PHYSICS 474: Introduction to Particle Physics

Homework 7 **Due:** noon Friday, March 23, 2018

1. **Decay Lifetime:** Consider a large collection $N(t)$ of an unstable particle species with decay rate Γ at any given time t.

   (a) [5 points] What is the probability of any given particle from the sample decaying between t and $t + dt$?

   (b) [5 points] Calculate the average lifetime of the particle.

   (c) [5 points] What is the half-life of the species?

2. **Dirac Delta Function:** The Dirac delta function is defined as

$$\delta(x) \;=\; \begin{cases} 0, & \text{if } x \neq 0 \\ \infty, & \text{if } x = 0 \end{cases} \qquad \text{and} \qquad \int_{-\infty}^{\infty} \delta(x)dx \;=\; 1\,. \tag{1}$$

   (a) [5 points] Show that $\delta(kx) = \frac{1}{|k|}\delta(x)$, where k is a real number.

   (b) [5 points] Now generalize it to an arbitrary function $f(x)$ with n poles, i.e. $f(x_i) = 0$ for $i = 1, 2, \cdots, n$, and show that

$$\delta(f(x)) \;=\; \sum_{i=1}^{n} \frac{1}{|f'(x_i)|}\delta(x - x_i)\,, \tag{2}$$

   where $f'(x) \equiv \frac{df}{dx}$. This is an important formula used in phase space integrals.

3. **Two-body Scattering:** Consider the scattering process $1 + 2 \to 3 + 4$.

   (a) [10 points] Show that

$$\sqrt{(p_1 \cdot p_2)^2 - (m_1 m_2 c^2)^2} \;=\; \begin{cases} |\mathbf{p}_1|(E_1 + E_2)/c & \text{in the center-of-mass frame} \\ |\mathbf{p}_1| m_2 c & \text{in the lab frame with particle 2 at rest}\,. \end{cases}$$

   (b) [15 points] Show the differential cross section in the lab frame, with particle 2 at rest and with particles 3 and 4 massless, is given by

$$\frac{d\sigma}{d\Omega} \;=\; \left(\frac{\hbar}{8\pi}\right)^2 \frac{S|\mathcal{M}|^2|\mathbf{p}_3|}{m_2|\mathbf{p}_1|(E_1 + m_2c^2 - |\mathbf{p}_1|c\cos\theta)}\,, \tag{3}$$

   where θ is the scattering angle for particle 3, S is the symmetry factor and $\mathcal{M}$ is the matrix element.