PHYSICS 474: Introduction to Particle Physics

Homework 2 Due: noon Friday, Feb 2, 2018

1. **Mandelstam variables:** In a two-body scattering process $A + B \to C + D$, it is convenient to define the Lorentz invariants, called the *Mandelstam variables*,

$$s \equiv \frac{(p_A + p_B)^2}{c^2}, \qquad t \equiv \frac{(p_A - p_C)^2}{c^2}, \qquad u \equiv \frac{(p_A - p_D)^2}{c^2}. \tag{1}$$

(a) [5 points] Show that $s + t + u = m_A^2 + m_B^2 + m_C^2 + m_D^2$.

(b) [5 points] Show that the total center-of-mass energy $E_{\text{tot}}^{\text{CM}} = \sqrt{s}\, c^2$.

(c) [10 points] For elastic scattering of identical particles, $A + A \to A + A$, show that

$$s = \frac{4(\mathbf{p}^2 + m^2c^2)}{c^2}, \qquad t = \frac{-2\mathbf{p}^2(1-\cos\theta)}{c^2}, \qquad u = \frac{-2\mathbf{p}^2(1+\cos\theta)}{c^2}, \tag{2}$$

where $\mathbf{p}$ is the CM momentum and θ is the scattering angle.

2. **Unitary and Hermitian matrices:**

(a) [4 points] Prove the matrix identity: $\det(e^A) = e^{\text{Tr}\, A}$ for any matrix A.

(b) [4 points] Show that any $n \times n$ unitary matrix U can be written as $U = e^{iH}$, where H is a $n \times n$ Hermitian matrix.

(b) [2 points] Show that if $U \in SU(n)$, i.e. if $\det U = 1$, then $\text{Tr}\, H = 0$.

3. **Useful identities for Pauli matrices:**

(a) [10 points] For any two vectors $\mathbf{a}$ and $\mathbf{b}$, show that $(\boldsymbol{\sigma}\cdot\mathbf{a})(\boldsymbol{\sigma}\cdot\mathbf{b}) = \mathbf{a}\cdot\mathbf{b} + i\boldsymbol{\sigma}\cdot(\mathbf{a}\times\mathbf{b})$, where $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ and $\sigma_{1,2,3}$ are the usual 2×2 Pauli matrices.

(b) [10 points] An element of $SU(2)$ can be written as $U(\theta) = e^{i(\boldsymbol{\theta}\cdot\boldsymbol{\sigma})/2}$. Using the result of (a), show that

$$U = \cos\frac{\theta}{2}\,\mathbb{I} + i(\hat{\boldsymbol{\theta}}\cdot\boldsymbol{\sigma})\sin\frac{\theta}{2}, \tag{3}$$

where $\mathbb{I}$ is the 2×2 identity matrix, θ is the magnitude of $\boldsymbol{\theta}$ and $\hat{\boldsymbol{\theta}} \equiv \boldsymbol{\theta}/\theta$ is the unit vector in the $\boldsymbol{\theta}$-direction.