PHYSICS 474: Introduction to Particle Physics

Homework 6 **Due:** noon Friday, March 2, 2018

1. **Meson Masses:**

   The mass of a meson made of light quarks and anti-quarks is given by

   $$M_{\text{meson}} \;=\; m_1 + m_2 + A\frac{\mathbf{S}_1 \cdot \mathbf{S}_2}{m_1 m_2}\,, \tag{1}$$

   where $m_{1,2}$ and $\mathbf{S}_{1,2}$ are the effective masses and spins of the constituent (anti)quarks, respectively, and A is a constant.

   (a) [5 points] Using the constituent quark masses $m_u = m_d = 308$ MeV$/c^2$, $m_s =$ 483 MeV$/c^2$, and the best-fit value for $A = \left(\frac{2m_u}{\hbar}\right)^2 159$ MeV$/c^2$, calculate the masses of the pseudo-scalar mesons $\pi, K^+, K^0, \eta, \eta'$ and the vector mesons $\rho, K^{*+}, K^{*0}, \omega, \phi$.

   [*Hint:* For linear combinations like $\eta = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$, first find the masses for pure $u\bar{u}$, $d\bar{d}$, $s\bar{s}$, and then think of the η as being $\frac{1}{6}u\bar{u}$, $\frac{1}{6}d\bar{d}$ and $\frac{4}{6}s\bar{s}$. Similarly, for η'.]

   (b) [5 points] Let's check how good is Eq. (1) for *heavier* quarks. Using $m_c =$ 1250 MeV$/c^2$, and the same values for $m_{u,d,s}$ and A as in part (a), calculate the masses of the 'charmed' pseudo-scalar mesons $\eta_c(c\bar{c})$, $D^0(c\bar{u})$, $D_s^+(c\bar{s})$, and the corresponding vector mesons $J/\psi(c\bar{c})$, $D^{*0}(c\bar{u})$, $D_s^{*+}(c\bar{s})$.

   (c) [5 points] Using $m_b = 4.5$ GeV$/c^2$, repeat the same exercise for the 'beauty' pseudo-scalar mesons $\eta_b(b\bar{b})$, $B^+(u\bar{b})$, $B^0(d\bar{b})$, $B_c^+(c\bar{b})$ and the corresponding vector mesons $\Upsilon(b\bar{b})$, $B^{*+}(u\bar{b})$, $B^{*0}(d\bar{b})$, $B_c^{*+}(c\bar{b})$.

   (d) [5 points] Compare all the masses found in parts (a)-(c) to the experimental values as given in the *Particle Data Book*.

   [*Hint:* You may not find some of the 'beauty' meson entries there, because they have not been discovered yet.]

2. **Baryon Masses:** We can write a mass formula similar to Eq. (1) for baryons:

   $$M_{\text{baryon}} \;=\; m_1 + m_2 + m_3 + A'\left[\frac{\mathbf{S}_1 \cdot \mathbf{S}_2}{m_1 m_2} + \frac{\mathbf{S}_1 \cdot \mathbf{S}_3}{m_1 m_3} + \frac{\mathbf{S}_2 \cdot \mathbf{S}_3}{m_2 m_3}\right]\,, \tag{2}$$

where $m_{1,2,3}$ and $\mathbf{S}_{1,2,3}$ are the effective masses and spins of the constituent three quarks, and A' is a constant.

(a) [15 points] Using Eq. (2), calculate the masses of the baryon decuplet $\Delta, \Sigma^*, \Xi^*, \Omega$ and octet N, Σ, Λ, Ξ. Note that for baryons, the best-fit constituent quark masses are $m_u = m_d = 363$ MeV/c^2, $m_s = 538$ MeV/c^2 and $A' = \left(\frac{2m_u}{\hbar}\right)^2 50$ MeV/c^2.

(b) [5 points] Compare all the masses found in part (a) to the experimental values as given in the *Particle Data Book*.

3. **Λ-baryon:** [10 points] Requiring that the different baryon wave functions must be orthonormal to each other and using the results for Λ' and Σ^0 discussed in class, show that the flavor part of the iso-singlet Λ-baryon is given by

$$\Lambda \; : \; \frac{1}{\sqrt{12}} \left[2(ud - du)s + (us - su)d - (ds - sd)u\right] \, . \tag{3}$$