PHYSICS 590 II: GROUP THEORY AND SYMMETRIES IN PHYSICS

Bhupal Dev | **Homework 12** (last one, yay!) | Due: 04/27/17

1. **Simple Roots:** We discussed the positive and simple roots for the four families of Lie algebra. Let's verify those formulas explicitly for some example cases. Here e^i's denote the unit vector in the ith direction.

   (a) How many roots does $SO(6)$ have? Show that the positive roots $e^1 - e^3$, $e^1 + e^3$ and $e^1 + e^2$ are not simple.

   (b) How many roots does $Sp(6)$ have? Show that the simple roots are $e^1 - e^2$, $e^2 - e^3$ and $2e^3$, i.e. the other roots are not simple.

   (c) Now do the same for $SO(7)$. Do you see any similarities with the $Sp(6)$ case?

2. **Mathematicians' Adjoint:** For any two elements X and Y of a Lie algebra, consider the linear mapping $Y \to \mathrm{adj}(X)Y \equiv [X, Y]$. Then prove that $\mathrm{adj}([X, Y]) = [\mathrm{adj}(X), \mathrm{adj}(Y)]$. *Hint:* You might want to call Jacobi for help.

3. **Root Diagram:** Using the general restrictions on the angle between and length of different roots, show that a root diagram for any Lie algebra cannot contain more than two different lengths. This remarkable feature enabled Cartan to classify all Lie algebras based on their root diagrams.