PHYSICS 474: Introduction to Particle Physics

Homework 4 **Due:** noon Friday, Feb 16, 2018

1. **Isospin in Nuclear Physics:**

   (a) [5 points] The α particle (used in Rutherford's scattering experiment) is a bound state of two protons and two neutrons, i.e. a ^{4}He nucleus. Given that we do not find either ^{4}H or ^{4}Li in nature, what can you say about the isospin of an α particle?

   (b) [5 points] Based on your result from part (a), explain why the reaction $D + D \to \alpha + \pi^0$ has never been observed. [Here D stands for deuteron (^{2}H).]

   (c) [5 points] Based on isospin arguments, would you expect a four-proton bound state (^{4}Be) to exist? What about a four-neutron bound state?

2. **Isospin in Particle Physics:** [15 points] Using the fact that strong interactions conserve isospin, find the ratio of cross sections for the following reactions:

   $$\text{(a) } \pi^- + p \to K^0 + \Sigma^0\,, \qquad \text{(b) } \pi^- + p \to K^+ + \Sigma^-\,, \qquad \text{(c) } \pi^+ + p \to K^+ + \Sigma^+\,.$$

   You can assume that the CM energy is such that either the $I = 3/2$ or $I = 1/2$ channel dominates. [*Hint:* See p.35-36 of Griffiths if you aren't sure what isospin multiplets K mesons and Σ baryons belong to.]

3. **Gell-Mann Matrices and $SU(3)$ Algebra:** Using the explicit form of the Gell-Mann matrices λ_a (with $a = 1, \cdots, 8$) discussed in class, show that

   (a) [10 points] $[T_a, T_b] = i f_{abc} T_c$ (sum over c implied), where $T_a = \lambda_a/2$ are the generators and f_{abc}'s are the structure constants of $SU(3)$. List the non-zero f_{abc}'s and their values. [*Hint:* If correctly done, you should find that all of them are real numbers, in agreement with the general result you proved in Homework 3, Problem 2(a).]

   (b) [10 points] The subsets $\{\lambda_1, \lambda_2, \lambda_3\}$, $\{\lambda_4, \lambda_5, [\lambda_4, \lambda_5]\}$ and $\{\lambda_6, \lambda_7, [\lambda_6, \lambda_7]\}$ generate $SU(2)$ sub-algebras. [*Hint:* $SU(2)$ algebra is given by the commutation relation $[J_i, J_j] = i\varepsilon_{ijk} J_k$.]