PHYSICS 543: GROUP THEORY AND SYMMETRIES IN PHYSICS

Homework 12 (Last one) Due: April 26, 2019

1. **Course Evaluation:** Please complete the course evaluation online to receive the full class participation credit (3% of the total).

2. **Weight Diagram:** Draw the weight diagram (in the $i_3 - y$ plane) for the decuplet $\mathbf{10}$ of $SU(3)$ and write down the coordinates of each lattice point.

3. **Roots of $\boldsymbol{SU(4)}$:** Calculate all the root vectors for $SU(4)$ using the $SU(2)$ subalgebra method and identify the simple roots.

4. **Symplectic Group $\boldsymbol{Sp(2n)}$:**

   (a) Prove that a $2n \times 2n$ matrix R satisfying the symplectic condition $R^T J R = J$, where $J = \begin{pmatrix} 0 & \mathbf{1} \\ -\mathbf{1} & 0 \end{pmatrix}$ implies det $R = +1$ (unlike in the $SO(n)$ case, where we need to impose this condition by hand).

   (b) Show that the Lie generator of the symplectic group is given by the $2n \times 2n$ Hermitian matrix H satisfying the condition $H^T = JHJ$.

   (c) Show that H can be written as a linear combination of the Hermitian traceless matrices $iA \otimes \mathbf{1}$ and $S_i \otimes \sigma_i$, where A is an arbitrary real $n \times n$ antisymmetric matrix, S_i (with $i = 1, 2, 3$) are arbitrary real $n \times n$ symmetric matrices and σ_i are the usual 2×2 Pauli matrices.