# PHYSICS 543: GROUP THEORY AND SYMMETRIES IN PHYSICS

**Homework 4** Due: February 15, 2019

1. **Number of Orthogonal Vectors:** Prove that in an $n$-dimensional complex vector space, there can be a maximum of $n$ linearly independent, mutually orthogonal vectors. We have used this in class to derive an important result that the number of irreducible representations of a finite group is equal to the number of its equivalence classes, i.e. the character table is always square.

2. **Characters of $A_4$:**

   (a) List the equivalence classes of $A_4$.

   (b) Find the dimensions of the irreducible representations of $A_4$.

   (c) Derive the character table for $A_4$.

   (d) How does the 4-dimensional regular representation of $A_4$ reduce?

3. **Characters of $S_4$:**

   (a) Find the dimensions of the irreducible representations of $S_4$.

   (b) Derive the character table for $S_4$.

   (c) How does the 4-dimensional regular representation of $S_4$ reduce?

   (d) What happens to the 2-dimensional representation of $S_4$ on restriction to $A_4$?