **D 72**, 052008 (2005); Heavy Flavor Averaging Group, Unitarity Triangle Working Group Summer 2005 (http://www.slac.stanford.edu/xorg/hfag/triangle/summer2005/).

[18] W. Grimus and M. N. Rebelo, *Phys. Rep.* **281**, 239 (1997).

[19] P. F. Harrison and H. R. Quinn, editors (BABAR Collab.), SLAC-R-0504 (2005).

[20] For a review, see for instance: C. Quigg, arXiv:hep-ph/0001145 (2000); G. Altarelli, arXiv:hep-ph/0011078 (2000).

[21] A. Sakharov, *JETP Lett.* **5**, 24 (1967).

[22] M. Trodden, arXiv:hep-ph/0411301 (2004).

[23] J. F. Gunion et al., *The Higgs Hunter's Guide*, Addison-Wesley (1990). For an excellent review, see A. Djouadi, arXiv:hep-ph/0503172 (2005); *ibid.* hep-ph/0503173 (2005).

[24] See for instance: R. N. Mohapatra, *Unification and Supersymmetry*, Springer-Verlag (1986).

[25] See for instance: M. Drees, R. M. Godbole, and P. Roy, *Theory and Phenomenology of Sparticles*, World Scientific (2004).

[26] For a review, see for instance: R. M. Godbole, arXiv:hep-ph/0205114 (2002).

[27] G. Weiglein et al. (LHC/LC Study Group), arXiv:hep-ph/0410364.

[28] For a brief review, see for instance: R. M. Godbole et al., arXiv:hep-ph/0404024 (2004).

[29] E. Accomando et al., (Workshop on CP Studies and Non- Standard Physics), CERN 2006-009 (2006).

[30] F. Abe et al. (CDF Collaboration), *Phys. Rev. Lett.* **74**, 2626 (1995); S. Abachi et al. (D0 Collaboration), *Phys. Rev. Lett.* **74**, 2632 (1995).

[31] I. Bigi et al., *Phys. Lett.* **B 181**, 157 (1986).

[32] R. M. Godbole, S. D. Rindani, and R. K. Singh, *JHEP* **12**, 021 (2006).

[33] G. Eilam, J. L. Hewett, and A. Soni, *Phys. Rev. Lett.* **67**, 1979 (1991).

[34] D. Atwood et al., *Phys. Rep.* **347**, 1 (2001).

[35] S. S. Biswal et al., *Phys. Rev.* **D 73**, 035001 (2006).

[36] B. A. Kniehl, *Nucl. Phys.* **B 352**, 1 (1991); *ibid.* **357**, 439 (1991).

[37] M. C. Gonzalez-Garcia, *Int. J. Mod. Phys.* **A 14**, 3121 (1999); V. Barger et al., *Phys. Rev.* **D 67**, 115001 (2003).

[38] C. J. C. Burgess and H. J. Schnitzer, *Nucl. Phys.* **B 228**, 454 (1983); C. N. Leung, S. T. Love, and S. Rao, *Z. Phys.* **C 31**, 433 (1986); W. Buchmuller and D. Wyler, *Nucl. Phys.* **B 268**, 621 (1986); K. Hagiwara et al., *Phys. Rev.* **D 48**, 2182 (1993).

[39] K. Whisnant et al., *Phys. Rev.* **D 56**, 467 (1997).

[40] J. M. Yang and B.-L. Young, *Phys. Rev.* **D 56**, 5907 (1997).

[41] G. J. Gounaris, D. T. Papadamou, and F. M. Renard, *Z. Phys.* **C 76**, 333 (1997).

[42] T. Han et al., *Phys. Rev.* **D 61**, 015006 (1999).

[43] K. J. F. Gaemers and G. J. Gounaris, *Phys. Lett.* **77B**, 379 (1978).

[44] A. Djouadi, J. Kalinowski, and P. M. Zerwas, *Z. Phys.* **C 54**, 255 (1992).

[45] A. Juste and G. Merino, `arXiv:hep-ph/9910301` (1999); H. Baer, S. Dawson, and L. Reina, *Phys. Rev.* **D 61**, 013002 (2000).

[46] S. Dawson and L. Reina, *Phys. Rev.* **D 59**, 054012 (1999).

[47] S. Dittmaier et al., *Phys. Lett.* **B 441**, 383 (1998).

[48] G. Belanger et al., *Phys. Lett.* **B 571**, 163 (2003); Y. Yu et al., *Phys. Lett.* **B 571**, 85 (2003); A. Denner et al., *Phys. Lett.* **B 575**, 290 (2003); *ibid. Nucl. Phys.* **B 680**, 85 (2004).

[49] S. Bar-Shalom et al., *Phys. Rev.* **D 53**, 1162 (1996).

[50] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics*, McGraw Hill (1964).

[51] H. E. Haber, `arXiv:hep-ph/9405376` (1994).

[52] C. Bouchiat and L. Michel, *Nucl. Phys.* **5**, 416 (1958); L. Michel, *Suppl. Nuovo. Cim.* **14**, 95 (1959).

[53] V. I. Borodulin, R. N. Rogalyov, and S. R. Slabospitsky, *COmpendium of RElations*, `arXiv:hep-ph/9507456` (1995).

[54] E. Byckling and K. Kajantie, *Particle Kinematics*, John Wiley (1973).

[55] J. F. Gunion, B. Grzadkowski, and X.-G. He, *Phys. Rev. Lett.* **77**, 5172 (1996).

[56] G. Moortgat-Pick et al., `arXiv:hep-ph/0507011` (2005).

[57] D. Chang, W.-Y. Keung, and I. Phillips, *Phys. Rev.* **D 48**, 3225 (1993); T. Plehn, D. Rainwater, and D. Zeppenfeld, *Phys. Rev. Lett.* **88** 051801 (2002).

[58] V. Barger and R. J. N. Phillips, *Collider Physics*, Addison-Wesley (1997).

[59] T. Z. Rizzo, *Phys. Rev.* **D 23**, 1987 (1981).

[60] V. Barger et al., *Phys. Rev.* **D 30**, 947 (1984).

[61] See for instance: H. B. Dwight, *Tables of Integrals and Other Mathematical Data*, 3rd ed., The Macmillan Company (1957).

[62] W. Bernreuther et al., `arXiv:hep-ph/0410197` (2004).

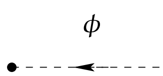
[63] A . Pilaftis and C. E. Wagner, *Nucl. Phys.* **B 533**, 3 (1999); S. Y. Choi, M. Drees, and J. S. Lee, *Phys. Lett.* **B 481**, 57 (2000).

APPENDIX A

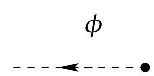
CONVENTION AND PARAMETERS USED

A.1 THE FEYNMAN RULES

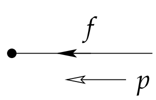
External Leg Contractions



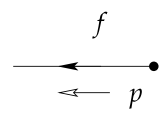
$= 1$



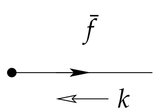
$= 1$



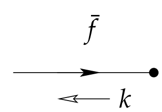
$= u^s(p)$



$= \overline{u}^s(p)$

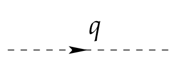


$= \overline{v}^s(k)$

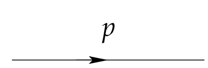


$= v^s(k)$

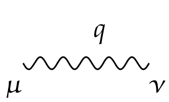
Propagators



$= \frac{i}{q^2 - m_\phi^2 + i\epsilon}$

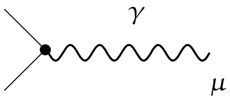


$= \frac{i(\not{p} + m)}{p^2 - m_f^2 + i\epsilon}$

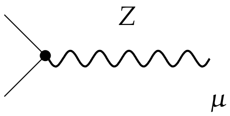


$= \frac{-i}{q^2 - m^2 + i\epsilon}\left(g_{\mu\nu} - (1-\zeta)\frac{q_\mu q_\nu}{q^2 - \zeta m^2 + i\epsilon}\right) = \frac{-i}{q^2 - m^2 + i\epsilon}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m^2 + i\epsilon}\right)$ (in unitary gauge $\zeta \to \infty$)

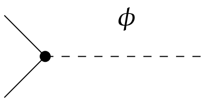
Vertices



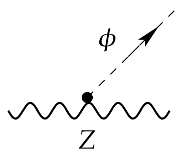
$\Gamma_{\gamma f\bar{f}} = ieQ_f\gamma_\mu$



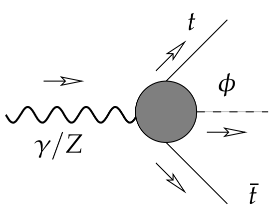
$\Gamma_{Zf\bar{f}} = \frac{ig_2}{2\cos\theta_W}\gamma_\mu[(1-\gamma_5)T_f^{3L} - 2Q_f\sin^2\theta_W)] + g_{f\bar{f}\phi}^{\text{eff}}$



$t\bar{t}\phi = -ig_2\frac{m_t}{2m_W}(a + ib\gamma_5)$



$\Gamma_{ZZ\phi} = -ig_{\mu\nu}c\frac{g_2 m_Z}{\cos\theta_W}$



$\Gamma_{Zt\bar{t}\phi} = g_{Zt\bar{t}\phi}^{\text{eff}}$ & $\Gamma_{\gamma t\bar{t}\phi} = g_{\gamma t\bar{t}\phi}^{\text{eff}}$

A.2 THE COMPHEP PARAMETERS

We have used the following *CompHEP* parameters in our numerical calculations:

$$
\begin{aligned}
m_t &= 174.3 \text{ GeV}\\
m_Z &= 91.1876 \text{ GeV}\\
m_H &= 115.0 \text{ GeV}\\
\Gamma_Z &= 2.43631 \text{ GeV}\\
\Gamma_t &= 1.54688 \text{ GeV}\\
\sin\theta_W &= 0.48076\\
e &= 0.31345\\
\pi &= 4\tan^{-1}(1)
\end{aligned}
$$

APPENDIX B

THE FEYNMAN AMPLITUDES OBTAINED BY HELICITY METHOD

```
F1RL[1,-1,1,1] =
(((-2*I)*E1*(E4 + mt + ptb)*Cos[thtb/2]*(Cos[phtb/2] + I*Sin[phtb/2])*(Cos[tht/2]*(a*(E3 + mt)*
(2*E1 - E4 + mt) - a*(-2*E1 + E4 + mt)*pt + I*b*(-2*E1*E3 + E3*E4 - 2*E1*mt + E3*mt + E4*mt + mt^2 -
2*E1*pt + E4*pt - mt*pt) + (a - I*b)*(E3 + mt + pt)*ptb*Cos[thtb]) + (a - I*b)*(E3 + mt + pt)*ptb*
(Cos[phtb] + I*Sin[phtb])*Sin[tht/2*Sin[thtb]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]))

F1RL[1,-1,1,-1] =
((2*E1*(E4 + mt - ptb)*(Cos[phtb/2] + I*Sin[phtb/2])*Sin[thtb/2]*(Cos[tht/2]*(-(b*(E3 + mt)*
(-2*E1 + E4 + mt)) + 2*b*E1*pt - b*E4*pt + b*mt*pt - I*a*((E4 - mt)*(E3 + mt) + (E4 + mt)*pt -
2*E1*(E3 + mt + pt)) + (I*a + b)*(E3 + mt + pt)*ptb*Cos[thtb]) + (I*a + b)*(E3 + mt + pt)*ptb*
(Cos[phtb] + I*Sin[phtb])*Sin[tht/2*Sin[thtb]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]))

F1RL[1,-1,-1,1] =
((2*E1*(E4 + mt + ptb)*Cos[thtb/2]*(Cos[phtb/2] + I*Sin[phtb/2])*(I*(a*(E3 + mt)*(2*E1 - E4 + mt) +
a*(-2*E1 + E4 + mt)*pt + I*b*(-2*E1*E3 + E3*E4 - 2*E1*mt + E3*mt + E4*mt + mt^2 + 2*E1*pt - E4*pt + mt*pt) +
(a - I*b)*(E3 + mt - pt)*ptb*Cos[thtb])*Sin[tht/2] + (a - I*b)*(E3 + mt - pt)*ptb*Cos[tht/2]*((-I)*
Cos[phtb] + Sin[phtb])*Sin[thtb]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]))

F1RL[1,-1,-1,-1] =
((2*E1*(E4 + mt - ptb)*(Cos[phtb/2] + I*Sin[phtb/2])*Sin[thtb/2]*((-I)*(a*(E3 + mt)*(2*E1 - E4 + mt) +
a*(-2*E1 + E4 + mt)*pt + I*b*(-2*E1*E3 + E3*E4 - 2*E1*mt + E3*mt + E4*mt + mt^2 + 2*E1*pt - E4*pt + mt*pt) +
(a - I*b)*(E3 + mt - pt)*ptb*Cos[thtb])*Sin[tht/2] + (I*a + b)*(E3 + mt - pt)*ptb*Cos[tht/2]*
(Cos[phtb] + I*Sin[phtb])*Sin[thtb]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]))

F1RR[1,-1,1,1] =
(((-2*I)*E1*(E4 + mt - ptb)*Cos[thtb/2]*(Cos[phtb/2] + I*Sin[phtb/2])*(Cos[tht/2]*(a*(E3 + mt)*
(2*E1 - E4 + mt) + a*(-2*E1 + E4 + mt)*pt - I*b*(-2*E1*E3 + E3*E4 - 2*E1*mt + E3*mt + E4*mt + mt^2 +
2*E1*pt - E4*pt + mt*pt) - (a + I*b)*(E3 + mt - pt)*ptb*Cos[thtb]) - (a + I*b)*(E3 + mt - pt)*ptb*
(Cos[phtb] + I*Sin[phtb])*Sin[tht/2*Sin[thtb]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]))

F1RR[1,-1,1,-1] =
((2*E1*(E4 + mt + ptb)*(Cos[phtb/2] + I*Sin[phtb/2])*Sin[thtb/2]*(Cos[tht/2]*(b*(E3 + mt)*
(-2*E1 + E4 + mt) + 2*b*E1*pt - b*E4*pt + b*mt*pt - I*a*((-2*E1 + E4 - mt)*(E3 + mt) -
(-2*E1 + E4 + mt)*pt) + ((-I)*a + b)*(E3 + mt - pt)*ptb*Cos[thtb]) + ((-I)*a + b)*(E3 + mt - pt)*ptb*
(Cos[phtb] + I*Sin[phtb])*Sin[tht/2*Sin[thtb]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]))

F1RR[1,-1,-1,1] =
((2*E1*(E4 + mt - ptb)*Cos[thtb/2]*(Cos[phtb/2] + I*Sin[phtb/2])*((b*(E3 + mt)*(-2*E1 + E4 + mt) -
2*b*E1*pt + b*E4*pt - b*mt*pt - I*a*((E4 - mt)*(E3 + mt) + (E4 + mt)*pt - 2*E1*(E3 + mt + pt)) +
((-I)*a + b)*(E3 + mt + pt)*ptb*Cos[thtb])*Sin[tht/2] + I*(a + I*b)*(E3 + mt + pt)*ptb*Cos[tht/2]*
(Cos[phtb] + I*Sin[phtb])*Sin[thtb]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]))

F1RR[1,-1,-1,-1] =
((2*E1*(E4 + mt + ptb)*(Cos[phtb/2] + I*Sin[phtb/2])*Sin[thtb/2]*((-(b*(E3 + mt)*(-2*E1 + E4 + mt)) +
```

```
2*b*E1*pt - b*E4*pt + b*mt*pt + I*a*((E4 - mt)*(E3 + mt) + (E4 + mt)*pt - 2*E1*(E3 + mt + pt)) +
I*(a + I*b)*(E3 + mt + pt)*ptb*Cos[thtb])*Sin[tht/2] + (a + I*b)*(E3 + mt + pt)*ptb*Cos[tht/2]*((-I)*
Cos[phtb] + Sin[phtb])*Sin[thtb]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]))

F2RL[1,-1,1,1] =
((2*E1*(E3 + mt - pt)*Cos[tht/2]*(Cos[phtb/2] - I*Sin[phtb/2])*((-I)*(a*(2*E1 - E3 + mt)*(E4 + mt) +
a*(-2*E1 + E3 + mt)*ptb + I*b*(-((-2*E1 + E3 + mt)*(E4 + mt)) - 2*E1*ptb + E3*ptb - mt*ptb) -
(a + I*b)*pt*(E4 + mt - ptb)*Cos[tht])*Cos[thtb/2]*(Cos[phtb] + I*Sin[phtb]) +
I*(a + I*b)*pt*(E4 + mt - ptb)*Sin[tht]*Sin[thtb/2]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]))

F2RL[1,-1,1,-1] =
((2*E1*(E3 + mt - pt)*Cos[tht/2]*(Cos[phtb/2] - I*Sin[phtb/2])*(I*(a + I*b)*pt*(E4 + mt + ptb)*Cos[thtb/2]*
Sin[tht] + (-2*b*E1*E4 + b*E3*E4 - 2*b*E1*mt + b*E3*mt + b*E4*mt + b*mt^2 - 2*b*E1*ptb + b*E3*ptb -
b*mt*ptb + I*a*(2*E1*E4 - E3*E4 + 2*E1*mt - E3*mt + E4*mt + mt^2 + 2*E1*ptb - E3*ptb - mt*ptb) + ((-I)*
a + b)*pt*(E4 + mt + ptb)*Cos[tht])*(Cos[phtb] + I*Sin[phtb])*Sin[thtb/2]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]))

F2RL[1,-1,-1,1] =
((2*E1*(E3 + mt + pt)*(Cos[phtb/2] - I*Sin[phtb/2])*Sin[tht/2]*(((E4 + mt)*((2*I)*a*E1 - 2*b*E1 - I*a*E3 +
b*E3 + I*a*mt + b*mt) - (2*I)*a*E1*ptb + 2*b*E1*ptb + I*a*E3*ptb - b*E3*ptb + I*a*mt*ptb + b*mt*ptb +
((-I)*a + b)*pt*(E4 + mt - ptb)*Cos[tht])*Cos[thtb/2]*(Cos[phtb] + I*Sin[phtb]) + ((-I)*a + b)*
pt*(E4 + mt - ptb)*Sin[tht]*Sin[thtb/2]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]))

F2RL[1,-1,-1,-1] =
((2*E1*(E3 + mt + pt)*(Cos[phtb/2] - I*Sin[phtb/2])*Sin[tht/2]*(((-I)*a + b)*pt*(E4 + mt + ptb)*
Cos[thtb/2]*Sin[tht] + (-(b*(-2*E1 + E3 + mt)*(E4 + mt)) + 2*b*E1*ptb - b*E3*ptb + b*mt*ptb -
I*a*(2*E1*E4 - E3*E4 + 2*E1*mt - E3*mt + E4*mt + mt^2 + 2*E1*ptb - E3*ptb - mt*ptb) + I*(a + I*b)*pt*
(E4 + mt + ptb)*Cos[tht])*(Cos[phtb] + I*Sin[phtb])*Sin[thtb/2]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]))

F2RR[1,-1,1,1] =
((2*E1*(E3 + mt + pt)*Cos[tht/2]*(Cos[phtb/2] - I*Sin[phtb/2])*((-2*b*E1*E4 + b*E3*E4 - 2*b*E1*mt + b*E3*
mt + b*E4*mt + b*mt^2 - 2*b*E1*ptb + b*E3*ptb - b*mt*ptb - I*a*(2*E1*E4 - E3*E4 + 2*E1*mt - E3*mt + E4*mt +
mt^2 + 2*E1*ptb - E3*ptb - mt*ptb) + ((-I)*a - b)*pt*(E4 + mt + ptb)*Cos[tht])*Cos[thtb/2]*(Cos[phtb] +
I*Sin[phtb]) + ((-I)*a - b)*pt*(E4 + mt + ptb)*Sin[tht]*Sin[thtb/2]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]))

F2RR[1,-1,1,-1] =
((2*E1*(E3 + mt + pt)*Cos[tht/2]*(Cos[phtb/2] - I*Sin[phtb/2])*(((-I)*a - b)*pt*(E4 + mt - ptb)*Cos[thtb/2]*
Sin[tht] + (b*(2*E1 - E3 - mt)*(E4 + mt) - 2*b*E1*ptb + b*E3*ptb - b*mt*ptb + I*a*(2*E1*E4 - E3*E4 +
2*E1*mt - E3*mt + E4*mt +mt^2 - 2*E1*ptb + E3*ptb + mt*ptb) + (I*a + b)*pt*(E4 + mt - ptb)*Cos[tht])*
(Cos[phtb] + I*Sin[phtb])*Sin[thtb/2]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]))

F2RR[1,-1,-1,1] =
((2*E1*(E3 + mt - pt)*(Cos[phtb/2] - I*Sin[phtb/2])*Sin[tht/2]*((-(b*(-2*E1 + E3 + mt)*(E4 + mt)) +
2*b*E1*ptb - b*E3*ptb + b*mt*ptb + I*a*(2*E1*E4 - E3*E4 + 2*E1*mt - E3*mt + E4*mt + mt^2 + 2*E1*ptb -
E3*ptb - mt*ptb) + (I*a + b)*pt*(E4 + mt + ptb)*Cos[tht])*Cos[thtb/2]*(Cos[phtb] + I*Sin[phtb]) +
(I*a + b)*pt*(E4 + mt + ptb)*Sin[tht]*Sin[thtb/2]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]))

F2RR[1,-1,-1,-1] =
((2*E1*(E3 + mt - pt)*(Cos[phtb/2] - I*Sin[phtb/2])*Sin[tht/2]*((I*a + b)*pt*(E4 + mt - ptb)*
Cos[thtb/2]*Sin[tht] + (a*(2*E1 - E3 + mt)*(E4 + mt) + a*(-2*E1 + E3 + mt)*ptb -
I*b*(-((-2*E1 + E3 + mt)*(E4 + mt)) - 2*E1*ptb + E3*ptb - mt*ptb) + (a - I*b)*pt*(E4 + mt - ptb)*
Cos[tht])*((-I)*Cos[phtb] + Sin[phtb])*Sin[thtb/2]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]))

F3RL[1,-1,1,1] =
(((I/2)*E1*(E3 + mt - pt)*(E4 + mt + ptb)*(Cos[phtb/2] - I*Sin[phtb/2])*(-2*Cos[tht/2]*Cos[thtb/2]*
(2*mz^2 + (pt - ptb)*(E3 + E4 + pt - ptb) + (E3 + E4 + pt - ptb)*(-(pt*Cos[tht]) + ptb*Cos[thtb]))*
(Cos[phtb] + I*Sin[phtb]) + 2*(E3 + E4 + pt - ptb)*(pt + pt*Cos[tht] - ptb*(1 + Cos[thtb])*(Cos[2*phtb] +
I*Sin[2*phtb]))*Sin[tht/2]*Sin[thtb/2]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]*mz^2))

F3RL[1,-1,1,-1] =
(((I/2)*E1*(E3 + mt - pt)*(E4 + mt - ptb)*(Cos[phtb/2] - I*Sin[phtb/2])*((E3 + E4 + pt + ptb)*
Cos[thtb/2]*((pt - 2*ptb*(-1 + Cos[thtb])*(Cos[2*phtb] + I*Sin[2*phtb]))*Sin[tht/2] + pt*Sin[(3*tht)/2]) +
2*Cos[tht/2]*(2*mz^2 + (pt + ptb)*(E3 + E4 + pt + ptb) + (E3 + E4 + pt + ptb)*(-(pt*Cos[tht]) +
ptb*Cos[thtb]))*(Cos[phtb] + I*Sin[phtb])*Sin[thtb/2]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]*mz^2))

F3RL[1,-1,-1,1] =
```

```
(((-I/2)*E1*(E3 + mt + pt)*(E4 + mt + ptb)*(Cos[phtb/2] - I*Sin[phtb/2])*(-2*Cos[thtb/2]*(2*mz^2 -
(E3 + E4 - pt - ptb)*(pt + ptb) + (E3 + E4 - pt - ptb)*(-(pt*Cos[tht]) + ptb*Cos[thtb]))*
(Cos[phtb] + I*Sin[phtb])*Sin[tht/2] + 2*(E3 + E4 - pt - ptb)*Cos[tht/2]*(pt - pt*Cos[tht] +
ptb*(1 + Cos[thtb])*(Cos[2*phtb] + I*Sin[2*phtb]))*Sin[thtb/2]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]*mz^2))

F3RL[1,-1,-1,-1] =
(((-I/2)*E1*(E3 + mt + pt)*(E4 + mt - ptb)*(Cos[phtb/2] - I*Sin[phtb/2])*(-2*(E3 + E4 - pt + ptb)*
Cos[tht/2]*Cos[thtb/2]*(-pt + pt*Cos[tht] - ptb*(-1 + Cos[thtb])*(Cos[2*phtb] + I*Sin[2*phtb])) +
2*(2*mz^2 - (pt - ptb)*(E3 + E4 - pt + ptb) + (E3 + E4 - pt + ptb)*(-(pt*Cos[tht]) + ptb*Cos[thtb]))*
(Cos[phtb] + I*Sin[phtb])*Sin[tht/2]*Sin[thtb/2]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]*mz^2))

F3RR[1,-1,1,1] =
(((-I/2)*E1*(E3 + mt + pt)*(E4 + mt - ptb)*(Cos[phtb/2] - I*Sin[phtb/2])*(2*Cos[tht/2]*Cos[thtb/2]*
(2*mz^2 - (pt - ptb)*(E3 + E4 - pt + ptb) + (E3 + E4 - pt + ptb)*(pt*Cos[tht] - ptb*Cos[thtb]))*
(Cos[phtb] + I*Sin[phtb]) + 2*(E3 + E4 - pt + ptb)*(pt + pt*Cos[tht] - ptb*(1 + Cos[thtb])*(Cos[2*phtb] +
I*Sin[2*phtb]))*Sin[tht/2]*Sin[thtb/2]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]*mz^2))

F3RR[1,-1,1,-1] =
(((I/2)*E1*(E3 + mt + pt)*(E4 + mt + ptb)*(Cos[phtb/2] - I*Sin[phtb/2])*(-2*(E3 + E4 - pt - ptb)*
Cos[thtb/2]*(pt + pt*Cos[tht] - ptb*(-1 + Cos[thtb])*(Cos[2*phtb] + I*Sin[2*phtb]))*Sin[tht/2] +
2*Cos[tht/2]*(2*mz^2 - (E3 + E4 - pt - ptb)*(pt + ptb) + (E3 + E4 - pt - ptb)*
(pt*Cos[tht] - ptb*Cos[thtb]))*(Cos[phtb] + I*Sin[phtb])*Sin[thtb/2]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]*mz^2))

F3RR[1,-1,-1,1] =
(((I/2)*E1*(E3 + mt - pt)*(E4 + mt - ptb)*(Cos[phtb/2] - I*Sin[phtb/2])*(2*Cos[thtb/2]*(2*mz^2 +
(pt + ptb)*(E3 + E4 + pt + ptb) + (E3 + E4 + pt + ptb)*(pt*Cos[tht] - ptb*Cos[thtb]))*
(Cos[phtb] + I*Sin[phtb])*Sin[tht/2] + 2*(E3 + E4 + pt + ptb)*Cos[tht/2]*(pt - pt*Cos[tht] +
ptb*(1 + Cos[thtb])*(Cos[2*phtb] +I*Sin[2*phtb]))*Sin[thtb/2]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]*mz^2))

F3RR[1,-1,-1,-1] =
(((I/2)*E1*(E3 + mt - pt)*(E4 + mt + ptb)*(Cos[phtb/2] - I*Sin[phtb/2])*(-2*(E3 + E4 + pt - ptb)*
Cos[tht/2]*Cos[thtb/2]*(-pt + pt*Cos[tht] - ptb*(-1 + Cos[thtb])*(Cos[2*phtb] + I*Sin[2*phtb])) -
2*(2*mz^2 + (pt - ptb)*(E3 + E4 + pt - ptb) + (E3 + E4 + pt - ptb)*(pt*Cos[tht] - ptb*Cos[thtb]))*
(Cos[phtb] + I*Sin[phtb])*Sin[tht/2]*Sin[thtb/2]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]*mz^2))

F1LL[-1,1,1,1] =
((2*E1*(E4 + mt + ptb)*(Cos[phtb/2] - I*Sin[phtb/2])*Sin[thtb/2]*((b*(E3 + mt)*(-2*E1 + E4 + mt) -
2*b*E1*pt + b*E4*pt - b*mt*pt + I*a*((E4 - mt)*(E3 + mt) + (E4 + mt)*pt - 2*E1*(E3 + mt + pt)) +
(I*a + b)*(E3 + mt + pt)*ptb*Cos[thtb])*Sin[tht/2] - (a - I*b)*(E3 + mt + pt)*ptb*Cos[tht/2]*
(I*Cos[phtb] + Sin[phtb])*Sin[thtb]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]))

F1LL[-1,1,1,-1] =
((2*E1*(E4 + mt - ptb)*Cos[thtb/2]*(Cos[phtb/2] - I*Sin[phtb/2])*((b*(E3 + mt)*(-2*E1 + E4 + mt) -
2*b*E1*pt + b*E4*pt - b*mt*pt + I*a*((E4 - mt)*(E3 + mt) + (E4 + mt)*pt - 2*E1*(E3 + mt + pt)) +
(I*a + b)*(E3 + mt + pt)*ptb*Cos[thtb])*Sin[tht/2] - (a - I*b)*(E3 + mt + pt)*ptb*Cos[tht/2]*
(I*Cos[phtb] + Sin[phtb])*Sin[thtb]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]))

F1LL[-1,1,-1,1] =
((2*E1*(E4 + mt + ptb)*(Cos[phtb/2] - I*Sin[phtb/2])*Sin[thtb/2]*(Cos[tht/2]*(b*(E3 + mt)*
(-2*E1 + E4 + mt) + 2*b*E1*pt - b*E4*pt + b*mt*pt + I*a*((-2*E1 + E4 - mt)*(E3 + mt) -
(-2*E1 + E4 + mt)*pt) + (I*a + b)*(E3 + mt - pt)*ptb*Cos[thtb]) + (I*a + b)*(E3 + mt - pt)*ptb*
(Cos[phtb] - I*Sin[phtb])*Sin[tht/2]*Sin[thtb]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]))

F1LL[-1,1,-1,-1] =
((2*E1*(E4 + mt - ptb)*Cos[thtb/2]*(Cos[phtb/2] - I*Sin[phtb/2])*(Cos[tht/2]*(b*(E3 + mt)*
(-2*E1 + E4 + mt) + 2*b*E1*pt - b*E4*pt + b*mt*pt + I*a*((-2*E1 + E4 - mt)*(E3 + mt) -
(-2*E1 + E4 + mt)*pt) + (I*a + b)*(E3 + mt - pt)*ptb*Cos[thtb]) + (I*a + b)*(E3 + mt - pt)*ptb*
(Cos[phtb] - I*Sin[phtb])*Sin[tht/2]*Sin[thtb]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]))

F1LR[-1,1,1,1] =
((2*E1*(E4 + mt - ptb)*(Cos[phtb/2] - I*Sin[phtb/2])*Sin[thtb/2]*((-I)*(a*(E3 + mt)*(2*E1 - E4 + mt) +
a*(-2*E1 + E4 + mt)*pt - I*b*(-2*E1*E3 + E3*E4 - 2*E1*mt + E3*mt + E4*mt + mt^2 + 2*E1*pt - E4*pt + mt*pt) +
(a + I*b)*(E3 + mt - pt)*ptb*Cos[thtb])*Sin[tht/2] + (a + I*b)*(E3 + mt - pt)*ptb*Cos[tht/2]*
(I*Cos[phtb] + Sin[phtb])*Sin[thtb]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]))
```

```
F1LR[-1,1,1,-1] =
((2*E1*(E4 + mt + ptb)*Cos[thtb/2]*(Cos[phtb/2] - I*Sin[phtb/2])*((-I)*(a*(E3 + mt)*(2*E1 - E4 + mt) +
a*(-2*E1 + E4 + mt)*pt - I*b*(-2*E1*E3 + E3*E4 - 2*E1*mt + E3*mt + E4*mt + mt^2 + 2*E1*pt - E4*pt + mt*pt) +
(a + I*b)*(E3 + mt - pt)*ptb*Cos[thtb])*Sin[tht/2] + (a + I*b)*(E3 + mt - pt)*ptb*Cos[tht/2]*
(I*Cos[phtb] + Sin[phtb])*Sin[thtb]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]))

F1LR[-1,1,-1,1] =
(((-2*I)*E1*(E4 + mt - ptb)*(Cos[phtb/2] - I*Sin[phtb/2])*Sin[thtb/2]*(Cos[tht/2]*(a*(E3 + mt)*
(2*E1 - E4 + mt) - a*(-2*E1 + E4 + mt)*pt + I*b*(-((E3 + mt)*(E4 + mt)) - E4*pt + mt*pt +
2*E1*(E3 + mt + pt)) + (a + I*b)*(E3 + mt + pt)*ptb*Cos[thtb]) + (a + I*b)*(E3 + mt + pt)*ptb*(Cos[phtb] -
I*Sin[phtb])*Sin[tht/2]*Sin[thtb]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]))

F1LR[-1,1,-1,-1] =
(((-2*I)*E1*(E4 + mt + ptb)*Cos[thtb/2]*(Cos[phtb/2] - I*Sin[phtb/2])*(Cos[tht/2]*(a*(E3 + mt)*
(2*E1 - E4 + mt) - a*(-2*E1 + E4 + mt)*pt + I*b*(-((E3 + mt)*(E4 + mt)) - E4*pt + mt*pt +
2*E1*(E3 + mt + pt)) + (a + I*b)*(E3 + mt + pt)*ptb*Cos[thtb]) + (a + I*b)*(E3 + mt + pt)*ptb*(Cos[phtb] -
I*Sin[phtb])*Sin[tht/2]*Sin[thtb]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]))

F2LL[-1,1,1,1] =
(((2*I)*E1*(E3 + mt - pt)*(Cos[phtb/2] - I*Sin[phtb/2])*Sin[tht/2]*((a + I*b)*pt*(E4 + mt - ptb)*
Cos[thtb/2]*(Cos[phtb] + I*Sin[phtb])*Sin[tht] - (a*(2*E1 - E3 + mt)*(E4 + mt) + a*(-2*E1 + E3 + mt)*ptb +
I*b*(-((-2*E1 + E3 + mt)*(E4 + mt)) - 2*E1*ptb + E3*ptb - mt*ptb) + (a + I*b)*pt*(E4 + mt - ptb)*Cos[tht])*
Sin[thtb/2]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]))

F2LL[-1,1,1,-1] =
((2*E1*(E3 + mt - pt)*(Cos[phtb/2] - I*Sin[phtb/2])*Sin[tht/2]*((-(b*(-2*E1 + E3 + mt)*(E4 + mt)) +
2*b*E1*ptb - b*E3*ptb + b*mt*ptb - I*a*(2*E1*E4 - E3*E4 + 2*E1*mt - E3*mt + E4*mt + mt^2 + 2*E1*ptb -
E3*ptb - mt*ptb) + ((-I)*a + b)*pt*(E4 + mt + ptb)*Cos[tht])*Cos[thtb/2] + ((-I)*a + b)*pt*(E4 + mt + ptb)*
(Cos[phtb] + I*Sin[phtb])*Sin[tht]*Sin[thtb/2]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]))

F2LL[-1,1,-1,1] =
(((2*I)*E1*(E3 + mt + pt)*Cos[tht/2]*(Cos[phtb/2] - I*Sin[phtb/2])*((a + I*b)*pt*(E4 + mt - ptb)*
Cos[thtb/2]*(Cos[phtb] + I*Sin[phtb])*Sin[tht] - (a*(2*E1 - E3 + mt)*(E4 + mt) + a*(-2*E1 + E3 + mt)*ptb +
I*b*(-((-2*E1 + E3 + mt)*(E4 + mt)) - 2*E1*ptb + E3*ptb - mt*ptb) + (a + I*b)*pt*(E4 + mt - ptb)*Cos[tht])*
Sin[thtb/2]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]))

F2LL[-1,1,-1,-1] =
((2*E1*(E3 + mt + pt)*Cos[tht/2]*(Cos[phtb/2] - I*Sin[phtb/2])*((-(b*(-2*E1 + E3 + mt)*(E4 + mt)) +
2*b*E1*ptb - b*E3*ptb + b*mt*ptb - I*a*(2*E1*E4 - E3*E4 +2*E1*mt - E3*mt + E4*mt + mt^2 + 2*E1*ptb -
E3*ptb - mt*ptb) + ((-I)*a + b)*pt*(E4 + mt + ptb)*Cos[tht])*Cos[thtb/2] + ((-I)*a + b)*pt*(E4 + mt + ptb)*
(Cos[phtb] + I*Sin[phtb])*Sin[tht]*Sin[thtb/2]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]))

F2LR[-1,1,1,1] =
((2*E1*(E3 + mt + pt)*(Cos[phtb/2] - I*Sin[phtb/2])*Sin[tht/2]*(((-I)*a - b)*pt*(E4 + mt + ptb)*Cos[thtb/2]*
(Cos[phtb] + I*Sin[phtb])*Sin[tht] + (-2*b*E1*E4 + b*E3*E4 - 2*b*E1*mt + b*E3*mt + b*E4*mt + b*mt^2 -
2*b*E1*ptb + b*E3*ptb -b*mt*ptb - I*a*(2*E1*E4 - E3*E4 + 2*E1*mt - E3*mt + E4*mt + mt^2 + 2*E1*ptb -
E3*ptb - mt*ptb) + (I*a + b)*pt*(E4 + mt + ptb)*Cos[tht])*Sin[thtb/2]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]))

F2LR[-1,1,1,-1] =
((2*E1*(E3 + mt + pt)*(Cos[phtb/2] - I*Sin[phtb/2])*Sin[tht/2]*(((E4 + mt)*((-2*I)*a*E1 - 2*b*E1 + I*a*E3 +
b*E3 - I*a*mt + b*mt) + (2*I)*a*E1*ptb + 2*b*E1*ptb - I*a*E3*ptb - b*E3*ptb - I*a*mt*ptb + b*mt*ptb +
(I*a + b)*pt*(E4 + mt - ptb)*Cos[tht])*Cos[thtb/2] + (I*a + b)*pt*(E4 + mt - ptb)*
(Cos[phtb] + I*Sin[phtb])*Sin[tht]*Sin[thtb/2]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]))

F2LR[-1,1,-1,1] =
((2*E1*(E3 + mt - pt)*Cos[tht/2]*(Cos[phtb/2] - I*Sin[phtb/2])*(((-I)*a - b)*pt*(E4 + mt + ptb)*Cos[thtb/2]*
(Cos[phtb] + I*Sin[phtb])*Sin[tht] + (-2*b*E1*E4 + b*E3*E4 - 2*b*E1*mt + b*E3*mt + b*E4*mt + b*mt^2 -
2*b*E1*ptb + b*E3*ptb - b*mt*ptb - I*a*(2*E1*E4 - E3*E4 + 2*E1*mt - E3*mt + E4*mt + mt^2 + 2*E1*ptb -
E3*ptb - mt*ptb) + (I*a + b)*pt*(E4 + mt + ptb)*Cos[tht])*Sin[thtb/2]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]))

F2LR[-1,1,-1,-1] =
((2*E1*(E3 + mt - pt)*Cos[tht/2]*(Cos[phtb/2] - I*Sin[phtb/2])*(((E4 + mt)*((-2*I)*a*E1 - 2*b*E1 + I*a*E3 +
b*E3 - I*a*mt + b*mt) + (2*I)*a*E1*ptb + 2*b*E1*ptb - I*a*E3*ptb - b*E3*ptb - I*a*mt*ptb + b*mt*ptb +
(I*a + b)*pt*(E4 + mt - ptb)*Cos[tht])*Cos[thtb/2] + (I*a + b)*pt*(E4 + mt - ptb)*
(Cos[phtb] + I*Sin[phtb])*Sin[tht]*Sin[thtb/2]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]))
```

```
F3LL[-1,1,1,1] =
(((I/2)*E1*(E3 + mt - pt)*(E4 + mt + ptb)*(Cos[(3*phtb)/2] - I*Sin[(3*phtb)/2])*(-2*(E3 + E4 + pt - ptb)*
Cos[tht/2]*Cos[thtb/2]*(ptb - ptb*Cos[thtb] + pt*(-1 + Cos[tht])*(Cos[2*phtb] + I*Sin[2*phtb])) -
2*(2*mz^2 + (pt - ptb)*(E3 + E4 + pt - ptb) + (E3 + E4 + pt - ptb)*(pt*Cos[tht] - ptb*Cos[thtb]))*
(Cos[phtb] + I*Sin[phtb])*Sin[tht/2]*Sin[thtb/2]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]*mz^2))

F3LL[-1,1,1,-1] =
(((-I/2)*E1*(E3 + mt - pt)*(E4 + mt - ptb)*(Cos[(3*phtb)/2] - I*Sin[(3*phtb)/2])*(2*Cos[thtb/2]*
(2*mz^2 + (pt + ptb)*(E3 + E4 + pt + ptb) + (E3 + E4 + pt + ptb)*(pt*Cos[tht] - ptb*Cos[thtb]))*
(Cos[phtb] + I*Sin[phtb])*Sin[tht/2] + 2*(E3 + E4 + pt + ptb)*Cos[tht/2]*(ptb + ptb*Cos[thtb] -
pt*(-1 + Cos[tht])*(Cos[2*phtb] + I*Sin[2*phtb]))*Sin[thtb/2]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]*mz^2))

F3LL[-1,1,-1,1] =
(((I/2)*E1*(E3 + mt + pt)*(E4 + mt + ptb)*(Cos[(3*phtb)/2] - I*Sin[(3*phtb)/2])*(2*(E3 + E4 - pt - ptb)*
Cos[thtb/2]*(ptb - ptb*Cos[thtb] + pt*(1 + Cos[tht])*(Cos[2*phtb] + I*Sin[2*phtb]))*Sin[tht/2] +
2*Cos[tht/2]*(-2*mz^2 + (E3 + E4 - pt - ptb)*(pt + ptb) + (E3 + E4 - pt - ptb)*(-(pt*Cos[tht]) +
ptb*Cos[thtb]))*(Cos[phtb] + I*Sin[phtb])*Sin[thtb/2]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]*mz^2))

F3LL[-1,1,-1,-1] =
(((I/2)*E1*(E3 + mt + pt)*(E4 + mt - ptb)*(Cos[(3*phtb)/2] - I*Sin[(3*phtb)/2])*(-2*Cos[tht/2]*Cos[thtb/2]*
(2*mz^2 - (pt - ptb)*(E3 + E4 - pt + ptb) + (E3 + E4 - pt + ptb)*(pt*Cos[tht] - ptb*Cos[thtb]))*(Cos[phtb] +
I*Sin[phtb]) - 2*(E3 + E4 - pt + ptb)*(-ptb - ptb*Cos[thtb] + pt*(1 + Cos[tht])*
(Cos[2*phtb] + I*Sin[2*phtb]))*Sin[tht/2]*Sin[thtb/2]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]*mz^2))

F3LR[-1,1,1,1] =
(((-I/2)*E1*(E3 + mt + pt)*(E4 + mt - ptb)*(Cos[(3*phtb)/2] - I*Sin[(3*phtb)/2])*(-2*(E3 + E4 - pt + ptb)*
Cos[tht/2]*Cos[thtb/2]*(ptb - ptb*Cos[thtb] + pt*(-1 + Cos[tht])*(Cos[2*phtb] + I*Sin[2*phtb])) +
2*(2*mz^2 - (pt - ptb)*(E3 + E4 - pt + ptb) + (E3 + E4 - pt + ptb)*(-(pt*Cos[tht]) + ptb*Cos[thtb]))*
(Cos[phtb] + I*Sin[phtb])*Sin[tht/2]*Sin[thtb/2]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]*mz^2))

F3LR[-1,1,1,-1] =
(((-I/2)*E1*(E3 + mt + pt)*(E4 + mt + ptb)*(Cos[(3*phtb)/2] - I*Sin[(3*phtb)/2])*(2*Cos[thtb/2]*
(2*mz^2 - (E3 + E4 - pt - ptb)*(pt + ptb) + (E3 + E4 - pt - ptb)*(-(pt*Cos[tht]) + ptb*Cos[thtb]))*
(Cos[phtb] + I*Sin[phtb])*Sin[tht/2] + 2*(E3 + E4 - pt - ptb)*Cos[tht/2]*(-ptb - ptb*Cos[thtb] +
pt*(-1 + Cos[tht])*(Cos[2*phtb] + I*Sin[2*phtb]))*Sin[thtb/2]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]*mz^2))

F3LR[-1,1,-1,1] =
(((-I/2)*E1*(E3 + mt - pt)*(E4 + mt - ptb)*(Cos[(3*phtb)/2] - I*Sin[(3*phtb)/2])*((E3 + E4 + pt + ptb)*
Cos[thtb/2]*(-2*ptb*(-1 + Cos[thtb])*Sin[tht/2] + pt*(Cos[2*phtb] + I*Sin[2*phtb])*(Sin[tht/2] +
Sin[(3*tht)/2])) + 2*Cos[tht/2]*(2*mz^2 + (pt + ptb)*(E3 + E4 + pt + ptb) + (E3 + E4 + pt + ptb)*
(-(pt*Cos[tht]) + ptb*Cos[thtb]))*(Cos[phtb] + I*Sin[phtb])*Sin[thtb/2]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]*mz^2))

F3LR[-1,1,-1,-1] =
(((I/2)*E1*(E3 + mt - pt)*(E4 + mt + ptb)*(Cos[(3*phtb)/2] - I*Sin[(3*phtb)/2])*(-2*Cos[tht/2]*Cos[thtb/2]*
(2*mz^2 + (pt - ptb)*(E3 + E4 + pt - ptb) + (E3 + E4 + pt - ptb)*(-(pt*Cos[tht]) + ptb*Cos[thtb]))*
(Cos[phtb] +I*Sin[phtb]) + 2*(E3 + E4 + pt - ptb)*(-ptb - ptb*Cos[thtb] + pt*(1 + Cos[tht])*(Cos[2*phtb] +
I*Sin[2*phtb]))*Sin[tht/2]*Sin[thtb/2]))/(Sqrt[E3 + mt]*Sqrt[E4 + mt]*mz^2))
```