PHYSICS 474: Introduction to Particle Physics

Homework 7 **Due: 11.30 03/02/2020**

1. **Pion Decay:** [5 points] Apply the Fermi Golden Rule to calculate the decay rate of $\pi^0 \to \gamma + \gamma$. Since we don't know the matrix element yet (will do it later when discussing QED), just assume that it is proportional to α (the fine structure constant), i.e. $\mathcal{M} = \alpha m_\pi c$ (because it must have the dimension of momentum for $1 \to 2$ decay). Compare your result with the experimental value of the mean lifetime $\tau_{\pi^0}^{\rm exp} = 8.5 \times 10^{-17}$s.

2. $2 \to 2$ **Scattering:** Consider the scattering process $1 + 2 \to 3 + 4$.

   (a) [10 points] Show that

   $$\sqrt{(p_1 \cdot p_2)^2 - (m_1 m_2 c^2)^2} \;=\; \begin{cases} |\mathbf{p}_1|(E_1 + E_2)/c & \text{in the center-of-mass frame} \\ |\mathbf{p}_1| m_2 c & \text{in the lab frame with particle 2 at rest} \end{cases}.$$

   (b) [15 points] Using the Fermi Golden Rule, show that the differential cross section in the lab frame, with particle 2 at rest and with particles 3 and 4 massless, is given by

   $$\frac{d\sigma}{d\Omega} \;=\; \left(\frac{\hbar}{8\pi}\right)^2 \frac{S|\mathcal{M}|^2 |\mathbf{p}_3|}{m_2 |\mathbf{p}_1| (E_1 + m_2 c^2 - |\mathbf{p}_1| c \cos\theta)}\,, \tag{1}$$

   where θ is the scattering angle for particle 3, S is the symmetry factor and $\mathcal{M}$ is the matrix element.

3. **Elastic Scattering:** Consider the elastic $2 \to 2$ scattering $1 + 2 \to 1 + 2$ in the lab frame with particle 2 at rest.

   (a) [15 points] Show that the differential cross section is given by

   $$\frac{d\sigma}{d\Omega} \;=\; \left(\frac{\hbar}{8\pi}\right)^2 \frac{S|\mathcal{M}|^2 |\mathbf{p}_3|^2}{m_2 |\mathbf{p}_1| \Big[(E_1 + m_2 c^2) |\mathbf{p}_3| - |\mathbf{p}_1| E_3 \cos\theta \Big]}\,. \tag{2}$$

   (b) [5 points] If the incident particle is massless (i.e. $m_1 = 0$), show that the result in part (a) simplifies to

   $$\frac{d\sigma}{d\Omega} \;=\; S \left(\frac{\hbar E_3}{8\pi m_2 c E_1}\right)^2 |\mathcal{M}|^2\,. \tag{3}$$