PHYSICS 543: GROUP THEORY AND SYMMETRIES IN PHYSICS

Homework 6 Due: March 1, 2019

1. **Young Tableaux:** The Young tableaux with n boxes are in one-to-one correspondence with the irreps of S_n. Use this information and the *hook length formula* discussed in class to decompose the permutation group S_5 into its irreps and verify that they satisfy the general formula

$$N(G) \;=\; \sum_r d_r^2\,, \tag{1}$$

where $N(G)$ is the order of the group and d_r is the dimension of the irrep r.

2. **Reality of Structure Constants:** For a Lie group, the commutator between any two generators can be written as a linear combination of the generators, i.e.

$$[T_a, T_b] \;=\; i f_{abc} T_c\,. \tag{2}$$

Show that for Hermitian generators T_a, the coefficients f_{abc} must be real. These are known as the *structure constants* of the corresponding Lie algebra, which essentially characterize the Lie group.

3. **Jacobi Identity:**

   (a) Show that X, Y, Z being three matrices (or operators or generators)

$$[X,[Y,Z]] + [Y,[Z,X]] + [Z,[X,Y]] \;=\; 0\,. \tag{3}$$

   This is the *Jacobi identity*, which often comes in handy while dealing with Lie algebra, and sometimes even used as part of the definition of Lie algebra.

   (b) *(Bonus)* In vector notation, Eq. (3) simply means for three vectors $\vec{a}$, $\vec{b}$, $\vec{c}$,

$$\vec{a}\times(\vec{b}\times\vec{c}) + \vec{b}\times(\vec{c}\times\vec{a}) + \vec{c}\times(\vec{a}\times\vec{b}) = 0. \tag{4}$$

   Using this, show that the geometrical meaning of Jacobi identity is contained in a simple statement you learned in high-school math, namely, the three altitudes of a triangle intersect at one point.