PHYSICS 543: GROUP THEORY AND SYMMETRIES IN PHYSICS

Midterm Exam   March 6, 2019   2 - 3 PM

1. Briefly explain whether each of the following statements is True or False.

    (a) [**3 points**] $Z_3 \subset Z_{13}$.

    (b) [**3 points**] $Z_2 \otimes Z_2$ is isomorphic to Z_4.

    (c) [**3 points**] Z_3 is isomorphic to A_3.

    (d) [**3 points**] Z_2 has a 2-dimensional irreducible representation.

    (e) [**3 points**] None of the irreducible representations of Z_n can be pseudoreal.

2. The *center* $C(G)$ of a group G (discrete or continuous) is the set of elements that commute with every element of G:

$$C(G) \;=\; \{a \in G \mid ag = ga \; \forall \; g \in G\} \; .$$

    Consider the dihedral group D_4 as an example:

    (a) [**4 points**] What is the center C of D_4?

    (b) [**4 points**] What is the quotient group D_4/C?

    (c) [**2 points**] Is D_4 isomorphic to $(D_4/C) \otimes C$?

3. For the permutation group S_3:

    (a) [**4 points**] Enumerate the equivalence classes, subgroups and invariant subgroups.

    (b) [**4 points**] Find the quotient groups of the invariant subgroups.

    (c) [**2 points**] Find the maximal invariant subgroup.

    (d) [**5 points**] Derive the character table.

    (e) [**5 points**] Do the reality checker on each of the irreps.

    (f) [**5 points**] How does the regular representation **6** break into the irreps?