## PHYSICS 543: GROUP THEORY AND SYMMETRIES IN PHYSICS

**Homework 3** Due: February 8, 2019

1. **Properties of Hermitian Matrix:**

   (a) Show that *all* eigenvalues of a finite-dimensional Hermitian matrix are *real*, and that it is always possible to find an orthonormal basis consisting of its eigenvectors. This is known as *spectral decomposition* and has important physics applications, e.g. in the construction of Hilbert space in quantum mechanics.

   (b) Show that for any matrix $M$, the eigenvalues of $M^\dagger M$ must be non-negative. This was used in the proof of the *unitarity theorem* in class.

2. **Similarity Transformation:** Find the similarity transformation which reduces the 2-dimensional representation of $Z_2$ given in class into diagonal form.

3. **Irreps of Abelian Group:** Prove that *all* irreducible representations of an Abelian group must be one-dimensional.