PHYSICS 474: Introduction to Particle Physics

Homework 11 **Due:** noon Friday, April 20, 2018

Electron-Proton Scattering: In this problem, you will fill in the intermediate steps in the *elastic* electron-proton scattering calculation discussed in class. Please refer to your class notes for details.

(a) [10 points] Show that $q_\mu K^{\mu\nu} = 0$, where

$$K^{\mu\nu} = -K_1 g^{\mu\nu} + \frac{K_2}{(Mc)^2} p^\mu p^\nu + \frac{K_4}{(Mc)^2} q^\mu q^\nu + \frac{K_5}{(Mc)^2}(p^\mu q^\nu + q^\mu p^\nu) \tag{1}$$

describes the photon-proton vertex, M is the proton rest-mass, and p^μ, q^μ are the 4-momenta of the incoming proton and virtual photon, respectively.

(b) [10 points] Show that in Eq. (1), not all K's are linearly independent, i.e.

$$K_4 = \frac{(Mc)^2}{q^2} K_1 + \frac{1}{4} K_2 \,, \quad \text{and} \quad K_5 = \frac{1}{2} K_2 \,. \tag{2}$$

(c) [10 points] Show that in the lab frame, ignoring the electron mass, we have

$$\langle |\mathcal{M}|^2 \rangle = \frac{g_e^4 c^2}{4EE' \sin^4 \frac{\theta}{2}} \left(2K_1 \sin^2 \frac{\theta}{2} + K_2 \cos^2 \frac{\theta}{2} \right) , \tag{3}$$

$$\text{and} \quad E' = \frac{E}{1 + \frac{2E}{Mc^2} \sin^2 \frac{\theta}{2}} \,, \tag{4}$$

where E, E' are the energies of the incoming and outgoing electrons respectively, and θ is the scattering angle.

(d) [10 points] Using Eq. (3) and the general formula for differential cross section discussed earlier, derive the *Rosenbluth formula*

$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha \hbar}{4ME \sin^2 \frac{\theta}{2}} \right)^2 \frac{2K_1 \sin^2 \frac{\theta}{2} + K_2 \cos^2 \frac{\theta}{2}}{1 + \frac{2E}{Mc^2} \sin^2 \frac{\theta}{2}} \,. \tag{5}$$

(e) [10 points] Show that for a 'point-like' proton, the form factors are simply given by

$$K_1 = -q^2 \,, \qquad K_2 = (2Mc)^2 \,. \tag{6}$$

This is a pretty good approximation at low energies ($E \ll Mc^2$).