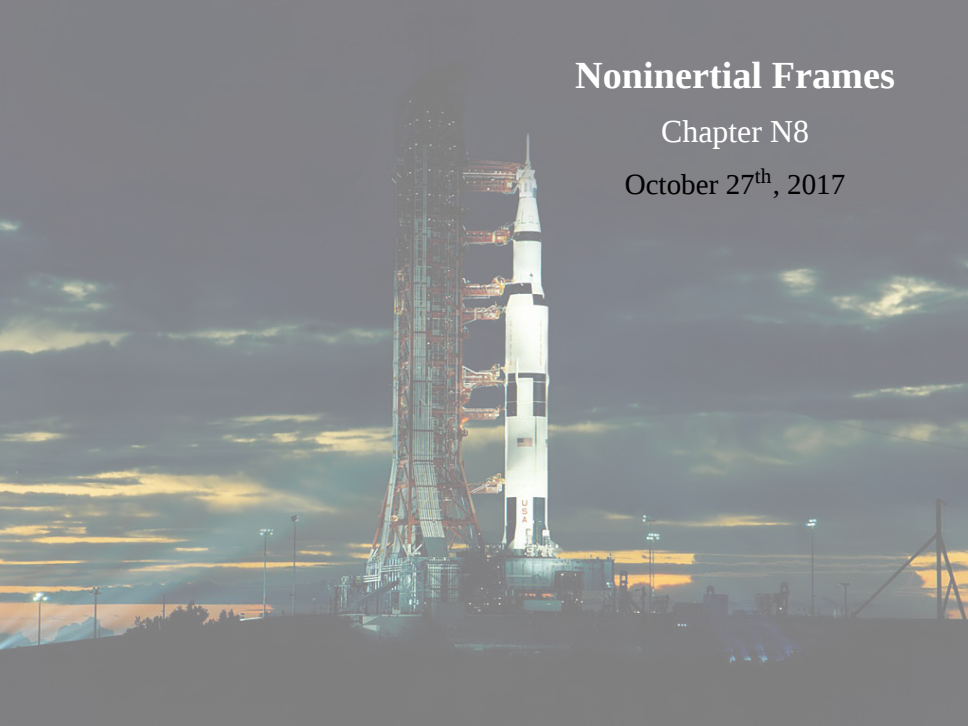


# Noninertial Frames

Chapter N8

October 27<sup>th</sup>, 2017



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- ▶ This chapter helps lay some groundwork for Unit R.

# Class Outline

1. Fictitious Forces
2. The Galilean Transformation
3. Inertial Reference Frames
4. Linearly Accelerating Frames
5. Circularly Accelerating Frames
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We will show that **these forces are not real forces, they are fictitious forces**

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## The Galilean Transformation

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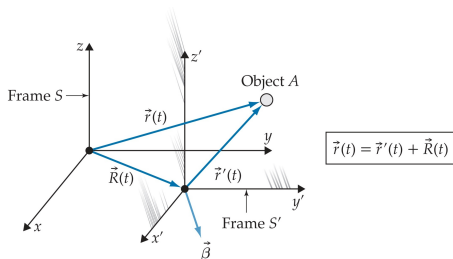
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How can we relate observations made in one reference frame with observations made in another?



Frame  $S'$  is moving at a relative “boost” velocity  $\vec{\beta}$  relative to frame  $S$ . The position vectors in the two frames at some time  $t$  are related by:  $\vec{r}(t) = \vec{r}'(t) + \vec{R}(t)$ .

# The Galilean Transformation

$$\vec{r}(t) = \vec{r}'(t) + \vec{R}(t)$$

Taking the time derivative of both sides of this equation and solving for the boosted-frame velocity yields:

$$\vec{r}'(t) = \vec{r}(t) - \vec{R}(t)$$

$$\vec{v}'(t) = \vec{v}(t) - \vec{\beta}(t)$$

This is the **Galilean velocity transformation equation**.

## The Galilean Transformation: Example

An airplane flies due east at a speed of 145 km/h relative to the ground. If there is a wind blowing east at 15 km/h, what is the plane's speed relative to the air (this is *air speed*, different from *ground speed*).

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First, we must decide what is frame  $S$  and  $S'$ , and what is  $\beta$ ?

We're given the plane's speed relative to the ground, so let this be  $v$  in  $S$ .

The “boosted” frame is the air frame, which is moving at  $\beta = +15$  km/h.

$$\begin{bmatrix} v'_x \\ v'_y \\ v'_z \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} - \begin{bmatrix} \beta_x \\ \beta_y \\ \beta_z \end{bmatrix}$$

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This makes sense intuitively!

## The Galilean Transformation: Two-Minute Problem

In a Western movie, a person shoots an arrow backward from a fleeing horse. If the velocity of the horse relative to the ground is 13 m/s west and the arrow's velocity relative to the horse is 38 m/s east, what is the arrow's velocity with respect to the ground?

- A. 41 m/s east
- B. 41 m/s west
- C. 25 m/s east
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Horse frame:  $S'$ , ground frame:  $S$ ,  $\beta = 13$  m/s west  $= -13$  m/s  $\hat{x}$ .

Arrow's velocity in the horse frame ( $S'$ ):  $\vec{v}' = 38$  m/s east  $= 38$  m/s  $\hat{x}$ .

$$\vec{v}' = \vec{v} - \vec{\beta}$$

$$\vec{v} = \vec{v}' + \vec{\beta}$$

$$v_x = v'_x + \beta_x$$

$$= (38 \text{ m/s}) + (-13 \text{ m/s}) = 25 \text{ m/s } (+x = \text{east})$$

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This equation implies that if frame  $S'$  moves at a constant velocity with respect to  $S$  (so that  $\vec{A} = 0$ ), the object's acceleration is the same in both frames:  $\vec{a}'(t) = \vec{a}(t)$

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**Noninertial Frames**  $\vec{A} \neq 0$ : isolated object will have some nonzero  
acceleration:  $\vec{a}' = -\vec{A} \neq 0$  and **Newton's first and  
second laws do not hold**

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- ▶ So the ball's horizontal acceleration seen by the camera is equal in magnitude and opposite in direction to the cart's acceleration relative to the Earth
- ▶ Now we don't need the magical force—it vanishes if we analyze the situation in an **inertial** reference frame.



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- ▶ Applying Newton’s laws in noninertial reference frames leads to nonsense!
- ▶ “Forces” that exist or don’t exist depending on one’s arbitrary choice of reference frame can’t be considered “real”.

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## Circularly Accelerating Frames

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- ▶ But we can see that this force is not real, and we can understand the movement of objects in circularly accelerating (noninertial) frames without the force, by analyzing the situation in an inertial frame.



## Circularly Accelerating Frames



# Noninertial Reference Frames

**Newton's laws do not apply in noninertial reference frames, and we can explain the motion of objects by observing the system in an inertial reference frame.**

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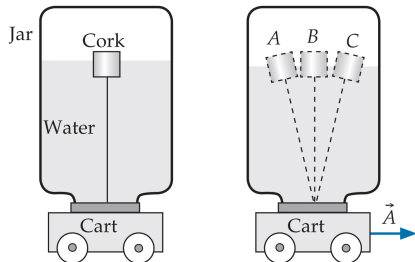
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- ▶ Call this “force” ( $-m\vec{A}$ ) a **frame-correction force**  $\vec{F}_{FC}$  (it’s only purpose is to compensate for the noninertial frame). It is proportional to the object’s mass, like the gravitational force ( $\vec{F}_g = m\vec{g}$ ).

## Two-Minute Problem

A cork floats in an inverted jar sitting on a cart, as shown in the diagram below. If we suddenly accelerate the cart to the right (as shown in the diagram) what will the cork do:

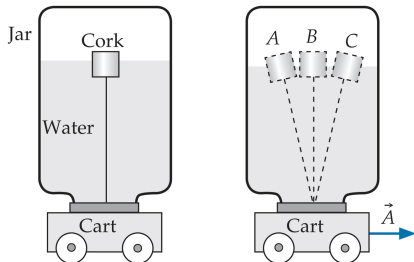
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## Two-Minute Problem: Solution

The water will slosh to the back of the cart and pile up, which pushes the cork forward.

Think of a jar mostly full of water, but with an air bubble at the top. Which way does the air bubble go when the jar is on a cart which accelerates forward?

The water goes *backward* and the air bubble goes *forward*.

**Or:** we can pretend Newton's laws work in this accelerated frame as long as we add an "effective gravitational force" vector that points *opposite* to the direction of the frame's acceleration relative to the ground. This vector points to the left, which adds to the real downward gravitational force, for a summed force that points back and down. The cork will float opposite to this: forward and up.

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Similarly a balloon in an accelerating car will move *forward* (video).

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Remember Chapter C4: a freely falling reference frame can be considered *inertial* if we ignore the external gravitational field. **Why?** Consider an inertial frame  $S$  in a gravitational field  $\vec{g}$  and consider a frame  $S'$  that is freely falling in that field:  $\vec{A} = \vec{g}$ .

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## Freely Falling Frames and Gravity

Remember Chapter C4: a freely falling reference frame can be considered *inertial* if we ignore the external gravitational field. **Why?** Consider an inertial frame  $S$  in a gravitational field  $\vec{g}$  and consider a frame  $S'$  that is freely falling in that field:  $\vec{A} = \vec{g}$ .

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Therefore an object in a freely-falling frame behaves as if Newton's second law is valid, if we ignore the gravitational field in which the frame falls. **So we can treat freely falling frames as inertial!**

# The Equivalence Principle

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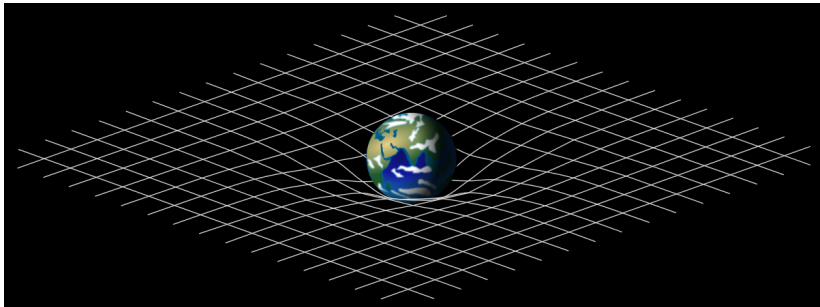
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The equivalence principle is the foundation of Einstein's theory of general relativity.

# The Equivalence Principle



Lattice analogy of the deformation of spacetime caused by a planetary mass.

# Gravity Probe B

**Missions Operations Center**

**World's roughest spheres**

**Probe during assembly**

**Vehicle on Booster**

**First results - Geodetic Precession**

**Gyroscopes and housing**

**Rotor topology**

**642 kilometers (~400 miles)**

**Guide Star IM Pegasi HR8703**

**Frame Dragging Precession: 39 marc-sec/year in Equatorial plane**

**Geodetic Precession: 6606 marc-sec/year in orbital plane**

**SQUID sensor and package**

**Guide Star tracking telescope**

**Detector Package**

**Launch 20 April 2004**

Initial Orbit Checkout (IOC)	128 days
Science Phase	353 days
Post-experiment tests	46 days

$$\dot{\Omega} = \frac{3GM}{2c^2 R^3} (R \times v) + \frac{GI}{c^2 R^3} \left[ \frac{3R}{R^2} (\omega \cdot R) - \omega \right]$$

**Gravity Probe B**

**NASA**

# Gravity Probe B



The 645-gallon GP-B flight dewar (liquid helium)

## Example Problem: N8R.1

You are kidnapped and put blindfolded in an elevator at the ground floor of a building. As the elevator starts, you notice that your weight seems to increase by 10% for 3 s, remain normal for 24 s, then decrease by 10% for 3 s. You are then taken out and put into a locked room. What is the approximate floor number your room is on?