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# PHYSICS 543: GROUP THEORY AND SYMMETRIES IN PHYSICS

## Homework 5

Due: February 22, 2019

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1. **Pseudoreal Representation:** Show that a representation can be pseudoreal only if its dimension is even.
2. **Dihedral and Quaternion Groups:** It turns out that the character table for the dihedral group  $D_4$  is exactly the same as that of the quaternion group  $\mathcal{Q}$  [Hint: See Eqs. (19) and (21) on p. 127-128 in Zee's book]. Now using the reality checker discussed in class, check explicitly which of the irreps are real, pseudo-real and complex in each case. Can you still say "A character table uniquely characterizes a group"?
3. **Square Roots of Identity:**
  - (a) Use the reality checker on the irreps of  $A_4$  and  $S_4$ .
  - (b) Find how many square roots of identity exist in each case. List them.
4. **Harmonic Oscillator:** In class, we discussed the harmonic motion of two equal masses connected by a frictionless spring and moving in one dimension. What happens for *unequal masses*? Just using group theory (and without solving the equations of motion), can you say if there is still a zero mode in this case? [Hint: You can solve the system of differential equations and verify your guess, if you like, though you don't need to.]