**PHYSICS 474: Introduction to Particle Physics**

**Homework 2** Due: 11.30 Monday, 1/27/2020

1. **Mandelstam variables:** In a two-body scattering process $A + B \to C + D$, it is convenient to define the Lorentz invariants, called the *Mandelstam variables*,

$$s \equiv \frac{(p_A + p_B)^2}{c^2}, \qquad t \equiv \frac{(p_A - p_C)^2}{c^2}, \qquad u \equiv \frac{(p_A - p_D)^2}{c^2}. \tag{1}$$

   (a) [5 points] Show that $s + t + u = m_A^2 + m_B^2 + m_C^2 + m_D^2$.

   (b) [5 points] Show that the total center-of-mass (CM) energy $E_{\rm tot}^{\rm CM} = \sqrt{s}\, c^2$.

   (c) [10 points] For elastic scattering of identical particles, $A + A \to A + A$, show that

$$s = \frac{4(\mathbf{p}^2 + m^2c^2)}{c^2}, \qquad t = \frac{-2\mathbf{p}^2(1 - \cos\theta)}{c^2}, \qquad u = \frac{-2\mathbf{p}^2(1 + \cos\theta)}{c^2}, \tag{2}$$

   where $\mathbf{p}$ is the CM momentum and $\theta$ is the scattering angle.

2. **Compton Scattering:** [10 points] A photon of wavelength $\lambda$ collides elastically with an electron and scatters at angle $\theta$. Find its outgoing wavelength.

3. **Unitary and Orthogonal Groups:**

   (a) [5 points] Show that the set $U(N)$ of all $N \times N$ unitary matrices constitutes a group.

   (b) [5 points] Show that the set $SU(N)$ of all $N \times N$ unitary matrices with unit determinant constitutes a subgroup of $U(N)$.

   (c) [5 points] Show that the set $SO(N)$ of all $N \times N$ real orthogonal matrices with unit determinant constitutes a subgroup of $SU(N)$.

4. **Rotation in 2-D:** [5 points] Consider a vector $\mathbf{p}$ in two dimensions, with its components $(p_x, p_y)$ with respect to the Cartesian axes $(x, y)$. What are its components $(p'_x, p'_y)$ in a coordinate system $(x', y')$ which is rotated counterclockwise by an angle $\theta$? Express your answer in terms of $p_x, p_y, \theta$. How is this transformation related to the $SO(2)$ group?