PHYSICS 590 II: GROUP THEORY AND SYMMETRIES IN PHYSICS

Bhupal Dev **Homework 3** Due: 02/09/17

1. **Orthogonal Vectors:** Prove that in an n-dimensional complex vector space, there can be a maximum of n mutually orthogonal vectors. We have used this in class to derive an important result that the number of irreps of a finite group is equal to the number of its equivalence classes, i.e. the character table is always square.

2. **Characters of Dihedral Groups:** Recall from class that the dihedral group D_n is the symmetry group of the n-sided regular polygon.

   (a) Work out the character table for D_4, the symmetry group of the square, and using this, check explicitly which of the irreps are real, pseudoreal or complex.

   (b) Compare the D_4 character table from (a) with the character table for Q_4, the quaternion group (you don't have to work out the Q_4 table, unless you want to, and can just look it up in your textbook). Do you see any similarities/differences? From this example, can you say whether a character table uniquely characterizes a group or not?

3. **Classical Harmonic Oscillator:** In class, we discussed the harmonic motion of two equal masses connected by a frictionless spring and moving in one dimension. What happens for *unequal masses*? Just using group theory (and without solving the equations of motion), can you say if there is still a zero mode in this case? Then you can solve the system and verify your guess.