PHYSICS 543: GROUP THEORY AND SYMMETRIES IN PHYSICS

Homework 2 Due: February 1, 2019

1. **2-cycles:** Show that for 2-cycles
$$(1a)(1b)(1a) \;=\; (ab)\,. \tag{1}$$

2. **Cycle structure:**

   (a) Show that the number of elements in the permutation group S_n with a given cycle structure is given by
   $$\frac{n!}{\Pi_j\, j^{n_j}\, n_j!}\,, \tag{2}$$
   where n_j is the number of j-cycles in the cycle structure.

   (b) Verify Eq. (2) by listing all possible cycle structures in S_5.

3. $\boldsymbol{S_n}$ **and** $\boldsymbol{A_n}$:

   (a) In class, we studied that $A_n \subset S_n$. Now show that S_n is isomorphic to a subgroup of A_{n+2}.

   (b) As an example of part (a), show explicitly how $S_3 \subset A_5$.

4. $\boldsymbol{S_4}$: Consider the permutation group S_4.

   (a) Enumerate the equivalence classes and subgroups of S_4.

   (b) Which of the subgroups are invariant ones?

   (c) Find the quotient groups of the invariant subgroups.

   (d) Show that A_4 is the maximal invariant subgroup of S_4.

5. $\boldsymbol{D_4}$: Consider the dihedral group D_4 which is the symmetry group of the square.

   (a) Enumerate the equivalence classes and subgroups of D_4.

   (b) Which of the subgroups are invariant ones?

   (c) Find the quotient groups of the invariant subgroups.

   (d) Is D_4 the direct product of some of its subgroups?