Circularly Constrained Motion

Chapter N7

October 25th, 2017

Announcements

- **Regrade requests**: If you think a grading error has occurred, give the exam to me *with an explanatory note attached to the front*. Be sure to give your specific reasons for the request.
  - *All regrade requests are subject to having the entire exam regraded*, meaning your score may increase, decrease, or stay the same.
- Regrade requests for Exam 1 must be turned in to me no later than the end of class on Friday October 27th.

Chapter N7: Key Ideas

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- This extends our work on uniform circular motion (chapter N1) to cover cases in which the object moves in a circle with non-constant speed
- This chapter lays important foundations for chapters N10 and N11.

Class Outline

1. Uniform Circular Motion
2. Unit Vectors
3. Nonuniform Circular Motion
4. Banking
5. Examples

Uniform Circular Motion

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In problems of uniform circular motion, we are often given the time T that it takes to go once around the circle, instead of the object's speed. In time T the object travels $2\pi R$, so if its speed is constant:

$$|\vec{v}| = \frac{2\pi R}{T}$$

Frequently Missed LearnSmart Question

Rank the following planets in terms of their orbital speed, with the largest orbital speed at the top and the smallest at the bottom.

Read about this

✔ Your answer is correct.

✔ 1 Planet orbiting a star at a distance of 1×10^{11} m, and the mass of the star is equal to the mass of the Sun

✔ 2 Planet orbiting a star at a distance of 1×10^{11} m, and the mass of the star is equal to half the mass of the Sun

✔ 3 Planet orbiting a star at a distance of 3×10^{11} m, and the mass of the star is equal to the mass of the Sun

✔ 4 Planet orbiting a star at a distance of 3×10^{11} m, and the mass of the star is equal to half the mass of the Sun

Challenge

OK

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- In column-vector form we'd write this vector:

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} -2.0 \text{ m/s} \\ 0 \\ 0 \end{bmatrix} = (-2.0 \text{ m/s}) \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

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- Most importantly, **a unit vector has a magnitude (length) equal to 1**.

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- For uniform circular motion, the origin is the circle's center. The acceleration of an object in uniform circular motion can be written:

$$\vec{a} = -\frac{|\vec{v}|^2}{R}\hat{r} \qquad \text{(minus sign means } \vec{a} \text{ is toward circle's center)}$$

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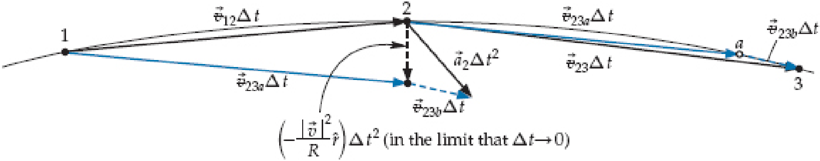
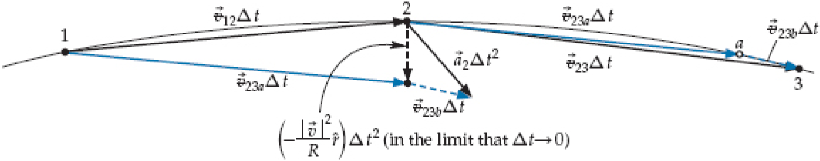
Nonuniform Circular Motion



Moore's derivation of acceleration in nonuniform circular motion using vectors.

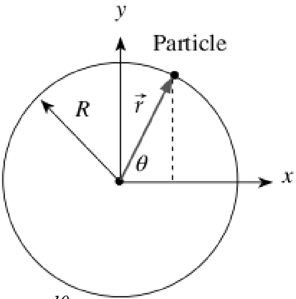
Nonuniform Circular Motion



Moore's derivation of acceleration in nonuniform circular motion using vectors.

We will derive this in a different way using calculus.

$\vec{a}$ for *non*uniform circular motion derivation



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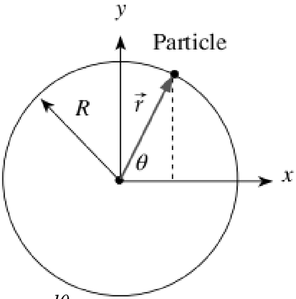

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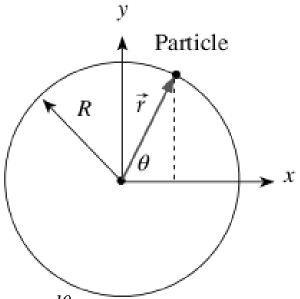

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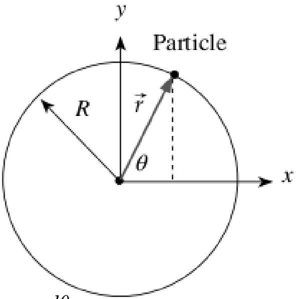

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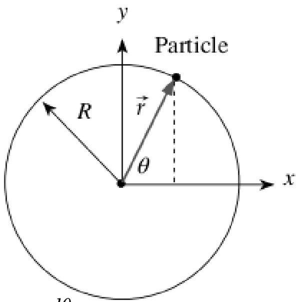

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$$\vec{a} = \frac{d|\vec{v}|}{dt}\hat{v} + \frac{|\vec{v}|^2}{R}(-\hat{r})$$

Nonuniform Circular Motion

$$\vec{a} = \frac{d|\vec{v}|}{dt}\hat{v} - \frac{|\vec{v}|^2}{R}\hat{r}$$

The **first term** is due to the change in speed so the acceleration associated with this term is in the direction of $\vec{v}$.

The direction of the **second term** we derived previously: towards the *center* of the circle. Because we define $\hat{r}$ to be increasing away from the center of the circle, we need to include a minus sign.

Nonuniform Circular Motion

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For motion in a circle:

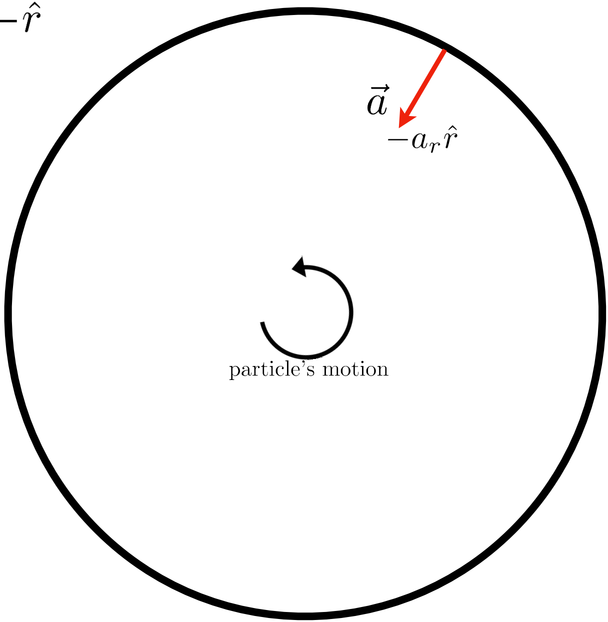
1. $\hat{v} \perp \hat{r}$
2. The magnitude of $\vec{a}$ is:

$$|\vec{a}| = \sqrt{\left(\frac{dv}{dt}\right)^2 + \left(\frac{v^2}{R}\right)^2} = \sqrt{|\vec{a}_v|^2 + |\vec{a}_r|^2}$$

3. If the speed v does not change: $\vec{a} = -(v^2/R)\hat{r} = -\omega^2 R\hat{r}$ (uniform circular motion)
4. Note the following three cases…

$$\vec{a} \equiv \frac{d|\vec{v}|}{dt}\hat{v} - \frac{|\vec{v}|^2}{R}\hat{r}$$

$$\vec{a} \equiv a_v\hat{v} - a_r\hat{r}$$



$$|\vec{v}| = \text{constant}$$

$$\frac{d|\vec{v}|}{dt} = 0$$

$$\vec{a} \equiv \frac{d|\vec{v}|}{dt}\hat{v} - \frac{|\vec{v}|^2}{R}\hat{r}$$

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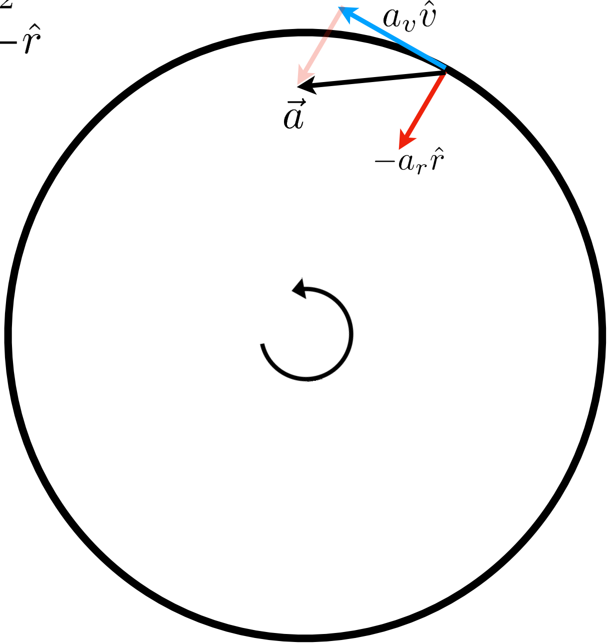

$|\vec{v}|$ increasing

$$\frac{d|\vec{v}|}{dt} > 0$$

$$\vec{a} \equiv \frac{d|\vec{v}|}{dt}\hat{v} - \frac{|\vec{v}|^2}{R}\hat{r}$$

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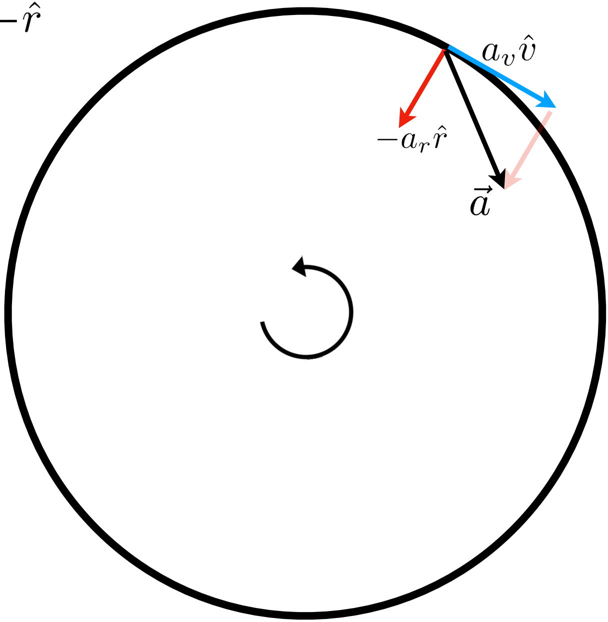
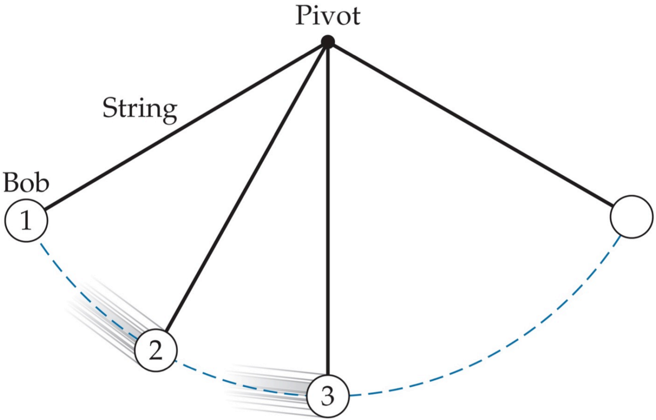


$|\vec{v}|$ decreasing

$$\frac{d|\vec{v}|}{dt} < 0$$

Two-Minute Problem

A pendulum bob swings from the end of a string. At point 1, the bob is at the extreme point of the swing and thus is instantaneously at rest. At point 3, the bob is directly below its suspension point and has its maximum speed.



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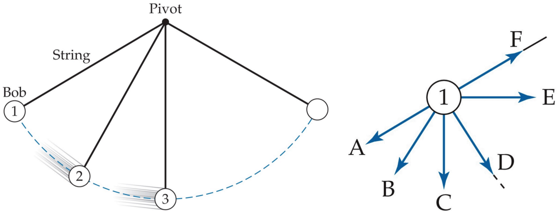

Which one of the arrows most closely indicates the direction of the bob's acceleration at point 1?

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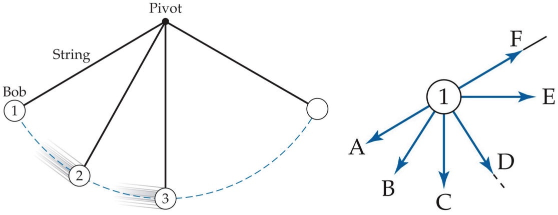

D: The bob's speed is zero at point 1, so its acceleration is due to dv/dt which is pointed closest to D.

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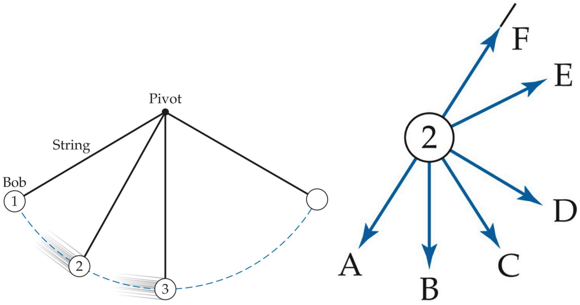

Which one of the arrows most closely indicates the direction of the bob's acceleration at point 2?

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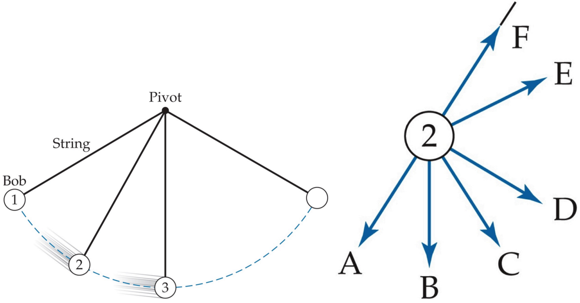

E: It's speed is nonzero and increasing, so one component of $\vec{a}$ points along the string, another points tangent to the blue dotted line. The sum of these two is E.

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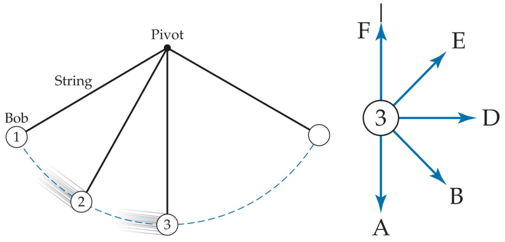

Which one of the arrows most closely indicates the direction of the bob's acceleration at point 3?

Two-Minute Problem

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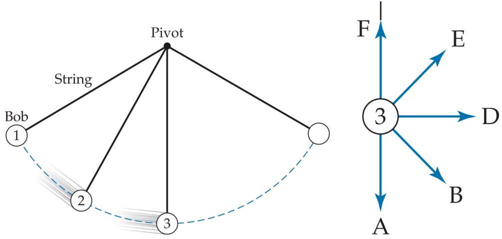

F: It's speed is nonzero (one component of $\vec{a}$ points along the string) and is at a maximum (just increasing before 3, just decreasing after 3) so $dv/dt = 0$. Therefore the only component is along the string: F.

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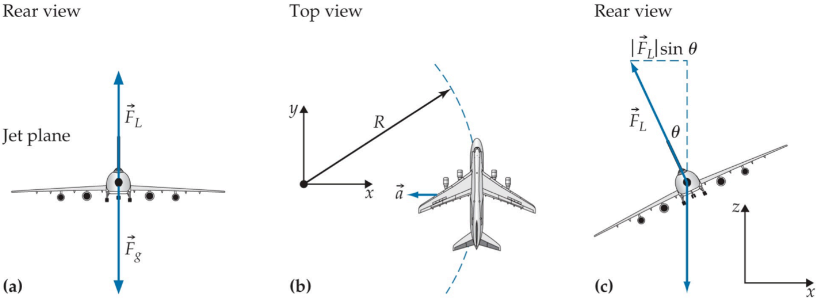
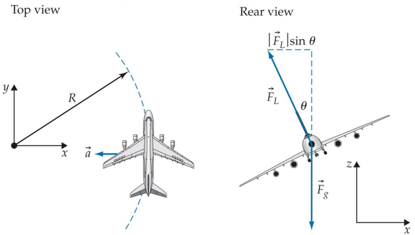
Circular Motion Example: Banking



A turning airplane accelerates toward the center of the circle defined by the turning radius. It's engines can't pivot! Tilting the wings at an angle θ so that the lift force creates a component of the lift force towards the circle center, but the vertical component must still support the plane's weight!

Circular Motion Example: Banking

What is the jet's banking angle?



$$\begin{bmatrix} -m|\vec{v}|^2/R \\ 0 \\ 0 \end{bmatrix} = m\vec{a} = \vec{F}_g + \vec{F}_L = \begin{bmatrix} 0 \\ 0 \\ -m|\vec{g}| \end{bmatrix} + \begin{bmatrix} -|\vec{F}_L|\sin\theta \\ 0 \\ |\vec{F}_L|\cos\theta \end{bmatrix}$$

Circular Motion Example: Banking

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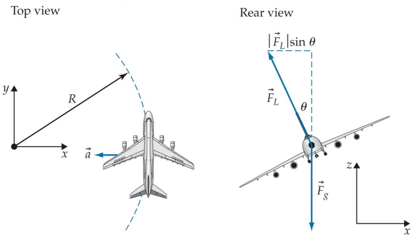
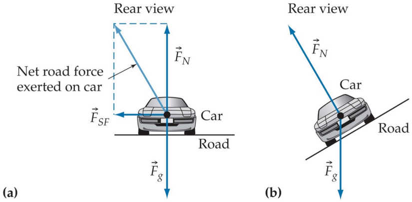


$$\frac{m|\vec{v}|^2}{R} = |\vec{F}_L|\sin\theta \qquad m|\vec{g}| = |\vec{F}_L|\cos\theta$$

Dividing these equations:

$$\frac{|\vec{v}|^2}{R|\vec{g}|} = \tan\theta \rightarrow \theta = \tan^{-1}\left(\frac{|\vec{v}|^2}{R|\vec{g}|}\right)$$

Car Banking



A car doesn't *need* to bank: the road exerts a static friction force on the tires to the left. But on a properly banked roadway, the car can use the leftward component of the normal force for the acceleration (safer under bad road conditions).

Class Outline

1. Uniform Circular Motion
2. Unit Vectors
3. Nonuniform Circular Motion
4. Banking
5. Examples

Solving Circularly Constrained Motion Problems

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- Because $\hat{r}$ and $\hat{v}$ keep changing as the object moves, your coordinate axes will only be useful for one instant

Example Problem N7M.8

You are riding in a 1650-kg car and approach a curve in the road with radius of 50 m. The roadbed is banked inward at a 10° angle.

- **(a)** Suppose the road is very icy, so that the coefficient of static friction is essentially zero. What is the maximum speed at which you can go around the curve?
- **(b)** Now suppose the road is dry and that the static friction coefficient between the tires and the asphalt road is 0.6. What is the max speed at which you can safely go around the curve?

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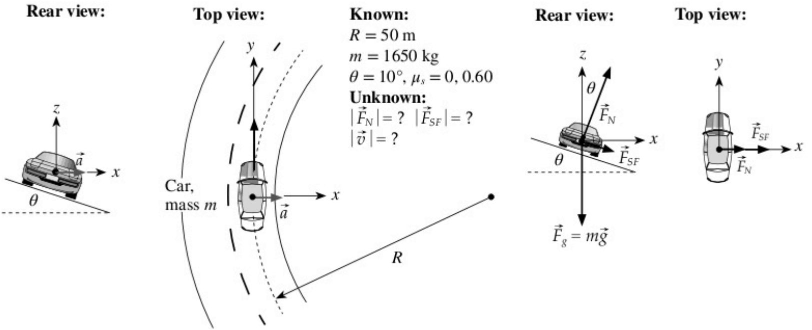

Demo: Loop-the-Loop

A ball of mass m and radius r starts at rest at the top of a hill of height H, rolls down the hill, and then goes around a vertical loop of radius R.

What is the minimum H for the ball to remain in contact with the track when it reaches the top of the loop (i.e. complete the loop without falling)?

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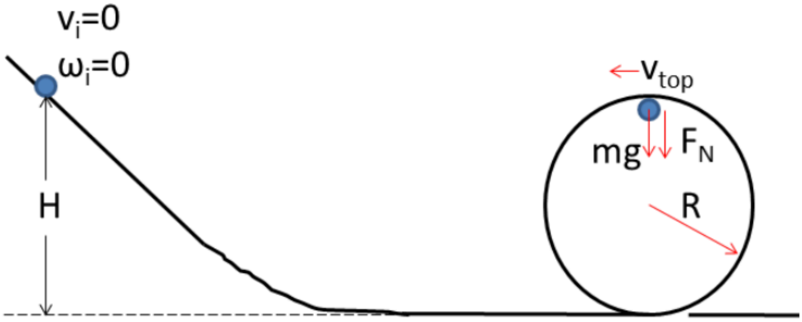

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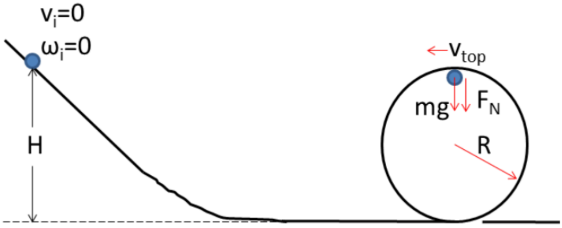

$R = 21$ cm, $H_{min,sphere} = 2.7R = 56.7$ cm,
$H_{min,sliding} = 2.5R = 52.5$ cm