PHYSICS 474: Introduction to Particle Physics

Final Exam **Max. Points: 100**

May 9, 2018 Max. Time: 2 hours

---

1. (a) [2.5 points] Given that the ground-state binding energy of hydrogen is 13.6 eV, what would it be for positronium (e^+e^- bound state)?

   (b) [2.5 points] The deuteron has mass of 1875.6 MeV/c^2 and binding energy of 2.2 MeV. Can we use Schrödinger's equation to describe the energy levels of the deuteron system?

   (c) [5 points] What is the degeneracy of the nth energy level for the hydrogen atom? Why is this different from the expectation based on spherical symmetry alone?

   (d) [5 points] Why the π^0 (which mostly decays to two photons) has a lifetime 9 orders of magnitude *smaller* than its charged siblings $\pi^\pm$ (which mostly decay to muons and neutrinos)?

   (e) [5 points] Why the $\pi^- \to \mu^- + \bar{\nu}_\mu$ decay rate is almost 10,000 times *larger* than the $\pi^- \to e^- + \bar{\nu}_e$ decay rate?

2. (a) [10 points] Using the meson mass formula

$$M_{\text{meson}} = m_1 + m_2 + A\frac{\mathbf{S}_1 \cdot \mathbf{S}_2}{m_1 m_2}\,, \tag{1}$$

   calculate the mass splitting between the pseudoscalar π and vector ρ mesons. You can use $m_u = m_d = 308$ MeV/c^2 for the constituent quark masses and $A = (2m_u/\hbar)^2 159$ MeV/c^2 for the constant in Eq. (1).

   (b) [10 points] Using the baryon mass formula

$$M_{\text{baryon}} = m_1 + m_2 + m_3 + A'\left[\frac{\mathbf{S}_1 \cdot \mathbf{S}_2}{m_1 m_2} + \frac{\mathbf{S}_2 \cdot \mathbf{S}_3}{m_2 m_3} + \frac{\mathbf{S}_1 \cdot \mathbf{S}_3}{m_1 m_3}\right]\,, \tag{2}$$

   calculate the mass splitting between the $j = 3/2$ (decuplet) Δ-baryons and $j = 1/2$ (octet) nucleons. You can use $m_u = m_d = 363$ MeV/c^2 for the constituent quark masses and $A' = (2m_u/\hbar)^2 50$ MeV/c^2 for the constant in Eq. (2).

3. (a) [10 points] Using the Feynman rules for strong interaction, write down the QCD matrix element for the process $d + \bar{u} \to d + \bar{u}$.

   (b) [10 points] Calculate the color factors for the $d\bar{u}$ scattering matrix for any of the octet configurations and also for the singlet configuration.

   (c) [5 points] What do you infer from the above results for the QCD potential describing the $q\bar{q}$ interaction?

4. (a) [10 points] Using the Feynman rules for weak interaction, write down the matrix element for the muon decay: $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$.

   (b) [10 points] Using Casimir's trick and the trace identities, show that the spin-averaged squared matrix element from part (a) is

$$\langle |\mathcal{M}|^2 \rangle \; = \; 2 \left(\frac{g_w}{M_W c} \right)^4 (p_1 \cdot p_2)(p_3 \cdot p_4) \, , \tag{3}$$

   where p_1, p_2, p_3, p_4 are respectively the 4-momenta of the muon, electron, muon neutrino and electron anti-neutrino.

   (c) [10 points] Show that in the rest frame of the muon and neglecting the electron mass, Eq. (3) can be written as

$$\langle |\mathcal{M}|^2 \rangle \; \simeq \; \left(\frac{g_w^2 m_\mu}{M_W^2 c} \right)^2 |\mathbf{p}_2| (m_\mu c - 2|\mathbf{p}_2|) \, . \tag{4}$$

   (d) [5 points] What are the minimum and maximum values of $|\mathbf{p}_2|$ in terms of the muon mass and electron energy?