PHYSICS 590 II: GROUP THEORY AND SYMMETRIES IN PHYSICS

Bhupal Dev **Homework 2** Due: 02/02/17

1. **Spectral Theorem:** Show that *all* eigenvalues of a finite-dimensional Hermitian matrix are *real*, and that it is always possible to find an orthonormal basis consisting of its eigenvectors. This is known as *spectral decomposition* and you might have already seen it in basic quantum mechanics (e.g. Hilbert space).

2. **Permutation Groups (Again!):**

   (a) From class, you know that $A_n \subset S_n$. Now show that S_n is isomorphic to a subgroup of A_{n+2}. Check this explicitly for $n = 3$, i.e. show that $S_3 \subset A_5$.

   (b) Show that A_4 is the maximal invariant subgroup of S_4.

3. **Irreps of Abelian Group:** Prove that all irreducible representations of an Abelian group are one-dimensional. (*Hint:* Use Schur's lemma.)